Computer algebra independent integration tests

Summer 2022 edition

3-Logarithms/61-3.2.3-u-log-e-f-a+b-x-^p-c+d-x-^q-^r-^s

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [108]. This is test number [61].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (108)	0.00 (0)
Rubi	98.15 (106)	1.85 (2)
Maxima	62.96 (68)	37.04 (40)
Fricas	37.96 (41)	62.04 (67)
Mupad	32.41 (35)	67.59 (73)
Giac	32.41 (35)	67.59 (73)
Maple	25.00 (27)	75.00 (81)
Sympy	18.52 (20)	81.48 (88)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

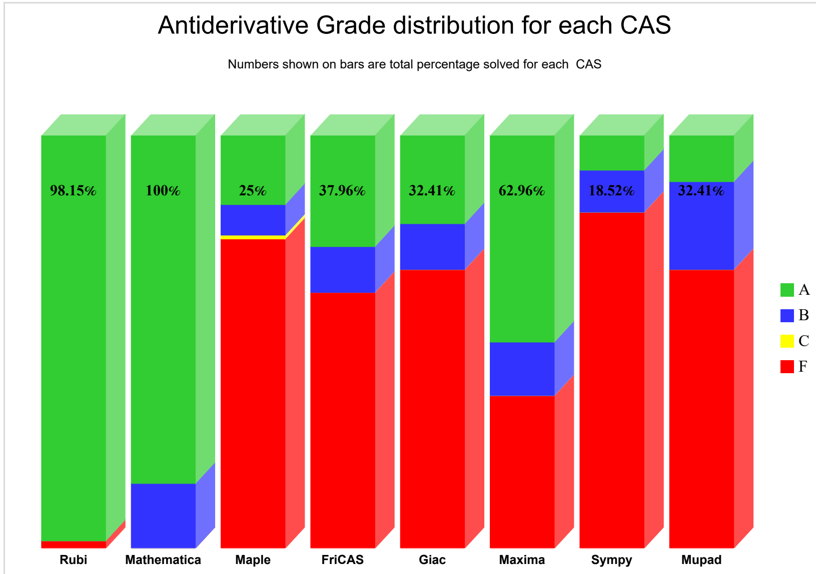
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

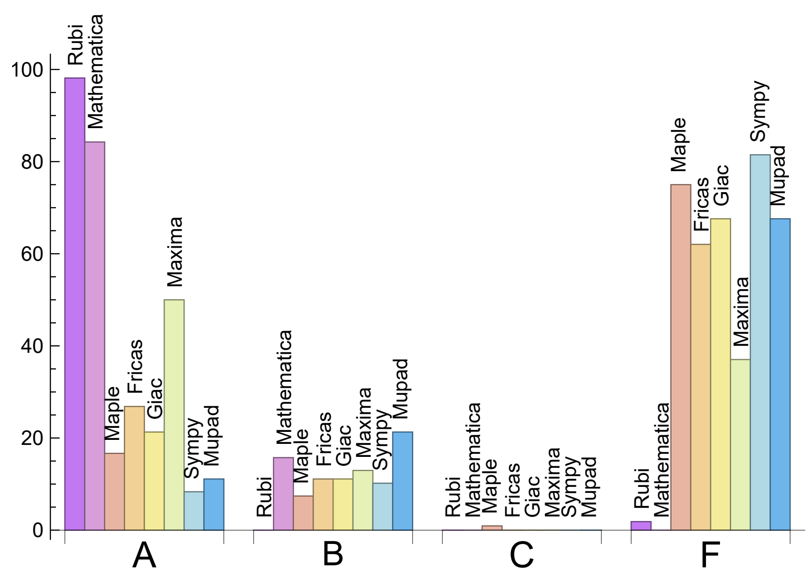
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.15	0.00	0.00	1.85
Mathematica	84.26	15.74	0.00	0.00
Maxima	50.00	12.96	0.00	37.04
Fricas	26.85	11.11	0.00	62.04
Giac	21.30	11.11	0.00	67.59
Maple	16.67	7.41	0.93	75.00
Mupad	N/A	21.30	0.00	67.59
Sympy	8.33	10.19	0.00	81.48

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	2	100.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	81	100.00 %	0.00 %	0.00 %
Fricas	67	95.52 %	4.48 %	0.00 %
Giac	73	90.41 %	9.59 %	0.00 %
Maxima	40	75.00 %	0.00 %	25.00 %
Sympy	88	31.82 %	53.41 %	14.77 %
Mupad	73	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

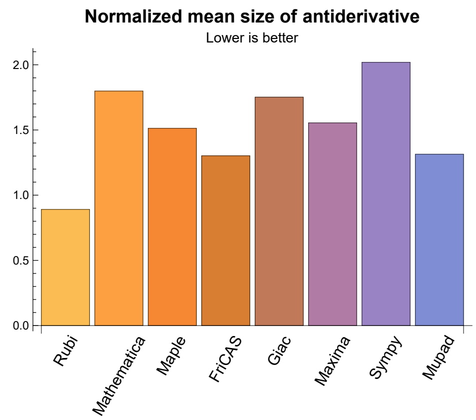
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

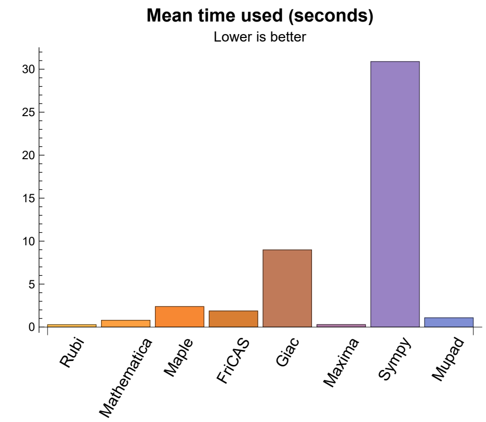
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.28	344.29	0.89	182.50	1.00
Mathematica	0.79	1155.40	1.80	165.00	0.96
Maple	2.39	133.74	1.51	36.00	1.03
Maxima	0.29	430.94	1.55	165.50	1.23
Fricas	1.88	171.63	1.30	49.00	1.22
Sympy	30.89	158.10	2.02	61.00	1.70
Giac	8.98	291.57	1.75	58.00	1.08
Mupad	1.08	261.03	1.31	44.00	1.00

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
2.0
1.5
1.0
0.5
0.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
30
25
20
15
10
5
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{54, 55, 60, 61, 62, 63, 65, 66, 70, 71, 72, 73\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {67}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

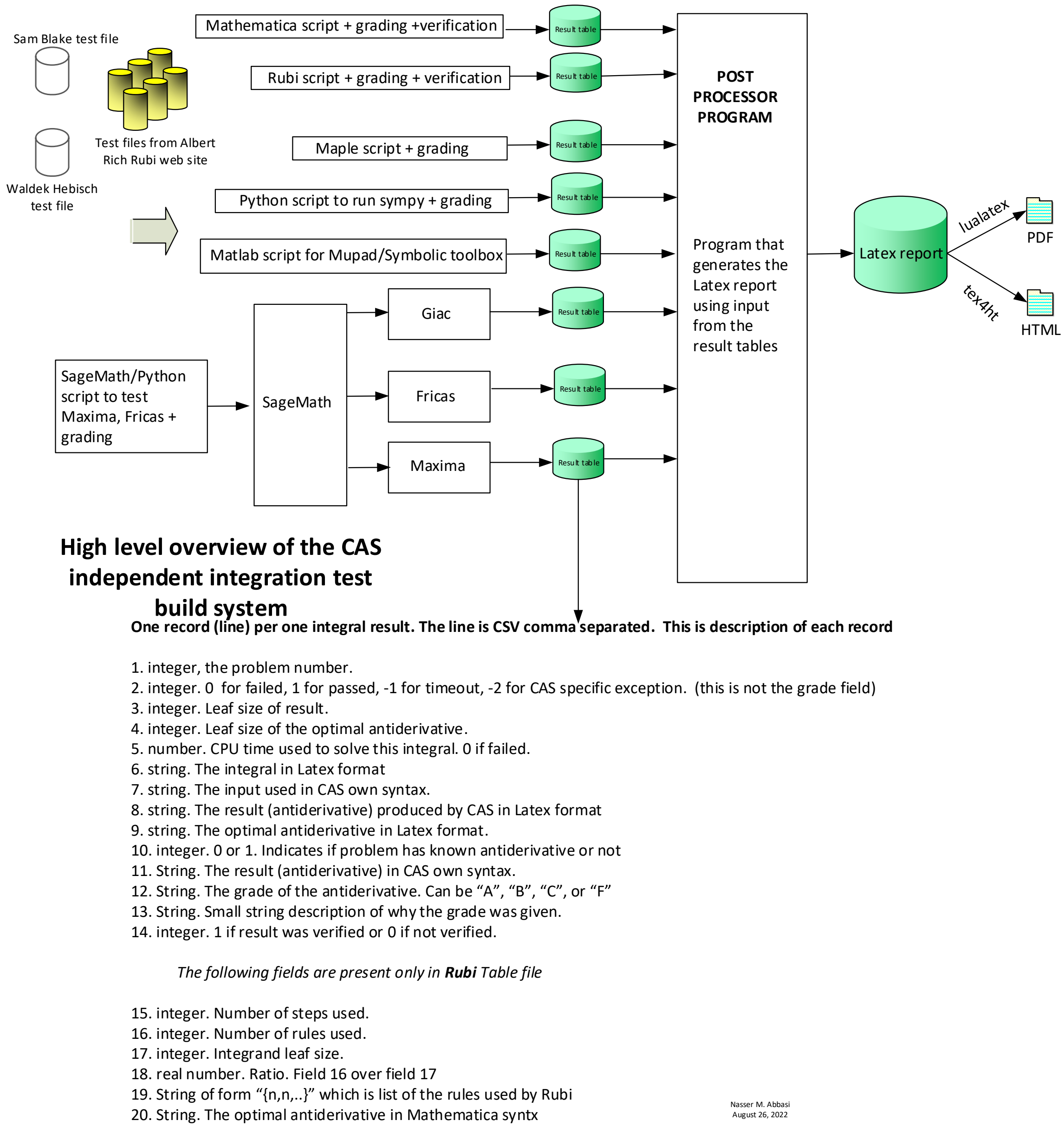
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108 }

B grade: { }

C grade: { }

F grade: { 74, 75 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 59, 60, 61, 62, 63, 64, 65, 66, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105 }

B grade: { 16, 17, 24, 40, 41, 42, 51, 56, 57, 58, 67, 68, 69, 92, 106, 107, 108 }

C grade: { }

F grade: { }

2.1.3 Maple

A grade: { 29, 54, 55, 60, 61, 62, 63, 65, 66, 70, 71, 72, 73, 89, 91, 96, 102, 106 }

B grade: { 64, 88, 93, 99, 103, 104, 105, 107 }

C grade: { 75 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 67, 68, 69, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 92, 94, 95, 97, 98, 100, 101, 108 }

2.1.4 Maxima

A grade: { 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 21, 22, 23, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 40, 41, 47, 48, 53, 54, 55, 59, 60, 61, 62, 63, 65, 66, 70, 71, 72, 73, 74, 75, 79, 81, 93, 94, 96, 97, 99, 100 }

B grade: { 7, 15, 24, 25, 34, 42, 44, 45, 46, 49, 50, 77, 88, 89 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 20, 39, 43, 51, 52, 56, 57, 58, 64, 67, 68, 69, 76, 78, 80, 82, 83, 84, 85, 86, 87, 90, 91, 92, 95, 98, 101, 102, 103, 104, 105, 106, 107, 108 }

2.1.5 FriCAS

A grade: { 10, 12, 28, 29, 43, 46, 47, 48, 49, 50, 54, 55, 60, 61, 62, 63, 65, 66, 70, 71, 72, 73, 74, 75, 89, 93, 96, 99, 106 }

B grade: { 7, 8, 9, 13, 14, 15, 25, 26, 27, 31, 44, 45 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 11, 16, 17, 18, 19, 20, 21, 22, 23, 24, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 51, 52, 53, 56, 57, 58, 59, 64, 67, 68, 69, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 107, 108 }

2.1.6 Sympy

A grade: { 47, 62, 63, 65, 66, 89, 93, 96, 99 }

B grade: { 9, 10, 12, 28, 29, 43, 44, 45, 46, 48, 50 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 64, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108 }

2.1.7 Giac

A grade: { 10, 12, 13, 28, 29, 31, 43, 47, 48, 54, 55, 60, 61, 62, 63, 65, 66, 70, 71, 72, 73, 96, 99 }

B grade: { 7, 8, 9, 14, 15, 32, 33, 34, 46, 49, 50, 93 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 11, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 30, 35, 36, 37, 38, 39, 40, 41, 42, 44, 45, 51, 52, 53, 56, 57, 58, 59, 64, 67, 68, 69, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108 }

2.1.8 Mupad

A grade: { 54, 55, 60, 61, 62, 63, 65, 66, 70, 71, 72, 73 }

B grade: { 7, 8, 9, 10, 12, 13, 14, 15, 25, 26, 27, 28, 29, 31, 32, 33, 34, 74, 75, 89, 93, 96, 99 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 11, 16, 17, 18, 19, 20, 21, 22, 23, 24, 30, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 64, 67, 68, 69, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	404	336	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.286	0.285	0.041	0.000	0.000	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	217	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.172	0.144	0.038	0.000	0.000	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	135	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.068	0.042	0.000	0.000	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	185	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.167	0.108	0.120	0.000	0.000	0.000	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	295	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.247	0.321	0.037	0.000	0.000	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	531	531	470	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.457	0.811	0.037	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	185	0	396	622	0	560	886
N.S.	1	1.00	0.92	0.00	1.97	3.09	0.00	2.79	4.41
time (sec)	N/A	0.077	0.221	0.154	0.296	0.413	0.000	175.994	0.697

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	154	0	286	467	0	407	501
N.S.	1	1.00	0.90	0.00	1.66	2.72	0.00	2.37	2.91
time (sec)	N/A	0.054	0.156	0.158	0.306	0.379	0.000	32.756	0.533

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	127	0	195	323	488	269	255
N.S.	1	1.00	0.89	0.00	1.36	2.26	3.41	1.88	1.78
time (sec)	N/A	0.043	0.102	0.182	0.299	0.384	213.713	7.761	0.469

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	105	0	119	195	325	148	128
N.S.	1	1.00	0.91	0.00	1.03	1.68	2.80	1.28	1.10
time (sec)	N/A	0.032	0.140	0.043	0.321	0.348	28.164	4.662	0.401

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	93	0	165	0	0	0	-1
N.S.	1	1.00	0.87	0.00	1.54	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.075	0.187	0.290	0.000	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	89	0	100	116	1112	112	99
N.S.	1	1.00	0.94	0.00	1.05	1.22	11.71	1.18	1.04
time (sec)	N/A	0.027	0.042	0.137	0.310	0.362	141.379	3.964	2.160

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	116	0	166	320	0	246	182
N.S.	1	1.00	0.86	0.00	1.23	2.37	0.00	1.82	1.35
time (sec)	N/A	0.047	0.180	0.165	0.284	0.418	0.000	5.202	2.389

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	141	0	290	575	0	469	346
N.S.	1	1.00	0.86	0.00	1.77	3.51	0.00	2.86	2.11
time (sec)	N/A	0.061	0.251	0.168	0.295	0.378	0.000	4.500	2.869

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	164	0	460	858	0	748	526
N.S.	1	1.00	0.85	0.00	2.38	4.45	0.00	3.88	2.73
time (sec)	N/A	0.077	0.247	0.183	0.306	0.375	0.000	4.772	3.712

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	920	920	2508	0	1423	0	0	0	-1
N.S.	1	1.00	2.73	0.00	1.55	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.610	1.732	0.199	0.345	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	805	805	1853	0	1073	0	0	0	-1
N.S.	1	1.00	2.30	0.00	1.33	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.464	1.250	0.212	0.314	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	686	686	1211	0	771	0	0	0	-1
N.S.	1	1.00	1.77	0.00	1.12	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.380	0.726	0.187	0.311	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	540	540	781	0	506	0	0	0	-1
N.S.	1	1.00	1.45	0.00	0.94	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.275	0.405	0.073	0.343	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	431	431	460	0	0	0	0	0	-1
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.345	0.125	0.173	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	465	465	411	0	394	0	0	0	-1
N.S.	1	1.00	0.88	0.00	0.85	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.267	0.449	0.147	0.311	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	632	632	872	0	757	0	0	0	-1
N.S.	1	1.00	1.38	0.00	1.20	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.343	0.811	0.153	0.352	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	764	764	1407	0	1254	0	0	0	-1
N.S.	1	1.00	1.84	0.00	1.64	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.438	1.165	0.159	0.402	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	884	884	2003	0	1818	0	0	0	-1
N.S.	1	1.00	2.27	0.00	2.06	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.537	1.778	0.169	0.470	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	275	0	625	943	0	0	1128
N.S.	1	1.00	0.82	0.00	1.87	2.82	0.00	0.00	3.38
time (sec)	N/A	0.133	0.216	0.156	0.287	0.414	0.000	0.000	1.034

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	231	0	432	677	0	0	641
N.S.	1	1.00	0.84	0.00	1.57	2.45	0.00	0.00	2.32
time (sec)	N/A	0.092	0.194	0.165	0.300	0.405	0.000	0.000	0.829

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	209	0	270	439	0	0	328
N.S.	1	1.00	0.96	0.00	1.24	2.01	0.00	0.00	1.50
time (sec)	N/A	0.071	0.161	0.136	0.287	0.374	0.000	0.000	0.652

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	120	0	144	240	502	176	153
N.S.	1	1.00	0.75	0.00	0.90	1.50	3.14	1.10	0.96
time (sec)	N/A	0.049	0.133	0.057	0.280	0.396	34.188	17.806	0.507

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	57	61	76	71	184	66	60
N.S.	1	1.00	0.93	1.00	1.25	1.16	3.02	1.08	0.98
time (sec)	N/A	0.011	0.043	0.080	0.289	0.467	6.665	4.227	0.218

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	163	0	187	0	0	0	-1
N.S.	1	1.00	1.10	0.00	1.26	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.059	0.167	0.279	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	93	0	124	276	0	190	152
N.S.	1	1.00	0.73	0.00	0.97	2.16	0.00	1.48	1.19
time (sec)	N/A	0.039	0.107	0.166	0.296	66.484	0.000	3.421	1.199

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	206	0	233	0	0	595	384
N.S.	1	1.00	1.02	0.00	1.15	0.00	0.00	2.95	1.90
time (sec)	N/A	0.085	0.281	0.166	0.295	0.000	0.000	4.814	2.709

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	254	0	457	0	0	1765	977
N.S.	1	1.00	0.98	0.00	1.76	0.00	0.00	6.79	3.76
time (sec)	N/A	0.113	0.488	0.165	0.345	0.000	0.000	5.307	5.479

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	480	0	777	0	0	3911	2215
N.S.	1	1.00	1.51	0.00	2.44	0.00	0.00	12.30	6.97
time (sec)	N/A	0.144	0.930	0.177	0.325	0.000	0.000	4.540	10.154

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2240	2240	1386	0	1801	0	0	0	-1
N.S.	1	1.00	0.62	0.00	0.80	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.704	1.983	0.198	0.359	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1645	1645	899	0	1125	0	0	0	-1
N.S.	1	1.00	0.55	0.00	0.68	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.165	1.210	0.199	0.338	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1063	1063	480	0	625	0	0	0	-1
N.S.	1	1.00	0.45	0.00	0.59	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.777	0.560	0.066	0.307	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	389	0	300	0	0	0	-1
N.S.	1	1.00	1.45	0.00	1.12	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.111	0.157	0.040	0.303	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1471	2096	1370	0	0	0	0	0	-1
N.S.	1	1.42	0.93	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.389	0.190	0.163	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	832	832	2930	0	747	0	0	0	-1
N.S.	1	1.00	3.52	0.00	0.90	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.599	0.773	0.165	0.345	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1304	1304	12086	0	1859	0	0	0	-1
N.S.	1	1.00	9.27	0.00	1.43	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.921	3.977	0.148	0.578	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1957	1957	47127	0	4734	0	0	0	-1
N.S.	1	1.00	24.08	0.00	2.42	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.426	6.334	0.183	1.203	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	0	56	94	38	-1
N.S.	1	1.00	1.00	0.00	0.00	1.33	2.24	0.90	-0.02
time (sec)	N/A	0.086	0.021	0.146	0.000	0.363	78.250	3.370	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	526	101	65	0	-1
N.S.	1	1.00	1.00	0.00	14.22	2.73	1.76	0.00	-0.03
time (sec)	N/A	0.069	0.008	0.107	0.325	0.354	6.435	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	268	74	65	0	-1
N.S.	1	1.00	1.00	0.00	7.24	2.00	1.76	0.00	-0.03
time (sec)	N/A	0.067	0.009	0.152	0.287	0.356	4.722	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	105	47	61	86	-1
N.S.	1	1.00	1.00	0.00	2.84	1.27	1.65	2.32	-0.03
time (sec)	N/A	0.046	0.006	0.111	0.288	0.351	4.595	5.022	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	0	36	30	54	31	-1
N.S.	1	1.00	1.00	0.00	1.06	0.88	1.59	0.91	-0.03
time (sec)	N/A	0.065	0.028	0.158	0.343	0.367	8.293	5.078	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	0	34	29	61	34	-1
N.S.	1	1.00	1.00	0.00	1.00	0.85	1.79	1.00	-0.03
time (sec)	N/A	0.066	0.010	0.123	0.352	0.329	87.976	4.859	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	80	59	0	85	-1
N.S.	1	1.00	1.00	0.00	2.16	1.59	0.00	2.30	-0.03
time (sec)	N/A	0.067	0.009	0.136	0.356	0.319	0.000	2.478	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	0	83	24	65	58	-1
N.S.	1	1.00	1.00	0.00	2.77	0.80	2.17	1.93	-0.03
time (sec)	N/A	0.035	0.007	0.117	0.522	0.349	3.201	2.759	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	958	0	0	0	0	0	-1
N.S.	1	1.00	2.34	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.330	6.131	0.275	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	436	0	0	0	0	0	-1
N.S.	1	1.00	1.42	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.208	0.949	0.286	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	166	0	205	0	0	0	-1
N.S.	1	1.00	0.97	0.00	1.19	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.072	0.162	0.287	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.035	0.241	0.272	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	1.397	0.293	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	1241	0	0	0	0	0	-1
N.S.	1	1.00	3.78	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.867	1.168	1.131	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	839	0	0	0	0	0	-1
N.S.	1	1.00	3.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.653	0.595	1.071	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	451	0	0	0	0	0	-1
N.S.	1	1.00	2.32	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.434	0.285	1.074	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	78	0	127	0	0	0	-1
N.S.	1	1.00	0.96	0.00	1.57	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.046	0.045	0.291	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.367	0.305	1.056	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.300	1.510	1.061	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.016	0.085	2.068	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.016	0.073	0.661	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	66	213	0	0	0	0	-1
N.S.	1	1.00	0.80	2.60	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.030	0.397	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.017	0.080	0.600	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.017	1.161	19.788	0.000	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	620	620	19087	0	0	0	0	0	-1
N.S.	1	1.00	30.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.560	21.353	0.280	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	496	496	3463	0	0	0	0	0	-1
N.S.	1	1.00	6.98	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.393	6.763	0.226	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	1408	0	0	0	0	0	-1
N.S.	1	1.00	3.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.311	1.706	0.277	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.385	1.255	0.240	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.617	0.011	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.373	0.412	2.705	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.353	0.365	1.504	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	A	A	F(-2)	F	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	0	44	0	59	50	0	0	44
N.S.	1	0.00	0.98	0.00	1.31	1.11	0.00	0.00	0.98
time (sec)	N/A	0.377	0.141	0.010	0.323	0.366	0.000	0.000	0.701

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	C	A	A	F(-2)	F	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	0	45	503	58	49	0	0	44
N.S.	1	0.00	1.00	11.18	1.29	1.09	0.00	0.00	0.98
time (sec)	N/A	0.361	0.059	11.256	0.384	0.344	0.000	0.000	0.537

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	560	560	461	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.517	0.264	0.182	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	550	550	467	0	1048	0	0	0	-1
N.S.	1	1.00	0.85	0.00	1.91	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.410	0.176	0.162	0.614	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	413	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.259	0.090	0.128	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	421	0	350	0	0	0	-1
N.S.	1	1.00	1.45	0.00	1.20	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.243	0.081	0.164	0.550	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	518	518	487	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.421	0.136	0.177	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	596	596	479	0	970	0	0	0	-1
N.S.	1	1.00	0.80	0.00	1.63	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.411	0.222	0.196	0.602	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1046	1046	1240	0	0	0	0	0	-1
N.S.	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.230	0.975	0.515	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	831	831	1105	0	0	0	0	0	-1
N.S.	1	1.00	1.33	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.806	2.334	0.514	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	685	685	669	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.488	1.775	0.530	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	401	401	604	0	0	0	0	0	-1
N.S.	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.535	0.286	0.530	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	800	800	625	0	0	0	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.718	0.736	0.521	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	995	995	721	0	0	0	0	0	-1
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.910	0.624	0.509	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	84	97	95	0	0	0	-1
N.S.	1	1.00	1.83	2.11	2.07	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.105	0.013	1.655	0.271	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	29	141	18	14	0	18
N.S.	1	1.00	1.00	1.45	7.05	0.90	0.70	0.00	0.90
time (sec)	N/A	0.045	0.055	0.563	0.281	0.342	0.048	0.000	0.257

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	76	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.011	0.478	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	110	238	0	0	0	0	-1
N.S.	1	1.00	0.73	1.59	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.161	0.028	3.429	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	559	0	0	0	0	0	-1
N.S.	1	1.00	3.49	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.320	0.271	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	28	54	25	29	20	153	25
N.S.	1	1.00	1.12	2.16	1.00	1.16	0.80	6.12	1.00
time (sec)	N/A	0.009	0.005	0.344	0.264	0.364	0.040	3.156	0.081

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	63	0	113	0	0	0	-1
N.S.	1	1.00	0.94	0.00	1.69	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.011	0.058	0.273	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	91	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.020	0.036	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	41	28	42	26	29	28
N.S.	1	1.00	1.00	1.46	1.00	1.50	0.93	1.04	1.00
time (sec)	N/A	0.005	0.003	0.214	0.270	0.325	0.048	3.957	0.148

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	106	0	118	0	0	0	-1
N.S.	1	1.00	1.58	0.00	1.76	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.013	0.060	0.280	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	98	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.020	0.036	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	59	28	41	26	29	28
N.S.	1	1.00	1.00	2.11	1.00	1.46	0.93	1.04	1.00
time (sec)	N/A	0.005	0.004	0.448	0.276	0.330	0.057	3.867	0.149

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	106	0	118	0	0	0	-1
N.S.	1	1.00	1.58	0.00	1.76	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.011	0.072	0.279	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.020	0.039	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	30	36	0	0	0	0	-1
N.S.	1	1.00	0.86	1.03	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.042	0.008	0.682	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	68	186	0	0	0	0	-1
N.S.	1	1.00	0.80	2.19	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.019	3.713	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	135	419	0	0	0	0	-1
N.S.	1	1.00	0.96	2.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.030	0.897	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	96	524	0	0	0	0	-1
N.S.	1	1.00	0.88	4.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.020	8.530	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	1636	338	0	266	0	0	-1
N.S.	1	1.00	8.02	1.66	0.00	1.30	0.00	0.00	-0.00
time (sec)	N/A	0.114	0.308	1.045	0.000	0.424	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	1080	813	0	0	0	0	-1
N.S.	1	1.00	3.35	2.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.306	0.149	0.933	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	433	433	908	0	0	0	0	0	-1
N.S.	1	1.00	2.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.364	0.511	1.151	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [108] had the largest ratio of [65]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	16	10	1.00	32	0.312
2	A	13	8	1.00	32	0.250
3	A	10	6	1.00	30	0.200
4	A	12	7	1.00	32	0.219
5	A	15	9	1.00	32	0.281
6	A	18	11	1.00	32	0.344
7	A	4	3	1.00	29	0.103
8	A	4	3	1.00	29	0.103
9	A	4	3	1.00	29	0.103
10	A	4	2	1.00	27	0.074
11	A	6	6	1.00	29	0.207
12	A	5	4	1.00	29	0.138
13	A	4	3	1.00	29	0.103
14	A	4	3	1.00	29	0.103
15	A	4	3	1.00	29	0.103
16	A	32	14	1.00	31	0.452
17	A	28	14	1.00	31	0.452
18	A	24	14	1.00	31	0.452
19	A	20	13	1.00	29	0.448
20	A	19	15	1.00	31	0.484
21	A	20	12	1.00	31	0.387
22	A	24	13	1.00	31	0.419
23	A	28	13	1.00	31	0.419
24	A	32	13	1.00	31	0.419
25	A	5	2	1.00	29	0.069

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	5	2	1.00	29	0.069
27	A	5	2	1.00	29	0.069
28	A	5	2	1.00	27	0.074
29	A	3	3	1.00	21	0.143
30	A	7	4	1.00	29	0.138
31	A	7	3	1.00	29	0.103
32	A	5	2	1.00	29	0.069
33	A	5	2	1.00	29	0.069
34	A	5	2	1.00	29	0.069
35	A	49	14	1.00	31	0.452
36	A	47	15	1.00	31	0.484
37	A	39	15	1.00	29	0.517
38	A	10	9	1.00	23	0.391
39	A	29	14	1.42	31	0.452
40	A	31	10	1.00	31	0.323
41	A	43	14	1.00	31	0.452
42	A	57	15	1.00	31	0.484
43	A	2	2	1.00	40	0.050
44	A	5	5	1.00	40	0.125
45	A	5	5	1.00	40	0.125
46	A	4	4	1.00	38	0.105
47	A	2	2	1.00	40	0.050
48	A	2	2	1.00	40	0.050
49	A	2	2	1.00	40	0.050
50	A	4	4	1.00	34	0.118
51	A	11	6	1.00	48	0.125
52	A	9	5	1.00	46	0.109
53	A	7	4	1.00	32	0.125
54	A	0	0	0.00	0	0.000
55	A	0	0	0.00	0	0.000
56	A	13	6	1.00	39	0.154
57	A	11	6	1.00	39	0.154
58	A	9	5	1.00	37	0.135
59	A	5	3	1.00	25	0.120
60	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	0	0	0.00	0	0.000
62	A	0	0	0.00	0	0.000
63	A	0	0	0.00	0	0.000
64	A	5	5	1.00	26	0.192
65	A	0	0	0.00	0	0.000
66	A	0	0	0.00	0	0.000
67	A	14	7	1.00	45	0.156
68	A	12	7	1.00	45	0.156
69	A	10	6	1.00	43	0.140
70	A	0	0	0.00	0	0.000
71	A	0	0	0.00	0	0.000
72	A	0	0	0.00	0	0.000
73	A	0	0	0.00	0	0.000
74	F	0	0	N/A	0.	N/A
75	F	0	0	N/A	0.	N/A
76	A	30	9	1.00	32	0.281
77	A	27	10	1.00	32	0.312
78	A	18	6	1.00	30	0.200
79	A	7	4	1.00	29	0.138
80	A	29	11	1.00	32	0.344
81	A	31	12	1.00	32	0.375
82	A	37	14	1.00	34	0.412
83	A	30	12	1.00	34	0.353
84	A	21	9	1.00	32	0.281
85	A	7	4	1.00	31	0.129
86	A	31	12	1.00	34	0.353
87	A	40	16	1.00	34	0.471
88	A	5	5	1.00	19	0.263
89	A	3	3	1.00	24	0.125
90	A	3	3	1.00	34	0.088
91	A	3	3	1.00	55	0.055
92	A	3	3	1.00	58	0.052
93	A	4	4	1.00	11	0.364
94	A	5	5	1.00	13	0.385
95	A	7	7	1.00	13	0.538

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	2	2	1.00	13	0.154
97	A	6	6	1.00	15	0.400
98	A	5	5	1.00	15	0.333
99	A	2	2	1.00	13	0.154
100	A	6	6	1.00	15	0.400
101	A	5	5	1.00	15	0.333
102	A	1	1	1.00	38	0.026
103	A	2	2	1.00	50	0.040
104	A	4	4	1.00	42	0.095
105	A	2	2	1.00	62	0.032
106	A	4	4	1.00	49	0.082
107	A	9	5	1.00	42	0.119
108	A	10	6	1.00	65	0.092

Chapter 3

Listing of integrals

Local contents

3.1 $\int \left(f+\frac{g}{x}\right)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal. Leaf size=404

$$-\frac{B(bc-ad)g^3n}{2acx}+Af^3x-\frac{1}{2}B\left(\frac{b^2}{a^2}-\frac{d^2}{c^2}\right)g^3n\log(x)+\frac{b^2Bg^3n\log(a+bx)}{2a^2}-3Bf^2gn\log(x)\log\left(1+\frac{bx}{a}\right)+\frac{B}{}$$

```
[Out] -1/2*B*(-a*d+b*c)*g^3*n/a/c/x+A*f^3*x-1/2*B*(b^2/a^2-d^2/c^2)*g^3*n*ln(x)+1
/2*b^2*B*g^3*n*ln(b*x+a)/a^2-3*B*f^2*g*n*ln(x)*ln(1+b*x/a)+B*f^3*(b*x+a)*ln
(e*((b*x+a)/(d*x+c))^n)/b-1/2*g^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/x^2+3*(-a
*d+b*c)*f*g^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/a/(d*x+c)/(a-c*(b*x+a
)/(d*x+c))+3*f^2*g*ln(x)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))-B*(-a*d+b*c)*f^3*n
*ln(d*x+c)/b/d-1/2*B*d^2*g^3*n*ln(d*x+c)/c^2+3*B*f^2*g*n*ln(x)*ln(1+d*x/c)+
3*B*(-a*d+b*c)*f*g^2*n*ln(a-c*(b*x+a)/(d*x+c))/a/c-3*B*f^2*g*n*polylog(2,-b
*x/a)+3*B*f^2*g*n*polylog(2,-d*x/c)
```

Rubi [A]
time = 0.29, antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2608, 2535, 31, 2547, 84, 2553, 2351, 2545, 2354, 2438}

$$-3Bf^2gn\text{PolyLog}\left(2,-\frac{bx}{a}\right)+3Bf^2gn\text{PolyLog}\left(2,-\frac{dx}{c}\right)-\frac{1}{2}Bg^3n\log(x)\left(\frac{b^2}{a^2}-\frac{d^2}{c^2}\right)+\frac{b^2Bg^3n\log(a+bx)}{2a^2}+3f^2g\log(x)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+\frac{3fg^2(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a(c+dx)\left(a-\frac{c(a+bx)}{c+dx}\right)}-\frac{g^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2x^2}+\frac{Bf^3(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{Bf^3n(bc-ad)\log(c+dx)}{bd}+\frac{3Bfg^2n(bc-ad)\log\left(a-\frac{c(a+bx)}{c+dx}\right)}{ac}-\frac{Bg^3n(bc-ad)}{2acx}-3Bf^2gn\log(x)\log\left(\frac{bx}{a}+1\right)+Af^3x-\frac{Bd^2g^3n\log(c+dx)}{2c^2}+3Bf^2gn\log(x)\log\left(\frac{dx}{c}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(f + g/x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -1/2*(B*(b*c - a*d)*g^3*n)/(a*c*x) + A*f^3*x - (B*(b^2/a^2 - d^2/c^2)*g^3*n
*Log[x])/2 + (b^2*B*g^3*n*Log[a + b*x])/(2*a^2) - 3*B*f^2*g*n*Log[x]*Log[1
+ (b*x)/a] + (B*f^3*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (g^3*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*x^2) + (3*(b*c - a*d)*f*g^2*(a + b*x
)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*(c + d*x)*(a - (c*(a + b*x))/(
c + d*x))) + 3*f^2*g*Log[x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*
c - a*d)*f^3*n*Log[c + d*x])/(b*d) - (B*d^2*g^3*n*Log[c + d*x])/(2*c^2) + 3
*B*f^2*g*n*Log[x]*Log[1 + (d*x)/c] + (3*B*(b*c - a*d)*f*g^2*n*Log[a - (c*(a
 + b*x))/(c + d*x)])/(a*c) - 3*B*f^2*g*n*PolyLog[2, -((b*x)/a)] + 3*B*f^2*g
*n*PolyLog[2, -((d*x)/c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
```

```
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2535

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rule 2545

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((A + B*Log[e*((
a + b*x)/(c + d*x))^n])/g), x] + (-Dist[b*B*(n/g), Int[Log[f + g*x]/(a + b*
x), x], x] + Dist[B*d*(n/g), Int[Log[f + g*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \left(f+\frac{g}{x}\right)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx &= \int \left(f^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{g^3\left(A+B\log\left(e\left(\frac{a}{c}\right.\right.\right.}{x^3}\right.\\
&= f^3\int\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx+\left(3f^2g\right)\int\frac{A+B}{}\\
&= Af^3x-\frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2x^2}-\frac{3fg^2\left(A+B\log\left(e\left(\frac{a}{c}\right.\right.\right.}{x}\\
&= Af^3x+\frac{Bf^3(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{g^3\left(A+B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.}{2x^2}\\
&= Af^3x+\frac{Bf^3(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{g^3\left(A+B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.}{2x^2}\\
&= -\frac{B(bc-ad)g^3n}{2acx}+Af^3x+\frac{3B(bc-ad)fg^2n\log(x)}{ac}-\frac{B(}{}\\
&= -\frac{B(bc-ad)g^3n}{2acx}+Af^3x+\frac{3B(bc-ad)fg^2n\log(x)}{ac}-\frac{B(}{}
\end{aligned}
$$

Mathematica [A]
time = 0.29, size = 336, normalized size = 0.83

$$
Af^3x+\frac{Bf^3(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2x^2}-\frac{3fg^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x}+3f^2g\log(x)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-\frac{B(bc-ad)f^3n\log(c+dx)}{bd}+\frac{3Bfg^2n((bc-ad)\log(x)-bc\log(a+bx)+ad\log(c+dx))}{ac}+\frac{Bg^3n((-b^2c^2x+a^2d^2x)\log(x)+b^2c^2x\log(a+bx)+a(-bc^2+acd-ad^2x\log(c+dx)))}{2a^2c^2x}-3Bf^2gn\left(\log(x)\left(\log\left(1+\frac{bx}{a}\right)-\log\left(1+\frac{dx}{c}\right)\right)+\mathrm{Li}_2\left(-\frac{bx}{a}\right)-\mathrm{Li}_2\left(-\frac{dx}{c}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g/x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] A*f^3*x + (B*f^3*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (g^3*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]))/(2*x^2) - (3*f*g^2*(A + B*Log[e*((a + b*x)
/(c + d*x))^n]))/x + 3*f^2*g*Log[x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
- (B*(b*c - a*d)*f^3*n*Log[c + d*x])/(b*d) + (3*B*f*g^2*n*((b*c - a*d)*Log[
x] - b*c*Log[a + b*x] + a*d*Log[c + d*x]))/(a*c) + (B*g^3*n*((-(b^2*c^2*x)
+ a^2*d^2*x)*Log[x] + b^2*c^2*x*Log[a + b*x] + a*(-(b*c^2) + a*c*d - a*d^2*
x*Log[c + d*x])))/(2*a^2*c^2*x) - 3*B*f^2*g*n*(Log[x]*(Log[1 + (b*x)/a] - L
og[1 + (d*x)/c]) + PolyLog[2, -((b*x)/a)] - PolyLog[2, -((d*x)/c)])
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \left(f+\frac{g}{x}\right)^{3} \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g/x)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int((f+g/x)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
```

```
[Out] B*f^3*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) - 3*B*f*g^2*n*(b*log(b*x + a)
/a - d*log(d*x + c)/c - (b*c - a*d)*log(x)/(a*c)) + 1/2*B*g^3*n*(b^2*log(b*
x + a)/a^2 - d^2*log(d*x + c)/c^2 - (b*c - a*d)/(a*c*x) - (b^2*c^2 - a^2*d^
2)*log(x)/(a^2*c^2)) + B*f^3*x*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + A*f
^3*x - 3*B*f^2*g*integrate(-(log((b*x + a)^n) - log((d*x + c)^n) + 1)/x, x)
 + 3*A*f^2*g*log(x) - 3*B*f*g^2*log((b*x/(d*x + c) + a/(d*x + c))^n*e)/x -
3*A*f*g^2/x - 1/2*B*g^3*log((b*x/(d*x + c) + a/(d*x + c))^n*e)/x^2 - 1/2*A*
g^3/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
```

```
[Out] integral((A*f^3*x^3 + 3*A*f^2*g*x^2 + 3*A*f*g^2*x + A*g^3 + (B*f^3*x^3 + 3*
B*f^2*g*x^2 + 3*B*f*g^2*x + B*g^3)*log(((b*x + a)/(d*x + c))^n*e))/x^3, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] integrate((B*log(((b*x + a)/(d*x + c))^n*e) + A)*(f + g/x)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(f+\frac{g}{x}\right)^3\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g/x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] int((f + g/x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

3.2 $\int \left(f+\frac{g}{x}\right)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal. Leaf size=263

$$Af^2x-2Bfgn\log(x)\log\left(1+\frac{bx}{a}\right)+\frac{Bf^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}+\frac{(bc-ad)g^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{a(c+dx)\left(a-\frac{c(a+bx)}{c+dx}\right)}$$

```
[Out] A*f^2*x-2*B*f*g*n*ln(x)*ln(1+b*x/a)+B*f^2*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)
/b+(-a*d+b*c)*g^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/a/(d*x+c)/(a-c*(b
*x+a)/(d*x+c))+2*f*g*ln(x)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))-B*(-a*d+b*c)*f^2
*n*ln(d*x+c)/b/d+2*B*f*g*n*ln(x)*ln(1+d*x/c)+B*(-a*d+b*c)*g^2*n*ln(a-c*(b*x
+a)/(d*x+c))/a/c-2*B*f*g*n*polylog(2,-b*x/a)+2*B*f*g*n*polylog(2,-d*x/c)
```

Rubi [A]
time = 0.17, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2608, 2535, 31, 2553, 2351, 2545, 2354, 2438}

$$-2BfgnPolyLog\left(2,-\frac{bx}{a}\right)+2BfgnPolyLog\left(2,-\frac{dx}{c}\right)+2fg\log(x)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+\frac{g^2(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a(c+dx)\left(a-\frac{c(a+bx)}{c+dx}\right)}+\frac{Bf^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{Bf^2n(bc-ad)\log(c+dx)}{bd}+\frac{Bg^2n(bc-ad)\log\left(a-\frac{c(a+bx)}{c+dx}\right)}{ac}-2Bfgn\log(x)\log\left(\frac{bx}{a}+1\right)+Af^2x+2Bfgn\log(x)\log\left(\frac{dx}{c}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(f + g/x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] A*f^2*x - 2*B*f*g*n*Log[x]*Log[1 + (b*x)/a] + (B*f^2*(a + b*x)*Log[e*((a +
b*x)/(c + d*x))^n])/b + ((b*c - a*d)*g^2*(a + b*x)*(A + B*Log[e*((a + b*x)/
(c + d*x))^n]))/(a*(c + d*x)*(a - (c*(a + b*x))/(c + d*x))) + 2*f*g*Log[x]*
(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c - a*d)*f^2*n*Log[c + d*x])
/(b*d) + 2*B*f*g*n*Log[x]*Log[1 + (d*x)/c] + (B*(b*c - a*d)*g^2*n*Log[a - (
c*(a + b*x))/(c + d*x)])/(a*c) - 2*B*f*g*n*PolyLog[2, -((b*x)/a)] + 2*B*f*g
*n*PolyLog[2, -((d*x)/c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2535

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rule 2545

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((A + B*Log[e*((
a + b*x)/(c + d*x))^n])/g), x] + (-Dist[b*B*(n/g), Int[Log[f + g*x]/(a + b*
x), x], x] + Dist[B*d*(n/g), Int[Log[f + g*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \left(f+\frac{g}{x}\right)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = \int \left(f^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{g^2\left(A+B\log\left(e\right.\right.}{x^2}\right.$$

$$= f^2\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx+(2fg)\int \frac{A+B}{}$$

$$= Af^2x-\frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x}+2fg\log(x)\left(A+B\right.$$

$$= Af^2x+\frac{Bf^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{g^2\left(A+B\log\left(e\right.\right.}{x}$$

$$= Af^2x+\frac{Bf^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{g^2\left(A+B\log\left(e\right.\right.}{x}$$

$$= Af^2x+\frac{B(bc-ad)g^2n\log(x)}{ac}-\frac{bBg^2n\log(a+bx)}{a}-2B$$

$$= Af^2x+\frac{B(bc-ad)g^2n\log(x)}{ac}-\frac{bBg^2n\log(a+bx)}{a}-2B$$

Mathematica [A]
time = 0.14, size = 217, normalized size = 0.83

$$Af^2x+\frac{Bf^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x}+2fg\log(x)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-\frac{B(bc-ad)f^2n\log(c+dx)}{bd}+\frac{Bg^2n((bc-ad)\log(x)-bc\log(a+bx)+ad\log(c+dx))}{ac}-2Bfgn\left(\log(x)\left(\log\left(1+\frac{bx}{a}\right)-\log\left(1+\frac{dx}{c}\right)\right)+\mathrm{Li}_2\left(-\frac{bx}{a}\right)-\mathrm{Li}_2\left(-\frac{dx}{c}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g/x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] A*f^2*x + (B*f^2*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (g^2*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]))/x + 2*f*g*Log[x]*(A + B*Log[e*((a + b*x)/(
c + d*x))^n]) - (B*(b*c - a*d)*f^2*n*Log[c + d*x])/(b*d) + (B*g^2*n*((b*c -
 a*d)*Log[x] - b*c*Log[a + b*x] + a*d*Log[c + d*x]))/(a*c) - 2*B*f*g*n*(Log
[x]*(Log[1 + (b*x)/a] - Log[1 + (d*x)/c]) + PolyLog[2, -((b*x)/a)] - PolyLo
g[2, -((d*x)/c)])
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \left(f+\frac{g}{x}\right)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g/x)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int((f+g/x)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
```

```
[Out] B*f^2*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) - B*g^2*n*(b*log(b*x + a)/a -
 d*log(d*x + c)/c - (b*c - a*d)*log(x)/(a*c)) + B*f^2*x*log((b*x/(d*x + c)
+ a/(d*x + c))^n*e) + A*f^2*x - 2*B*f*g*integrate(-(log((b*x + a)^n) - log(
(d*x + c)^n) + 1)/x, x) + 2*A*f*g*log(x) - B*g^2*log((b*x/(d*x + c) + a/(d*
x + c))^n*e)/x - A*g^2/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
```

```
[Out] integral((A*f^2*x^2 + 2*A*f*g*x + A*g^2 + (B*f^2*x^2 + 2*B*f*g*x + B*g^2)*l
og(((b*x + a)/(d*x + c))^n*e))/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A + B\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right)\right)\left(fx + g\right)^{2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Integral((A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))*(f*x + g)**2/x**2,
 x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] integrate((B*log(((b*x + a)/(d*x + c))^n*e) + A)*(f + g/x)^2, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \left(f+\frac{g}{x}\right)^2 \left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g/x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
[Out] int((f + g/x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

3.3 $\int \left(f+\frac{g}{x}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal. Leaf size=143

$$Afx-Bgn\log(x)\log\left(1+\frac{bx}{a}\right)+\frac{Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}+g\log(x)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-\frac{B(bc$$

```
[Out] A*f*x-B*g*n*ln(x)*ln(1+b*x/a)+B*f*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/b+g*ln(
x)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))-B*(-a*d+b*c)*f*n*ln(d*x+c)/b/d+B*g*n*ln(
x)*ln(1+d*x/c)-B*g*n*polylog(2,-b*x/a)+B*g*n*polylog(2,-d*x/c)
```

Rubi [A]
time = 0.10, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2608, 2535, 31, 2545, 2354, 2438}

$$-Bgn\text{PolyLog}\left(2,-\frac{bx}{a}\right)+Bgn\text{PolyLog}\left(2,-\frac{dx}{c}\right)+g\log(x)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+\frac{Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{Bfn(bc-ad)\log(c+dx)}{bd}-Bgn\log(x)\log\left(\frac{bx}{a}+1\right)+Afx+Bgn\log(x)\log\left(\frac{dx}{c}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(f + g/x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] A*f*x - B*g*n*Log[x]*Log[1 + (b*x)/a] + (B*f*(a + b*x)*Log[e*((a + b*x)/(c
+ d*x))^n])/b + g*Log[x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c -
 a*d)*f*n*Log[c + d*x])/(b*d) + B*g*n*Log[x]*Log[1 + (d*x)/c] - B*g*n*PolyL
og[2, -((b*x)/a)] + B*g*n*PolyLog[2, -((d*x)/c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2535

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)/(c + d*x)
```

```
)^n])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rule 2545

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((A + B*Log[e*((
a + b*x)/(c + d*x))^n])/g), x] + (-Dist[b*B*(n/g), Int[Log[f + g*x]/(a + b*
x), x], x] + Dist[B*d*(n/g), Int[Log[f + g*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \left(f+\frac{g}{x}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx &= \int \left(f\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{g\left(A+B\log\left(e\left(\frac{a+b}{c+c}\right.\right.\right.}{x}\right.\\
&= f\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx+g\int \frac{A+B\log\left(e\right.}{x}\\
&= Afx+g\log(x)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+(Bf)\int \mathrm{lo}\\
&= Afx+\frac{Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}+g\log(x)\left(A+B\log\right.\\
&= Afx+\frac{Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}+g\log(x)\left(A+B\log\right.\\
&= Afx-Bgn\log(x)\log\left(1+\frac{bx}{a}\right)+\frac{Bf(a+bx)\log\left(e\left(\frac{a+b}{c+d}\right.\right.}{b}\\
&= Afx-Bgn\log(x)\log\left(1+\frac{bx}{a}\right)+\frac{Bf(a+bx)\log\left(e\left(\frac{a+b}{c+d}\right.\right.}{b}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 135, normalized size = 0.94

$$Afx+\frac{Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}+g\log(x)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-\frac{B(bc-ad)fn\log(c+dx)}{bd}-Bgn\left(\log(x)\left(\log\left(1+\frac{bx}{a}\right)-\log\left(1+\frac{dx}{c}\right)\right)+\mathrm{Li}_2\left(-\frac{bx}{a}\right)-\mathrm{Li}_2\left(-\frac{dx}{c}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g/x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] A*f*x + (B*f*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b + g*Log[x]*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c - a*d)*f*n*Log[c + d*x])/(b*d) -
B*g*n*(Log[x]*(Log[1 + (b*x)/a] - Log[1 + (d*x)/c]) + PolyLog[2, -((b*x)/a)
] - PolyLog[2, -((d*x)/c)])
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \left(f+\frac{g}{x}\right)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g/x)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int((f+g/x)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
```

```
[Out] B*f*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*f*x*log((b*x/(d*x + c) + a/
(d*x + c))^n*e) + A*f*x - B*g*integrate(-(log((b*x + a)^n) - log((d*x + c)^
n) + 1)/x, x) + A*g*log(x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
```

```
[Out] integral((A*f*x + A*g + (B*f*x + B*g)*log(((b*x + a)/(d*x + c))^n*e))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)\right)(fx+g)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Integral((A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))*(f*x + g)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] integrate((B*log(((b*x + a)/(d*x + c))^n*e) + A)*(f + g/x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(f+\frac{g}{x}\right)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g/x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] int((f + g/x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

3.4 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+\frac{g}{x}}\,dx$

Optimal. Leaf size=217

$$\frac{Ax}{f}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf}-\frac{B(bc-ad)n\log(c+dx)}{bdf}+\frac{Bgn\log\left(\frac{f(a+bx)}{af-bg}\right)\log(g+fx)}{f^2}-\frac{g(A+B\log\left(e(\right.}{}$$

```
[Out] A*x/f+B*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/b/f-B*(-a*d+b*c)*n*ln(d*x+c)/b/d/
f+B*g*n*ln(f*(b*x+a)/(a*f-b*g))*ln(f*x+g)/f^2-g*(A+B*ln(e*((b*x+a)/(d*x+c))
^n))*ln(f*x+g)/f^2-B*g*n*ln(f*(d*x+c)/(c*f-d*g))*ln(f*x+g)/f^2+B*g*n*polylo
g(2,-b*(f*x+g)/(a*f-b*g))/f^2-B*g*n*polylog(2,-d*(f*x+g)/(c*f-d*g))/f^2
```

Rubi [A]
time = 0.17, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2608, 2535, 31, 2545, 2441, 2440, 2438}

$$\frac{BgnPolyLog\left(2,-\frac{b(fx+g)}{af-bg}\right)}{f^2}-\frac{BgnPolyLog\left(2,-\frac{d(fx+g)}{cf-dg}\right)}{f^2}-\frac{g\log(fx+g)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{f^2}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf}-\frac{Bn(bc-ad)\log(c+dx)}{bdf}+\frac{Bgn\log(fx+g)\log\left(\frac{f(a+bx)}{af-bg}\right)}{f^2}+\frac{Ax}{f}-\frac{Bgn\log(fx+g)\log\left(\frac{f(c+dx)}{cf-dg}\right)}{f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g/x),x]
```

```
[Out] (A*x)/f + (B*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(b*f) - (B*(b*c - a*
d)*n*Log[c + d*x])/(b*d*f) + (B*g*n*Log[(f*(a + b*x))/(a*f - b*g)]*Log[g +
f*x])/f^2 - (g*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[g + f*x])/f^2 - (
B*g*n*Log[(f*(c + d*x))/(c*f - d*g)]*Log[g + f*x])/f^2 + (B*g*n*PolyLog[2,
-((b*(g + f*x))/(a*f - b*g))])/f^2 - (B*g*n*PolyLog[2, -((d*(g + f*x))/(c*f
 - d*g))])/f^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2535

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rule 2545

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((A + B*Log[e*((
a + b*x)/(c + d*x))^n])/g), x] + (-Dist[b*B*(n/g), Int[Log[f + g*x]/(a + b*
x), x], x] + Dist[B*d*(n/g), Int[Log[f + g*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+\frac{g}{x}}\,dx = \int \left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f} - \frac{g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f(g+fx)}\right)\,dx$$

$$= \frac{\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx}{f} - \frac{g\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g+fx}\,dx}{f}$$

$$= \frac{Ax}{f} - \frac{g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(g+fx)}{f^2} + \frac{B\int \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\,dx}{f} + \frac{(Bgn}{}$$

$$= \frac{Ax}{f} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf} - \frac{g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(g+fx)}{f^2} - \frac{(B}{}$$

$$= \frac{Ax}{f} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf} - \frac{B(bc-ad)n\log(c+dx)}{bdf} - \frac{g\left(A+B\log\left(e\right.\right.}{}$$

$$= \frac{Ax}{f} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf} - \frac{B(bc-ad)n\log(c+dx)}{bdf} + \frac{Bgn\log\left(\frac{f(a+b}{af-b}\right.}{f}$$

$$= \frac{Ax}{f} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf} - \frac{B(bc-ad)n\log(c+dx)}{bdf} + \frac{Bgn\log\left(\frac{f(a+b}{af-b}\right.}{f}$$

$$= \frac{Ax}{f} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf} - \frac{B(bc-ad)n\log(c+dx)}{bdf} + \frac{Bgn\log\left(\frac{f(a+b}{af-b}\right.}{f}$$

Mathematica [A]
time = 0.11, size = 185, normalized size = 0.85

$$\frac{Afx + \frac{Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} - \frac{B(bc-ad)fn\log(c+dx)}{bd} - g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(g+fx) + Bgn\left(\left(\log\left(\frac{f(a+bx)}{af-bg}\right) - \log\left(\frac{f(c+dx)}{cf-dg}\right)\right)\log(g+fx) + \text{Li}_2\left(\frac{b(g+fx)}{-af+bg}\right) - \text{Li}_2\left(\frac{d(g+fx)}{-cf+dg}\right)\right)}{f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g/x),x]
```

```
[Out] (A*f*x + (B*f*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (B*(b*c - a*d)*
f*n*Log[c + d*x])/(b*d) - g*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[g +
f*x] + B*g*n*((Log[(f*(a + b*x))/(a*f - b*g)] - Log[(f*(c + d*x))/(c*f - d*
g)])*Log[g + f*x] + PolyLog[2, (b*(g + f*x))/(-(a*f) + b*g)] - PolyLog[2, (
d*(g + f*x))/(-(c*f) + d*g)]))/f^2
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{f+\frac{g}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(f+g/x),x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(f+g/x),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x),x, algorithm="maxima")
```

```
[Out] A*(x/f - g*log(f*x + g)/f^2) - B*integrate(-(x*log((b*x + a)^n) - x*log((d*
x + c)^n) + x)/(f*x + g), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x),x, algorithm="fricas")
```

```
[Out] integral((B*x*log(((b*x + a)/(d*x + c))^n*e) + A*x)/(f*x + g), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(f+g/x),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x),x, algorithm="giac")
```

```
[Out] integrate((B*log(((b*x + a)/(d*x + c))^n*e) + A)/(f + g/x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{f+\frac{g}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g/x),x)
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g/x), x)
```

3.5 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^2}\,dx$

Optimal. Leaf size=322

$$\frac{Ax}{f^2}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^2}-\frac{g^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^2(af-bg)(g+fx)}-\frac{B(bc-ad)n\log(c+dx)}{bdf^2}+\frac{2Bgn\log\left(\right.}{}$$

```
[Out] A*x/f^2+B*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/b/f^2-g^2*(b*x+a)*(A+B*ln(e*((b
*x+a)/(d*x+c))^n))/f^2/(a*f-b*g)/(f*x+g)-B*(-a*d+b*c)*n*ln(d*x+c)/b/d/f^2+2
*B*g*n*ln(f*(b*x+a)/(a*f-b*g))*ln(f*x+g)/f^3-2*g*(A+B*ln(e*((b*x+a)/(d*x+c)
)^n))*ln(f*x+g)/f^3-2*B*g*n*ln(f*(d*x+c)/(c*f-d*g))*ln(f*x+g)/f^3+B*(-a*d+b
*c)*g^2*n*ln((f*x+g)/(d*x+c))/f^2/(a*f-b*g)/(c*f-d*g)+2*B*g*n*polylog(2,-b*
(f*x+g)/(a*f-b*g))/f^3-2*B*g*n*polylog(2,-d*(f*x+g)/(c*f-d*g))/f^3
```

Rubi [A]
time = 0.25, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2608, 2535, 31, 2553, 2351, 2545, 2441, 2440, 2438}

$$\frac{2Bgn\text{PolyLog}\left(2,-\frac{b(fx+g)}{af-bg}\right)}{f^3}-\frac{2Bgn\text{PolyLog}\left(2,-\frac{d(fx+g)}{cf-dg}\right)}{f^3}-\frac{2g\log(fx+g)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{f^3}-\frac{g^2(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{f^2(fx+g)(af-bg)}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^2}+\frac{Bg^2n(bc-ad)\log\left(\frac{fx+g}{c+dx}\right)}{f^2(af-bg)(cf-dg)}-\frac{Bn(bc-ad)\log(c+dx)}{bdf^2}+\frac{2Bgn\log(fx+g)\log\left(\frac{f(a+bx)}{af-bg}\right)}{f^3}+\frac{Ax}{f^2}-\frac{2Bgn\log(fx+g)\log\left(\frac{f(c+dx)}{cf-dg}\right)}{f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g/x)^2,x]
```

```
[Out] (A*x)/f^2 + (B*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(b*f^2) - (g^2*(a
+ b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(f^2*(a*f - b*g)*(g + f*x))
- (B*(b*c - a*d)*n*Log[c + d*x])/(b*d*f^2) + (2*B*g*n*Log[(f*(a + b*x))/(a*
f - b*g)]*Log[g + f*x])/f^3 - (2*g*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*L
og[g + f*x])/f^3 - (2*B*g*n*Log[(f*(c + d*x))/(c*f - d*g)]*Log[g + f*x])/f^
3 + (B*(b*c - a*d)*g^2*n*Log[(g + f*x)/(c + d*x)])/(f^2*(a*f - b*g)*(c*f -
d*g)) + (2*B*g*n*PolyLog[2, -((b*(g + f*x))/(a*f - b*g))])/f^3 - (2*B*g*n*P
olyLog[2, -((d*(g + f*x))/(c*f - d*g))])/f^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2535

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rule 2545

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((A + B*Log[e*((
a + b*x)/(c + d*x))^n])/g), x] + (-Dist[b*B*(n/g), Int[Log[f + g*x]/(a + b*
x), x], x] + Dist[B*d*(n/g), Int[Log[f + g*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
```

```
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^2}\,dx = \int \left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f^2} + \frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^2(g+fx)^2} - \frac{2g\left(A+B\log\left(e\left(\right.\right.\right.}{f^2(g+fx}\right.$$

$$= \frac{\int\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx}{f^2} - \frac{(2g)\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g+fx}\,dx}{f^2} + \frac{g^2\int\frac{A+B\log\left(e\left(\right.\right.}{(g+f}}{f^2}$$

$$= \frac{Ax}{f^2} - \frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^3(g+fx)} - \frac{2g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(g+fx)}{f^3} +$$

$$= \frac{Ax}{f^2} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^2} - \frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^3(g+fx)} - \frac{2g\left(A+B\,\mathrm{lo}\right.}{}$$

$$= \frac{Ax}{f^2} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^2} - \frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^3(g+fx)} - \frac{B(bc-ad)n}{bd}$$

$$= \frac{Ax}{f^2} - \frac{bBg^2n\log(a+bx)}{f^3(af-bg)} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^2} - \frac{g^2\left(A+B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.}{f^3(g+fx)}$$

$$= \frac{Ax}{f^2} - \frac{bBg^2n\log(a+bx)}{f^3(af-bg)} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^2} - \frac{g^2\left(A+B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.}{f^3(g+fx)}$$

$$= \frac{Ax}{f^2} - \frac{bBg^2n\log(a+bx)}{f^3(af-bg)} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^2} - \frac{g^2\left(A+B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.}{f^3(g+fx)}$$

Mathematica [A]
time = 0.32, size = 295, normalized size = 0.92

$$\frac{Afx+\frac{Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g+fx}-\frac{B(bc-ad)fn\log(c+dx)}{bd}-2g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(g+fx)+\frac{Bg^2n(b(-cf+dg)\log(a+bx)+d(af-bg)\log(c+dx)+(bc-ad)f\log(g+fx))}{(af-bg)(cf-dg)}+2Bgn\left(\left(\log\left(\frac{f(a+bx)}{af-bg}\right)-\log\left(\frac{f(c+dx)}{cf-dg}\right)\right)\log(g+fx)+\mathrm{Li}_2\left(\frac{b(g+fx)}{-af+bg}\right)-\mathrm{Li}_2\left(\frac{d(g+fx)}{-cf+dg}\right)\right)}{f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g/x)^2,x]
```

```
[Out] (A*f*x + (B*f*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (g^2*(A + B*Log
[e*((a + b*x)/(c + d*x))^n]))/(g + f*x) - (B*(b*c - a*d)*f*n*Log[c + d*x])/
(b*d) - 2*g*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[g + f*x] + (B*g^2*n*
(b*(-(c*f) + d*g)*Log[a + b*x] + d*(a*f - b*g)*Log[c + d*x] + (b*c - a*d)*f
```

```
*Log[g + f*x]))/((a*f - b*g)*(c*f - d*g)) + 2*B*g*n*((Log[(f*(a + b*x))/(a*
f - b*g)] - Log[(f*(c + d*x))/(c*f - d*g)])*Log[g + f*x] + PolyLog[2, (b*(g
 + f*x))/(-(a*f) + b*g)] - PolyLog[2, (d*(g + f*x))/(-(c*f) + d*g)]))/f^3
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{\left(f+\frac{g}{x}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^2,x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^2,x, algorithm="maxima")
```

```
[Out] -A*(g^2/(f^4*x + f^3*g) - x/f^2 + 2*g*log(f*x + g)/f^3) - B*integrate(-(x^2
*log((b*x + a)^n) - x^2*log((d*x + c)^n) + x^2)/(f^2*x^2 + 2*f*g*x + g^2),
x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^2,x, algorithm="fricas")
```

```
[Out] integral((B*x^2*log(((b*x + a)/(d*x + c))^n*e) + A*x^2)/(f^2*x^2 + 2*f*g*x
+ g^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(f+g/x)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^2,x, algorithm="giac")
[Out] integrate((B*log(((b*x + a)/(d*x + c))^n*e) + A)/(f + g/x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{{\left(f+\frac{g}{x}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g/x)^2,x)
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g/x)^2, x)
```

3.6 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^3}\,dx$

Optimal. Leaf size=531

$$\frac{Ax}{f^3}+\frac{B(bc-ad)g^3n}{2f^3(af-bg)(cf-dg)(g+fx)}-\frac{b^2Bg^3n\log(a+bx)}{2f^4(af-bg)^2}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^3}+\frac{g^3\left(A+B\log\left(e\left(\frac{a+b}{c+d}\right.\right.\right.}{2f^4(g+fx)^2}$$

```
[Out] A*x/f^3+1/2*B*(-a*d+b*c)*g^3*n/f^3/(a*f-b*g)/(c*f-d*g)/(f*x+g)-1/2*b^2*B*g^
3*n*ln(b*x+a)/f^4/(a*f-b*g)^2+B*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/b/f^3+1/2
*g^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/f^4/(f*x+g)^2-3*g^2*(b*x+a)*(A+B*ln(e*
((b*x+a)/(d*x+c))^n))/f^3/(a*f-b*g)/(f*x+g)-B*(-a*d+b*c)*n*ln(d*x+c)/b/d/f^
3+1/2*B*d^2*g^3*n*ln(d*x+c)/f^4/(c*f-d*g)^2+1/2*B*(-a*d+b*c)*g^3*(a*d*f+b*c
*f-2*b*d*g)*n*ln(f*x+g)/f^3/(a*f-b*g)^2/(c*f-d*g)^2+3*B*g*n*ln(f*(b*x+a)/(a
*f-b*g))*ln(f*x+g)/f^4-3*g*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(f*x+g)/f^4-3*
B*g*n*ln(f*(d*x+c)/(c*f-d*g))*ln(f*x+g)/f^4+3*B*(-a*d+b*c)*g^2*n*ln((f*x+g)
/(d*x+c))/f^3/(a*f-b*g)/(c*f-d*g)+3*B*g*n*polylog(2,-b*(f*x+g)/(a*f-b*g))/f
^4-3*B*g*n*polylog(2,-d*(f*x+g)/(c*f-d*g))/f^4
```

Rubi [A]
time = 0.46, antiderivative size = 531, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2608, 2535, 31, 2547, 84, 2553, 2351, 2545, 2441, 2440, 2438}

$$\frac{3BgnPolyLog\left(2,-\frac{b(fx+g)}{af-bg}\right)}{f^4}-\frac{3BgnPolyLog\left(2,-\frac{d(fx+g)}{cf-dg}\right)}{f^4}+\frac{g^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2f^4(fx+g)^2}-\frac{3g\log(fx+g)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{f^4}-\frac{3g^2(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{f^3(fx+g)(af-bg)}-\frac{b^2Bg^3n\log(a+bx)}{2f^4(af-bg)^2}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^3}+\frac{Bg^3n(bc-ad)}{2f^3(fx+g)(af-bg)(cf-dg)}+\frac{Bg^3n(bc-ad)\log(fx+g)(adf+bcf-2bdg)}{2f^3(af-bg)^2(cf-dg)^2}+\frac{3Bg^2n(bc-ad)\log\left(\frac{fx+g}{c+dx}\right)}{f^3(af-bg)(cf-dg)}-\frac{Bn(bc-ad)\log(c+dx)}{bdf^3}+\frac{3Bgn\log(fx+g)\log\left(\frac{f(a+bx)}{af-bg}\right)}{f^4}+\frac{Ax}{f^3}+\frac{Bd^2g^3n\log(c+dx)}{2f^4(cf-dg)^2}-\frac{3Bgn\log(fx+g)\log\left(\frac{f(c+dx)}{cf-dg}\right)}{f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g/x)^3,x]
```

```
[Out] (A*x)/f^3 + (B*(b*c - a*d)*g^3*n)/(2*f^3*(a*f - b*g)*(c*f - d*g)*(g + f*x))
 - (b^2*B*g^3*n*Log[a + b*x])/(2*f^4*(a*f - b*g)^2) + (B*(a + b*x)*Log[e*((
a + b*x)/(c + d*x))^n])/(b*f^3) + (g^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n
]))/(2*f^4*(g + f*x)^2) - (3*g^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]))/(f^3*(a*f - b*g)*(g + f*x)) - (B*(b*c - a*d)*n*Log[c + d*x])/(b*d*f
^3) + (B*d^2*g^3*n*Log[c + d*x])/(2*f^4*(c*f - d*g)^2) + (B*(b*c - a*d)*g^3
*(b*c*f + a*d*f - 2*b*d*g)*n*Log[g + f*x])/(2*f^3*(a*f - b*g)^2*(c*f - d*g)
^2) + (3*B*g*n*Log[(f*(a + b*x))/(a*f - b*g)]*Log[g + f*x])/f^4 - (3*g*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n])*Log[g + f*x])/f^4 - (3*B*g*n*Log[(f*(c +
 d*x))/(c*f - d*g)]*Log[g + f*x])/f^4 + (3*B*(b*c - a*d)*g^2*n*Log[(g + f*x
)/(c + d*x)])/(f^3*(a*f - b*g)*(c*f - d*g)) + (3*B*g*n*PolyLog[2, -((b*(g +
 f*x))/(a*f - b*g))])/f^4 - (3*B*g*n*PolyLog[2, -((d*(g + f*x))/(c*f - d*g)
)])/f^4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2535

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rule 2545

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((A + B*Log[e*((
```

```
a + b*x)/(c + d*x))^n])/g), x] + (-Dist[b*B*(n/g), Int[Log[f + g*x]/(a + b*
x), x], x] + Dist[B*d*(n/g), Int[Log[f + g*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^3}\,dx = \int \left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f^3} - \frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^3(g+fx)^3} + \frac{3g^2\left(A+B\log\left(e\right.\right.}{f^3(g+fx}\right.$$

$$= \frac{\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx}{f^3} - \frac{(3g)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g+fx}\,dx}{f^3} + \frac{(3g^2)\int \frac{A+B\log}{(g}}{f}$$

$$= \frac{Ax}{f^3} + \frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2f^4(g+fx)^2} - \frac{3g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^4(g+fx)} - \frac{3g(A+B}{}$$

$$= \frac{Ax}{f^3} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^3} + \frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2f^4(g+fx)^2} - \frac{3g^2(A+B\,l}{f^4(g}$$

$$= \frac{Ax}{f^3} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^3} + \frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2f^4(g+fx)^2} - \frac{3g^2(A+B\,l}{f^4(g}$$

$$= \frac{Ax}{f^3} + \frac{B(bc-ad)g^3n}{2f^3(af-bg)(cf-dg)(g+fx)} - \frac{b^2Bg^3n\log(a+bx)}{2f^4(af-bg)^2} - \frac{3bBg^2n\log(}{f^4(af-}$$

$$= \frac{Ax}{f^3} + \frac{B(bc-ad)g^3n}{2f^3(af-bg)(cf-dg)(g+fx)} - \frac{b^2Bg^3n\log(a+bx)}{2f^4(af-bg)^2} - \frac{3bBg^2n\log(}{f^4(af-}$$

$$= \frac{Ax}{f^3} + \frac{B(bc-ad)g^3n}{2f^3(af-bg)(cf-dg)(g+fx)} - \frac{b^2Bg^3n\log(a+bx)}{2f^4(af-bg)^2} - \frac{3bBg^2n\log(}{f^4(af-}$$

<u>Mathematica</u> [A]
time = 0.81, size = 470, normalized size = 0.89

$$\frac{2Afx + \frac{2Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} + \frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(g+fx)^2} - \frac{6g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g+fx} - \frac{2B(bc-ad)fn\log(c+dx)}{bd} - 6g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(g+fx) + \frac{6Bg^2n(b(-cf+dg)\log(a+bx)+d(af-bg)\log(c+dx)+(bc-ad)f\log(g+fx))}{(af-bg)(cf-dg)} + B(bc-ad)g^3n\left(-\frac{b^2\log(a+bx)}{(bc-ad)(af-bg)^2} + \frac{\frac{d^2\log(c+dx)}{bc-ad} + \frac{f\left(\frac{(af-bg)(cf-dg)}{g+fx}+(bcf+adf-2bdg)\log(g+fx)\right)}{(af-bg)^2}}{(cf-dg)^2}\right) + 6Bgn\left(\left(\log\left(\frac{f(a+bx)}{af-bg}\right) - \log\left(\frac{f(c+dx)}{cf-dg}\right)\right)\log(g+fx) + \text{Li}_2\left(\frac{b(g+fx)}{-af+bg}\right) - \text{Li}_2\left(\frac{d(g+fx)}{-cf+dg}\right)\right)}{2f^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g/x)^3,x]
```

```
[Out] (2*A*f*x + (2*B*f*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b + (g^3*(A + B
*Log[e*((a + b*x)/(c + d*x))^n]))/(g + f*x)^2 - (6*g^2*(A + B*Log[e*((a + b
*x)/(c + d*x))^n]))/(g + f*x) - (2*B*(b*c - a*d)*f*n*Log[c + d*x])/(b*d) -
6*g*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[g + f*x] + (6*B*g^2*n*(b*(-(
c*f) + d*g)*Log[a + b*x] + d*(a*f - b*g)*Log[c + d*x] + (b*c - a*d)*f*Log[g
 + f*x]))/((a*f - b*g)*(c*f - d*g)) + B*(b*c - a*d)*g^3*n*(-((b^2*Log[a + b
*x])/((b*c - a*d)*(a*f - b*g)^2)) + ((d^2*Log[c + d*x])/(b*c - a*d) + (f*((
(a*f - b*g)*(c*f - d*g))/(g + f*x) + (b*c*f + a*d*f - 2*b*d*g)*Log[g + f*x]
))/(a*f - b*g)^2)/(c*f - d*g)^2) + 6*B*g*n*((Log[(f*(a + b*x))/(a*f - b*g)]
```

```
 - Log[(f*(c + d*x))/(c*f - d*g)])*Log[g + f*x] + PolyLog[2, (b*(g + f*x))/
(-(a*f) + b*g)] - PolyLog[2, (d*(g + f*x))/(-(c*f) + d*g)]))/(2*f^4)
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{A + B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{\left(f + \frac{g}{x}\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^3,x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*A*((6*f*g^2*x + 5*g^3)/(f^6*x^2 + 2*f^5*g*x + f^4*g^2) - 2*x/f^3 + 6*g
*log(f*x + g)/f^4) - B*integrate(-(x^3*log((b*x + a)^n) - x^3*log((d*x + c)
^n) + x^3)/(f^3*x^3 + 3*f^2*g*x^2 + 3*f*g^2*x + g^3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^3,x, algorithm="fricas")
```

```
[Out] integral((B*x^3*log(((b*x + a)/(d*x + c))^n*e) + A*x^3)/(f^3*x^3 + 3*f^2*g*
x^2 + 3*f*g^2*x + g^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(f+g/x)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^3,x, algorithm="giac")
[Out] integrate((B*log(((b*x + a)/(d*x + c))^n*e) + A)/(f + g/x)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{{\left(f+\frac{g}{x}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g/x)^3,x)
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g/x)^3, x)
```

3.7 $\int (a+bx)^4 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx$

Optimal. Leaf size=201

$$-\frac{(bc-ad)^4qrx}{5d^4}+\frac{(bc-ad)^3qr(a+bx)^2}{10bd^3}-\frac{(bc-ad)^2qr(a+bx)^3}{15bd^2}+\frac{(bc-ad)qr(a+bx)^4}{20bd}-\frac{pr(a+bx)^5}{25b}-\frac{qr(a+}{2}$$

```
[Out] -1/5*(-a*d+b*c)^4*q*r*x/d^4+1/10*(-a*d+b*c)^3*q*r*(b*x+a)^2/b/d^3-1/15*(-a*
d+b*c)^2*q*r*(b*x+a)^3/b/d^2+1/20*(-a*d+b*c)*q*r*(b*x+a)^4/b/d-1/25*p*r*(b*
x+a)^5/b-1/25*q*r*(b*x+a)^5/b+1/5*(-a*d+b*c)^5*q*r*ln(d*x+c)/b/d^5+1/5*(b*x
+a)^5*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b
```

Rubi [A]
time = 0.08, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2581, 32, 45}

$$\frac{qr(bc-ad)^5\log(c+dx)}{5bd^5}-\frac{qrx(bc-ad)^4}{5d^4}+\frac{qr(a+bx)^2(bc-ad)^3}{10bd^3}-\frac{qr(a+bx)^3(bc-ad)^2}{15bd^2}+\frac{(a+bx)^5\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5b}+\frac{qr(a+bx)^4(bc-ad)}{20bd}-\frac{pr(a+bx)^5}{25b}-\frac{qr(a+bx)^5}{25b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -1/5*((b*c - a*d)^4*q*r*x)/d^4 + ((b*c - a*d)^3*q*r*(a + b*x)^2)/(10*b*d^3)
 - ((b*c - a*d)^2*q*r*(a + b*x)^3)/(15*b*d^2) + ((b*c - a*d)*q*r*(a + b*x)^
4)/(20*b*d) - (p*r*(a + b*x)^5)/(25*b) - (q*r*(a + b*x)^5)/(25*b) + ((b*c -
 a*d)^5*q*r*Log[c + d*x])/(5*b*d^5) + ((a + b*x)^5*Log[e*(f*(a + b*x)^p*(c
+ d*x)^q)^r])/(5*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
```

```
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^4 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx = \frac{(a+bx)^5 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5b} - \frac{1}{5}(pr) \int (a+bx)^4$$

$$= -\frac{pr(a+bx)^5}{25b} + \frac{(a+bx)^5 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5b} -$$

$$= -\frac{(bc-ad)^4 qrx}{5d^4} + \frac{(bc-ad)^3 qr(a+bx)^2}{10bd^3} - \frac{(bc-ad)^2 qr(a}{15bd^2}$$

Mathematica [A]
time = 0.22, size = 185, normalized size = 0.92

$$\frac{-\frac{r\left(60bd(bc-ad)^4(p+5q)x-60b^2(bc-ad)^3(2p+5q)(c+dx)^2+40b^3(bc-ad)^2(3p+5q)(c+dx)^3-15b^4(bc-ad)(4p+5q)(c+dx)^4+12b^5(p+q)(c+dx)^5-60(bc-ad)^5q\log(c+dx)\right)}{60d^5}+(a+bx)^5\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] (-1/60*(r*(60*b*d*(b*c - a*d)^4*(p + 5*q)*x - 60*b^2*(b*c - a*d)^3*(2*p + 5
*q)*(c + d*x)^2 + 40*b^3*(b*c - a*d)^2*(3*p + 5*q)*(c + d*x)^3 - 15*b^4*(b*
c - a*d)*(4*p + 5*q)*(c + d*x)^4 + 12*b^5*(p + q)*(c + d*x)^5 - 60*(b*c - a
*d)^5*q*Log[c + d*x]))/d^5 + (a + b*x)^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^
r])/(5*b)
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int (bx+a)^4 \ln \left(e(f(bx+a)^p (dx+c)^q)^r\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

```
[Out] int((b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(186) = 372$.
time = 0.30, size = 396, normalized size = 1.97

$$\frac{1}{5}\left(b^4x^5+5\,ab^3x^4+10\,a^2b^2x^3+10\,a^3bx^2+5\,a^4x\right)\log\left(\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)+\frac{\left(\frac{60\,a^5 fp\log(bx+a)}{b}-\frac{12\,b^4d^4f(p+q)x^5+15\,\left(ab^3d^4f(4\,p+5\,q)-b^4cd^3fq\right)x^4+20\,\left(2\,a^2b^2d^4f(3\,p+5\,q)+b^4c^2d^2fq-5\,ab^3cd^3fq\right)x^3+30\,\left(2\,a^3bd^4f(2\,p+5\,q)-b^4c^3dfq+5\,ab^3c^2d^2fq-10\,a^2b^2cd^3fq\right)x^2+60\,\left(a^4d^4f(p+5\,q)+b^4c^4fq-5\,ab^3c^3dfq+10\,a^2b^2c^2d^2fq-10\,a^3bcd^3fq\right)x}{d^4}+\frac{60\,\left(b^4c^5fq-5\,ab^3c^4dfq+10\,a^2b^2c^3d^2fq-10\,a^3bc^2d^3fq+5\,a^4cd^4fq\right)\log(dx+c)}{d^5}\right)r}{300\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] 1/5*(b^4*x^5 + 5*a*b^3*x^4 + 10*a^2*b^2*x^3 + 10*a^3*b*x^2 + 5*a^4*x)*log((
(b*x + a)^p*(d*x + c)^q*f)^r*e) + 1/300*(60*a^5*f*p*log(b*x + a)/b - (12*b^
4*d^4*f*(p + q)*x^5 + 15*(a*b^3*d^4*f*(4*p + 5*q) - b^4*c*d^3*f*q)*x^4 + 20
*(2*a^2*b^2*d^4*f*(3*p + 5*q) + b^4*c^2*d^2*f*q - 5*a*b^3*c*d^3*f*q)*x^3 +
30*(2*a^3*b*d^4*f*(2*p + 5*q) - b^4*c^3*d*f*q + 5*a*b^3*c^2*d^2*f*q - 10*a^
2*b^2*c*d^3*f*q)*x^2 + 60*(a^4*d^4*f*(p + 5*q) + b^4*c^4*f*q - 5*a*b^3*c^3*
d*f*q + 10*a^2*b^2*c^2*d^2*f*q - 10*a^3*b*c*d^3*f*q)*x)/d^4 + 60*(b^4*c^5*f
*q - 5*a*b^3*c^4*d*f*q + 10*a^2*b^2*c^3*d^2*f*q - 10*a^3*b*c^2*d^3*f*q + 5*
a^4*c*d^4*f*q)*log(d*x + c)/d^5)*r/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 622 vs. $2(186) = 372$.
time = 0.41, size = 622, normalized size = 3.09

$$\frac{12\,(5\,b^5d^5-(b^5d^5p+b^5d^5q)r)x^5+15\,(20\,ab^4d^5-(4\,ab^4d^5p-(b^5cd^4-5\,ab^4d^5)q)r)x^4+20\,(30\,a^2b^3d^5-(6\,a^2b^3d^5p+(b^5c^2d^3-5\,ab^4cd^4+10\,a^2b^3d^5)q)r)x^3+30\,(20\,a^3b^2d^5-(4\,a^3b^2d^5p-(b^5c^3d^2-5\,ab^4c^2d^3+10\,a^2b^3cd^4-10\,a^3b^2d^5)q)r)x^2+60\,(5\,a^4bd^5-(a^4bd^5p+(b^5c^4d-5\,ab^4c^3d^2+10\,a^2b^3c^2d^3-10\,a^3b^2cd^4+5\,a^4bd^5)q)r)x+60\,(b^5d^5prx^5+5\,ab^4d^5prx^4+10\,a^2b^3d^5prx^3+10\,a^3b^2d^5prx^2+5\,a^4bd^5prx+a^5d^5pr)\log\left(bx+a\right)+60\,(b^5d^5qrx^5+5\,ab^4d^5qrx^4+10\,a^2b^3d^5qrx^3+10\,a^3b^2d^5qrx^2+5\,a^4bd^5qrx+(b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4)qr)\log\left(dx+c\right)+60\,(b^5d^5rx^5+5\,ab^4d^5rx^4+10\,a^2b^3d^5rx^3+10\,a^3b^2d^5rx^2+5\,a^4bd^5rx)\log\left(f\right)}{300\,bd^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] 1/300*(12*(5*b^5*d^5 - (b^5*d^5*p + b^5*d^5*q)*r)*x^5 + 15*(20*a*b^4*d^5 -
(4*a*b^4*d^5*p - (b^5*c*d^4 - 5*a*b^4*d^5)*q)*r)*x^4 + 20*(30*a^2*b^3*d^5 -
 (6*a^2*b^3*d^5*p + (b^5*c^2*d^3 - 5*a*b^4*c*d^4 + 10*a^2*b^3*d^5)*q)*r)*x^
3 + 30*(20*a^3*b^2*d^5 - (4*a^3*b^2*d^5*p - (b^5*c^3*d^2 - 5*a*b^4*c^2*d^3
+ 10*a^2*b^3*c*d^4 - 10*a^3*b^2*d^5)*q)*r)*x^2 + 60*(5*a^4*b*d^5 - (a^4*b*d
^5*p + (b^5*c^4*d - 5*a*b^4*c^3*d^2 + 10*a^2*b^3*c^2*d^3 - 10*a^3*b^2*c*d^4
 + 5*a^4*b*d^5)*q)*r)*x + 60*(b^5*d^5*p*r*x^5 + 5*a*b^4*d^5*p*r*x^4 + 10*a^
2*b^3*d^5*p*r*x^3 + 10*a^3*b^2*d^5*p*r*x^2 + 5*a^4*b*d^5*p*r*x + a^5*d^5*p*
r)*log(b*x + a) + 60*(b^5*d^5*q*r*x^5 + 5*a*b^4*d^5*q*r*x^4 + 10*a^2*b^3*d^
5*q*r*x^3 + 10*a^3*b^2*d^5*q*r*x^2 + 5*a^4*b*d^5*q*r*x + (b^5*c^5 - 5*a*b^4
*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4)*q*r)*log(
d*x + c) + 60*(b^5*d^5*r*x^5 + 5*a*b^4*d^5*r*x^4 + 10*a^2*b^3*d^5*r*x^3 + 1
0*a^3*b^2*d^5*r*x^2 + 5*a^4*b*d^5*r*x)*log(f))/(b*d^5)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 560 vs. $2(186) = 372$.

time = 175.99, size = 560, normalized size = 2.79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] 1/5*a^5*p*r*log(b*x + a)/b - 1/25*(b^4*p*r + b^4*q*r - 5*b^4*r*log(f) - 5*b
^4)*x^5 - 1/20*(4*a*b^3*d*p*r - b^4*c*q*r + 5*a*b^3*d*q*r - 20*a*b^3*d*r*lo
g(f) - 20*a*b^3*d)*x^4/d - 1/15*(6*a^2*b^2*d^2*p*r + b^4*c^2*q*r - 5*a*b^3*
c*d*q*r + 10*a^2*b^2*d^2*q*r - 30*a^2*b^2*d^2*r*log(f) - 30*a^2*b^2*d^2)*x^
3/d^2 + 1/5*(b^4*p*r*x^5 + 5*a*b^3*p*r*x^4 + 10*a^2*b^2*p*r*x^3 + 10*a^3*b*
p*r*x^2 + 5*a^4*p*r*x)*log(b*x + a) + 1/5*(b^4*q*r*x^5 + 5*a*b^3*q*r*x^4 +
10*a^2*b^2*q*r*x^3 + 10*a^3*b*q*r*x^2 + 5*a^4*q*r*x)*log(d*x + c) - 1/10*(4
*a^3*b*d^3*p*r - b^4*c^3*q*r + 5*a*b^3*c^2*d*q*r - 10*a^2*b^2*c*d^2*q*r + 1
0*a^3*b*d^3*q*r - 20*a^3*b*d^3*r*log(f) - 20*a^3*b*d^3)*x^2/d^3 - 1/5*(a^4*
d^4*p*r + b^4*c^4*q*r - 5*a*b^3*c^3*d*q*r + 10*a^2*b^2*c^2*d^2*q*r - 10*a^3
*b*c*d^3*q*r + 5*a^4*d^4*q*r - 5*a^4*d^4*r*log(f) - 5*a^4*d^4)*x/d^4 + 1/5*
(b^4*c^5*q*r - 5*a*b^3*c^4*d*q*r + 10*a^2*b^2*c^3*d^2*q*r - 10*a^3*b*c^2*d^
3*q*r + 5*a^4*c*d^4*q*r)*log(-d*x - c)/d^5
```

Mupad [B]
time = 0.70, size = 886, normalized size = 4.41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(a + b*x)^4,x)
```

```
[Out] log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(a^4*x + (b^4*x^5)/5 + 2*a^3*b*x^2 + a
*b^3*x^4 + 2*a^2*b^2*x^3) - x^4*((b^3*r*(5*a*d*p + b*c*p + 6*a*d*q))/(20*d)
 - (b^3*r*(p + q)*(5*a*d + 5*b*c))/(100*d)) + x^3*((((b^3*r*(5*a*d*p + b*c*
p + 6*a*d*q))/(5*d) - (b^3*r*(p + q)*(5*a*d + 5*b*c))/(25*d))*(5*a*d + 5*b*
c))/(15*b*d) - (a*b^2*r*(2*a*d*p + b*c*p + 3*a*d*q))/(3*d) + (a*b^3*c*r*(p
+ q))/(15*d)) - x*((a^3*r*(a*d*p + 2*b*c*p + 3*a*d*q))/d - ((5*a*d + 5*b*c)
*(((5*a*d + 5*b*c)*((((b^3*r*(5*a*d*p + b*c*p + 6*a*d*q))/(5*d) - (b^3*r*(p
 + q)*(5*a*d + 5*b*c))/(25*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*r*(2*a*d*p
 + b*c*p + 3*a*d*q))/d + (a*b^3*c*r*(p + q))/(5*d)))/(5*b*d) - (a*c*((b^3*r
*(5*a*d*p + b*c*p + 6*a*d*q))/(5*d) - (b^3*r*(p + q)*(5*a*d + 5*b*c))/(25*d
)))/(b*d) + (2*a^2*b*r*(a*d*p + b*c*p + 2*a*d*q))/d))/(5*b*d) + (a*c*((((b^
3*r*(5*a*d*p + b*c*p + 6*a*d*q))/(5*d) - (b^3*r*(p + q)*(5*a*d + 5*b*c))/(2
5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*r*(2*a*d*p + b*c*p + 3*a*d*q))/d +
(a*b^3*c*r*(p + q))/(5*d)))/(b*d)) - x^2*(((5*a*d + 5*b*c)*((((b^3*r*(5*a*d
*p + b*c*p + 6*a*d*q))/(5*d) - (b^3*r*(p + q)*(5*a*d + 5*b*c))/(25*d))*(5*a
*d + 5*b*c))/(5*b*d) - (a*b^2*r*(2*a*d*p + b*c*p + 3*a*d*q))/d + (a*b^3*c*r
```

```
*(p + q))/(5*d)))/(10*b*d) - (a*c*((b^3*r*(5*a*d*p + b*c*p + 6*a*d*q))/(5*d
) - (b^3*r*(p + q)*(5*a*d + 5*b*c))/(25*d)))/(2*b*d) + (a^2*b*r*(a*d*p + b*
c*p + 2*a*d*q))/d) + (log(c + d*x)*((b^4*c^5*q*r)/5 + a^4*c*d^4*q*r + 2*a^2
*b^2*c^3*d^2*q*r - a*b^3*c^4*d*q*r - 2*a^3*b*c^2*d^3*q*r))/d^5 - (b^4*r*x^5
*(p + q))/25 + (a^5*p*r*log(a + b*x))/(5*b)
```

3.8 $\int (a+bx)^3 \log\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx$

Optimal. Leaf size=172

$$\frac{(bc-ad)^3qrx}{4d^3}-\frac{(bc-ad)^2qr(a+bx)^2}{8bd^2}+\frac{(bc-ad)qr(a+bx)^3}{12bd}-\frac{pr(a+bx)^4}{16b}-\frac{qr(a+bx)^4}{16b}-\frac{(bc-ad)^4qr\log}{4bd^4}$$

```
[Out] 1/4*(-a*d+b*c)^3*q*r*x/d^3-1/8*(-a*d+b*c)^2*q*r*(b*x+a)^2/b/d^2+1/12*(-a*d+
b*c)*q*r*(b*x+a)^3/b/d-1/16*p*r*(b*x+a)^4/b-1/16*q*r*(b*x+a)^4/b-1/4*(-a*d+
b*c)^4*q*r*ln(d*x+c)/b/d^4+1/4*(b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b
```

Rubi [A]
time = 0.05, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2581, 32, 45}

$$-\frac{qr(bc-ad)^4\log(c+dx)}{4bd^4}+\frac{qrx(bc-ad)^3}{4d^3}-\frac{qr(a+bx)^2(bc-ad)^2}{8bd^2}+\frac{(a+bx)^4\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b}+\frac{qr(a+bx)^3(bc-ad)}{12bd}-\frac{pr(a+bx)^4}{16b}-\frac{qr(a+bx)^4}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] ((b*c - a*d)^3*q*r*x)/(4*d^3) - ((b*c - a*d)^2*q*r*(a + b*x)^2)/(8*b*d^2) +
 ((b*c - a*d)*q*r*(a + b*x)^3)/(12*b*d) - (p*r*(a + b*x)^4)/(16*b) - (q*r*(
a + b*x)^4)/(16*b) - ((b*c - a*d)^4*q*r*Log[c + d*x])/(4*b*d^4) + ((a + b*x
)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^3 \log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \frac{(a+bx)^4\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b} - \frac{1}{4}(pr)\int (a+bx)^3\,$$

$$= -\frac{pr(a+bx)^4}{16b} + \frac{(a+bx)^4\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b} - $$

$$= \frac{(bc-ad)^3qrx}{4d^3} - \frac{(bc-ad)^2qr(a+bx)^2}{8bd^2} + \frac{(bc-ad)qr(a+bx}{12bd}$$

Mathematica [A]
time = 0.16, size = 154, normalized size = 0.90

$$\frac{\frac{r\left(12bd(bc-ad)^3(p+4q)x-18b^2(bc-ad)^2(p+2q)(c+dx)^2+4b^3(bc-ad)(3p+4q)(c+dx)^3-3b^4(p+q)(c+dx)^4-12(bc-ad)^4q\log(c+dx)\right)}{12d^4}+(a+bx)^4\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] ((r*(12*b*d*(b*c - a*d)^3*(p + 4*q)*x - 18*b^2*(b*c - a*d)^2*(p + 2*q)*(c +
 d*x)^2 + 4*b^3*(b*c - a*d)*(3*p + 4*q)*(c + d*x)^3 - 3*b^4*(p + q)*(c + d*
x)^4 - 12*(b*c - a*d)^4*q*Log[c + d*x]))/(12*d^4) + (a + b*x)^4*Log[e*(f*(a
 + b*x)^p*(c + d*x)^q)^r])/(4*b)
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int (bx+a)^3 \ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

```
[Out] int((b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

Maxima [A]
time = 0.31, size = 286, normalized size = 1.66

$$\frac{1}{4}\left(b^3x^4+4\,ab^2x^3+6\,a^2bx^2+4\,a^3x\right)\log\left(\left(((bx+a)^p(dx+c)^qf\right)^re\right)+\frac{\left(\frac{12\,a^4fp\log(bx+a)}{b}-\frac{3\,b^3d^3f(p+q)x^4+4\,(ab^2d^3f(3\,p+4\,q)-b^3cd^2fq)x^3+6\,(3\,a^2bd^3f(p+2\,q)+b^3c^2dfq-4\,ab^2cd^2fq)x^2+12\,(a^3d^3f(p+4\,q)-b^3c^3fq+4\,ab^2c^2dfq-6\,a^2bcd^2fq)x}{d^3}-\frac{12\,(b^3c^4fq-4\,ab^2c^3dfq+6\,a^2bc^2d^2fq-4\,a^3cd^3fq)\log(dx+c)}{d^4}\right)r}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] 1/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)*log(((b*x + a)^p*(d*x +
 c)^q*f)^r*e) + 1/48*(12*a^4*f*p*log(b*x + a)/b - (3*b^3*d^3*f*(p + q)*x^4
```

```
+ 4*(a*b^2*d^3*f*(3*p + 4*q) - b^3*c*d^2*f*q)*x^3 + 6*(3*a^2*b*d^3*f*(p + 2
*q) + b^3*c^2*d*f*q - 4*a*b^2*c*d^2*f*q)*x^2 + 12*(a^3*d^3*f*(p + 4*q) - b^
3*c^3*f*q + 4*a*b^2*c^2*d*f*q - 6*a^2*b*c*d^2*f*q)*x)/d^3 - 12*(b^3*c^4*f*q
 - 4*a*b^2*c^3*d*f*q + 6*a^2*b*c^2*d^2*f*q - 4*a^3*c*d^3*f*q)*log(d*x + c)/
d^4)*r/f
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 467 vs. $2(159) = 318$.
time = 0.38, size = 467, normalized size = 2.72

$$\frac{3\,{\left(4\,b^{4}d^{4}-{\left(b^{4}d^{4}p+b^{4}d^{4}q\right)}r\right)}x^{4}+4\,{\left(12\,ab^{3}d^{4}-{\left(3\,ab^{3}d^{4}p-{\left(b^{4}cd^{3}-4\,ab^{3}d^{4}\right)}q\right)}r\right)}x^{3}+6\,{\left(12\,a^{2}b^{2}d^{4}-{\left(3\,a^{2}b^{2}d^{4}p+{\left(b^{4}c^{2}d^{2}-4\,ab^{3}cd^{3}+6\,a^{2}b^{2}d^{4}\right)}q\right)}r\right)}x^{2}+12\,{\left(4\,a^{3}bd^{4}-{\left(a^{3}bd^{4}p-{\left(b^{4}c^{3}d-4\,ab^{3}c^{2}d^{2}+6\,a^{2}b^{2}cd^{3}-4\,a^{3}bd^{4}\right)}q\right)}r\right)}x+12\,{\left(b^{4}d^{4}prx^{4}+4\,ab^{3}d^{4}prx^{3}+6\,a^{2}b^{2}d^{4}prx^{2}+4\,a^{3}bd^{4}prx+a^{4}d^{4}pr\right)}\log\left(bx+a\right)+12\,{\left(b^{4}d^{4}qrx^{4}+4\,ab^{3}d^{4}qrx^{3}+6\,a^{2}b^{2}d^{4}qrx^{2}+4\,a^{3}bd^{4}qrx-{\left(b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}\right)}qr\right)}\log\left(dx+c\right)+12\,{\left(b^{4}d^{4}rx^{4}+4\,ab^{3}d^{4}rx^{3}+6\,a^{2}b^{2}d^{4}rx^{2}+4\,a^{3}bd^{4}rx\right)}\log\left(f\right)}{48\,bd^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] 1/48*(3*(4*b^4*d^4 - (b^4*d^4*p + b^4*d^4*q)*r)*x^4 + 4*(12*a*b^3*d^4 - (3*
a*b^3*d^4*p - (b^4*c*d^3 - 4*a*b^3*d^4)*q)*r)*x^3 + 6*(12*a^2*b^2*d^4 - (3*
a^2*b^2*d^4*p + (b^4*c^2*d^2 - 4*a*b^3*c*d^3 + 6*a^2*b^2*d^4)*q)*r)*x^2 + 1
2*(4*a^3*b*d^4 - (a^3*b*d^4*p - (b^4*c^3*d - 4*a*b^3*c^2*d^2 + 6*a^2*b^2*c*
d^3 - 4*a^3*b*d^4)*q)*r)*x + 12*(b^4*d^4*p*r*x^4 + 4*a*b^3*d^4*p*r*x^3 + 6*
a^2*b^2*d^4*p*r*x^2 + 4*a^3*b*d^4*p*r*x + a^4*d^4*p*r)*log(b*x + a) + 12*(b
^4*d^4*q*r*x^4 + 4*a*b^3*d^4*q*r*x^3 + 6*a^2*b^2*d^4*q*r*x^2 + 4*a^3*b*d^4*
q*r*x - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3)*q*r)*
log(d*x + c) + 12*(b^4*d^4*r*x^4 + 4*a*b^3*d^4*r*x^3 + 6*a^2*b^2*d^4*r*x^2
+ 4*a^3*b*d^4*r*x)*log(f))/(b*d^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(159) = 318$.
time = 32.76, size = 407, normalized size = 2.37

$$\frac{a^{4}pr\log\left(bx+a\right)}{4\,b}-\frac{1}{16}\,{\left(b^{3}pr+b^{3}qr-4\,b^{3}r\log\left(f\right)-4\,b^{3}\right)}x^{4}-\frac{{\left(3\,ab^{2}dpr-b^{3}cqr+4\,ab^{2}dqr-12\,ab^{2}dr\log\left(f\right)-12\,ab^{2}d\right)}x^{3}}{12\,d}+\frac{1}{4}\,{\left(b^{3}prx^{4}+4\,ab^{2}prx^{3}+6\,a^{2}bprx^{2}+4\,a^{3}prx\right)}\log\left(bx+a\right)+\frac{1}{4}\,{\left(b^{3}qrx^{4}+4\,ab^{2}qrx^{3}+6\,a^{2}bqrx^{2}+4\,a^{3}qrx\right)}\log\left(dx+c\right)-\frac{{\left(3\,a^{2}bd^{2}pr+b^{3}c^{2}qr-4\,ab^{2}cdqr+6\,a^{2}bd^{2}qr-12\,a^{2}bd^{2}r\log\left(f\right)-12\,a^{2}bd^{2}\right)}x^{2}}{8\,d^{2}}-\frac{{\left(a^{3}d^{3}pr-b^{3}c^{3}qr+4\,ab^{2}c^{2}dqr-6\,a^{2}bcd^{2}qr+4\,a^{3}d^{3}qr-4\,a^{3}d^{3}r\log\left(f\right)-4\,a^{3}d^{3}\right)}x}{4\,d^{3}}-\frac{{\left(b^{3}c^{4}qr-4\,ab^{2}c^{3}dqr+6\,a^{2}bc^{2}d^{2}qr-4\,a^{3}cd^{3}qr\right)}\log\left(-dx-c\right)}{4\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] 1/4*a^4*p*r*log(b*x + a)/b - 1/16*(b^3*p*r + b^3*q*r - 4*b^3*r*log(f) - 4*b
^3)*x^4 - 1/12*(3*a*b^2*d*p*r - b^3*c*q*r + 4*a*b^2*d*q*r - 12*a*b^2*d*r*lo
```

```
g(f) - 12*a*b^2*d)*x^3/d + 1/4*(b^3*p*r*x^4 + 4*a*b^2*p*r*x^3 + 6*a^2*b*p*r
*x^2 + 4*a^3*p*r*x)*log(b*x + a) + 1/4*(b^3*q*r*x^4 + 4*a*b^2*q*r*x^3 + 6*a
^2*b*q*r*x^2 + 4*a^3*q*r*x)*log(d*x + c) - 1/8*(3*a^2*b*d^2*p*r + b^3*c^2*q
*r - 4*a*b^2*c*d*q*r + 6*a^2*b*d^2*q*r - 12*a^2*b*d^2*r*log(f) - 12*a^2*b*d
^2)*x^2/d^2 - 1/4*(a^3*d^3*p*r - b^3*c^3*q*r + 4*a*b^2*c^2*d*q*r - 6*a^2*b*
c*d^2*q*r + 4*a^3*d^3*q*r - 4*a^3*d^3*r*log(f) - 4*a^3*d^3)*x/d^3 - 1/4*(b^
3*c^4*q*r - 4*a*b^2*c^3*d*q*r + 6*a^2*b*c^2*d^2*q*r - 4*a^3*c*d^3*q*r)*log(
-d*x - c)/d^4
```

Mupad [B]
time = 0.53, size = 501, normalized size = 2.91

$$x^2\left(\frac{\left(\frac{b^2r(4adp+bcp+5adq)}{4d}-\frac{b^2r(p+q)(4ad+4bc)}{16d}\right)(4ad+4bc)}{8bd}-\frac{abr(3adp+2bcp+5adq)}{4d}+\frac{ab^2cr(p+q)}{8d}\right)-x^3\left(\frac{b^2r(4adp+bcp+5adq)}{12d}-\frac{b^2r(p+q)(4ad+4bc)}{48d}\right)+\ln\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\left(a^3x+\frac{3a^2bx^2}{2}+ab^2x^3+\frac{b^3x^4}{4}\right)-x\left(\frac{(4ad+4bc)\left(\frac{\left(\frac{b^2r(4adp+bcp+5adq)}{4d}-\frac{b^2r(p+q)(4ad+4bc)}{16d}\right)(4ad+4bc)}{4bd}-\frac{abr(3adp+2bcp+5adq)}{2d}+\frac{ab^2cr(p+q)}{4d}\right)}{4bd}+\frac{a^2r(2adp+3bcp+5adq)}{2d}-\frac{ac\left(\frac{b^2r(4adp+bcp+5adq)}{4d}-\frac{b^2r(p+q)(4ad+4bc)}{16d}\right)}{bd}\right)-\frac{\ln(c+dx)\left(-4qra^3cd^3+6qra^2bc^2d^2-4qrab^2c^3d+qrb^3c^4\right)}{4d^4}-\frac{b^3rx^4(p+q)}{16}+\frac{a^4pr\ln(a+bx)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(a + b*x)^3,x)
```

```
[Out] x^2*((((b^2*r*(4*a*d*p + b*c*p + 5*a*d*q))/(4*d) - (b^2*r*(p + q)*(4*a*d +
4*b*c))/(16*d))*(4*a*d + 4*b*c))/(8*b*d) - (a*b*r*(3*a*d*p + 2*b*c*p + 5*a*
d*q))/(4*d) + (a*b^2*c*r*(p + q))/(8*d)) - x^3*((b^2*r*(4*a*d*p + b*c*p + 5
*a*d*q))/(12*d) - (b^2*r*(p + q)*(4*a*d + 4*b*c))/(48*d)) + log(e*(f*(a + b
*x)^p*(c + d*x)^q)^r)*(a^3*x + (b^3*x^4)/4 + (3*a^2*b*x^2)/2 + a*b^2*x^3) -
 x*(((4*a*d + 4*b*c)*((((b^2*r*(4*a*d*p + b*c*p + 5*a*d*q))/(4*d) - (b^2*r*
(p + q)*(4*a*d + 4*b*c))/(16*d))*(4*a*d + 4*b*c))/(4*b*d) - (a*b*r*(3*a*d*p
 + 2*b*c*p + 5*a*d*q))/(2*d) + (a*b^2*c*r*(p + q))/(4*d)))/(4*b*d) + (a^2*r
*(2*a*d*p + 3*b*c*p + 5*a*d*q))/(2*d) - (a*c*((b^2*r*(4*a*d*p + b*c*p + 5*a
*d*q))/(4*d) - (b^2*r*(p + q)*(4*a*d + 4*b*c))/(16*d)))/(b*d)) - (log(c + d
*x)*(b^3*c^4*q*r - 4*a^3*c*d^3*q*r - 4*a*b^2*c^3*d*q*r + 6*a^2*b*c^2*d^2*q*
r))/(4*d^4) - (b^3*r*x^4*(p + q))/16 + (a^4*p*r*log(a + b*x))/(4*b)
```

3.9 $\int (a+bx)^2 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx$

Optimal. Leaf size=143

$$-\frac{(bc-ad)^2qrx}{3d^2}+\frac{(bc-ad)qr(a+bx)^2}{6bd}-\frac{pr(a+bx)^3}{9b}-\frac{qr(a+bx)^3}{9b}+\frac{(bc-ad)^3qr\log(c+dx)}{3bd^3}+(a+bx)^3\,\mathrm{lo}$$

```
[Out] -1/3*(-a*d+b*c)^2*q*r*x/d^2+1/6*(-a*d+b*c)*q*r*(b*x+a)^2/b/d-1/9*p*r*(b*x+a
)^3/b-1/9*q*r*(b*x+a)^3/b+1/3*(-a*d+b*c)^3*q*r*ln(d*x+c)/b/d^3+1/3*(b*x+a)^
3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b
```

Rubi [A]
time = 0.04, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2581, 32, 45}

$$\frac{qr(bc-ad)^3\log(c+dx)}{3bd^3}-\frac{qrx(bc-ad)^2}{3d^2}+\frac{(a+bx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b}+\frac{qr(a+bx)^2(bc-ad)}{6bd}-\frac{pr(a+bx)^3}{9b}-\frac{qr(a+bx)^3}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -1/3*((b*c - a*d)^2*q*r*x)/d^2 + ((b*c - a*d)*q*r*(a + b*x)^2)/(6*b*d) - (p
*r*(a + b*x)^3)/(9*b) - (q*r*(a + b*x)^3)/(9*b) + ((b*c - a*d)^3*q*r*Log[c
+ d*x])/(3*b*d^3) + ((a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*b
)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int (a+bx)^2 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx &= \frac{(a+bx)^3 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b} - \frac{1}{3}(pr)\int (a+bx)^2 \\
&= -\frac{pr(a+bx)^3}{9b} + \frac{(a+bx)^3 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b} - \\
&= -\frac{(bc-ad)^2 qrx}{3d^2} + \frac{(bc-ad)qr(a+bx)^2}{6bd} - \frac{pr(a+bx)^3}{9b} -
\end{aligned}
$$

Mathematica [**A**]
time = 0.10, size = 127, normalized size = 0.89

$$
\frac{-\frac{r\left(6bd(bc-ad)^2(p+3q)x-3b^2(bc-ad)(2p+3q)(c+dx)^2+2b^3(p+q)(c+dx)^3-6(bc-ad)^3q\log(c+dx)\right)}{6d^3}+(a+bx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] (-1/6*(r*(6*b*d*(b*c - a*d)^2*(p + 3*q)*x - 3*b^2*(b*c - a*d)*(2*p + 3*q)*(
c + d*x)^2 + 2*b^3*(p + q)*(c + d*x)^3 - 6*(b*c - a*d)^3*q*Log[c + d*x]))/d
^3 + (a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*b)
```

Maple [**F**]
time = 0.18, size = 0, normalized size = 0.00

$$
\int (bx+a)^2 \ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

```
[Out] int((b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

Maxima [**A**]
time = 0.30, size = 195, normalized size = 1.36

$$
\frac{1}{3}\left(b^2x^3+3\,abx^2+3\,a^2x\right)\log\left(\left(\left(bx+a\right)^p\left(dx+c\right)^q f\right)^r e\right)+\frac{\left(\frac{6\,a^3fp\log(bx+a)}{b}-\frac{2\,b^2d^2f(p+q)x^3+3\,\left(abd^2f(2\,p+3\,q)-b^2cdfq\right)x^2+6\,\left(a^2d^2f(p+3\,q)+b^2c^2fq-3\,abcdfq\right)x}{d^2}+\frac{6\,\left(b^2c^3fq-3\,abc^2dfq+3\,a^2cd^2fq\right)\log(dx+c)}{d^3}\right)r}{18\,f}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e) +
1/18*(6*a^3*f*p*log(b*x + a)/b - (2*b^2*d^2*f*(p + q)*x^3 + 3*(a*b*d^2*f*(2
```

```
*p + 3*q) - b^2*c*d*f*q)*x^2 + 6*(a^2*d^2*f*(p + 3*q) + b^2*c^2*f*q - 3*a*b
*c*d*f*q)*x)/d^2 + 6*(b^2*c^3*f*q - 3*a*b*c^2*d*f*q + 3*a^2*c*d^2*f*q)*log(
d*x + c)/d^3)*r/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(132) = 264$.
time = 0.38, size = 323, normalized size = 2.26

$$\frac{2\,{\left(3\,b^{3} d^{3} - {\left(b^{3} d^{3} p + b^{3} d^{3} q\right)} r\right)} x^{3} + 3\,{\left(6\,a b^{2} d^{3} - {\left(2\,a b^{2} d^{3} p - {\left(b^{3} c d^{2} - 3\,a b^{2} d^{3}\right)} q\right)} r\right)} x^{2} + 6\,{\left(3\,a^{2} b d^{3} - {\left(a^{2} b d^{3} p + {\left(b^{3} c^{2} d - 3\,a b^{2} c d^{2} + 3\,a^{2} b d^{3}\right)} q\right)} r\right)} x + 6\,{\left(b^{3} d^{3} p r x^{3} + 3\,a b^{2} d^{3} p r x^{2} + 3\,a^{2} b d^{3} p r x + a^{3} d^{3} p r\right)} \log\left(b x + a\right) + 6\,{\left(b^{3} d^{3} q r x^{3} + 3\,a b^{2} d^{3} q r x^{2} + 3\,a^{2} b d^{3} q r x + {\left(b^{3} c^{3} - 3\,a b^{2} c^{2} d + 3\,a^{2} b c d^{2}\right)} q r\right)} \log\left(d x + c\right) + 6\,{\left(b^{3} d^{3} r x^{3} + 3\,a b^{2} d^{3} r x^{2} + 3\,a^{2} b d^{3} r x\right)} \log\left(f\right)}{18\,b d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] 1/18*(2*(3*b^3*d^3 - (b^3*d^3*p + b^3*d^3*q)*r)*x^3 + 3*(6*a*b^2*d^3 - (2*a
*b^2*d^3*p - (b^3*c*d^2 - 3*a*b^2*d^3)*q)*r)*x^2 + 6*(3*a^2*b*d^3 - (a^2*b*
d^3*p + (b^3*c^2*d - 3*a*b^2*c*d^2 + 3*a^2*b*d^3)*q)*r)*x + 6*(b^3*d^3*p*r*
x^3 + 3*a*b^2*d^3*p*r*x^2 + 3*a^2*b*d^3*p*r*x + a^3*d^3*p*r)*log(b*x + a) +
 6*(b^3*d^3*q*r*x^3 + 3*a*b^2*d^3*q*r*x^2 + 3*a^2*b*d^3*q*r*x + (b^3*c^3 -
3*a*b^2*c^2*d + 3*a^2*b*c*d^2)*q*r)*log(d*x + c) + 6*(b^3*d^3*r*x^3 + 3*a*b
^2*d^3*r*x^2 + 3*a^2*b*d^3*r*x)*log(f))/(b*d^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(124) = 248$.
time = 213.71, size = 488, normalized size = 3.41

$$\begin{cases} a^{2} x \log{\left(e \left(a^{p} c^{q} f\right)^{r} \right)} & \text{for}\: b = 0 \wedge d = 0 \\\frac{a^{3} \log{\left(e \left(c^{q} f \left(a + b x\right)^{p}\right)^{r} \right)}}{3 b} - \frac{a^{2} p r x}{3} + a^{2} x \log{\left(e \left(c^{q} f \left(a + b x\right)^{p}\right)^{r} \right)} - \frac{a b p r x^{2}}{3} + a b x^{2} \log{\left(e \left(c^{q} f \left(a + b x\right)^{p}\right)^{r} \right)} - \frac{b^{2} p r x^{3}}{9} + \frac{b^{2} x^{3} \log{\left(e \left(c^{q} f \left(a + b x\right)^{p}\right)^{r} \right)}}{3} & \text{for}\: d = 0 \\a^{2} \left(\frac{c \log{\left(e \left(a^{p} f \left(c + d x\right)^{q}\right)^{r} \right)}}{d} - q r x + x \log{\left(e \left(a^{p} f \left(c + d x\right)^{q}\right)^{r} \right)}\right) & \text{for}\: b = 0 \\- \frac{a^{3} q r \log{\left(\frac{c}{d} + x \right)}}{3 b} + \frac{a^{3} \log{\left(e \left(f \left(a + b x\right)^{p} \left(c + d x\right)^{q}\right)^{r} \right)}}{3 b} + \frac{a^{2} c q r \log{\left(\frac{c}{d} + x \right)}}{d} - \frac{a^{2} p r x}{3} - a^{2} q r x + a^{2} x \log{\left(e \left(f \left(a + b x\right)^{p} \left(c + d x\right)^{q}\right)^{r} \right)} - \frac{a b c^{2} q r \log{\left(\frac{c}{d} + x \right)}}{d^{2}} + \frac{a b c q r x}{d} - \frac{a b p r x^{2}}{3} - \frac{a b q r x^{2}}{2} + a b x^{2} \log{\left(e \left(f \left(a + b x\right)^{p} \left(c + d x\right)^{q}\right)^{r} \right)} + \frac{b^{2} c^{3} q r \log{\left(\frac{c}{d} + x \right)}}{3 d^{3}} - \frac{b^{2} c^{2} q r x}{3 d^{2}} + \frac{b^{2} c q r x^{2}}{6 d} - \frac{b^{2} p r x^{3}}{9} - \frac{b^{2} q r x^{3}}{9} + \frac{b^{2} x^{3} \log{\left(e \left(f \left(a + b x\right)^{p} \left(c + d x\right)^{q}\right)^{r} \right)}}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Piecewise((a**2*x*log(e*(a**p*c**q*f)**r), Eq(b, 0) & Eq(d, 0)), (a**3*log(
e*(c**q*f*(a + b*x)**p)**r)/(3*b) - a**2*p*r*x/3 + a**2*x*log(e*(c**q*f*(a
+ b*x)**p)**r) - a*b*p*r*x**2/3 + a*b*x**2*log(e*(c**q*f*(a + b*x)**p)**r)
- b**2*p*r*x**3/9 + b**2*x**3*log(e*(c**q*f*(a + b*x)**p)**r)/3, Eq(d, 0)),
 (a**2*(c*log(e*(a**p*f*(c + d*x)**q)**r)/d - q*r*x + x*log(e*(a**p*f*(c +
d*x)**q)**r)), Eq(b, 0)), (-a**3*q*r*log(c/d + x)/(3*b) + a**3*log(e*(f*(a
+ b*x)**p*(c + d*x)**q)**r)/(3*b) + a**2*c*q*r*log(c/d + x)/d - a**2*p*r*x/
3 - a**2*q*r*x + a**2*x*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r) - a*b*c**2*
q*r*log(c/d + x)/d**2 + a*b*c*q*r*x/d - a*b*p*r*x**2/3 - a*b*q*r*x**2/2 + a
*b*x**2*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r) + b**2*c**3*q*r*log(c/d + x
)/(3*d**3) - b**2*c**2*q*r*x/(3*d**2) + b**2*c*q*r*x**2/(6*d) - b**2*p*r*x*
*3/9 - b**2*q*r*x**3/9 + b**2*x**3*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)/
3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(132) = 264.
time = 7.76, size = 269, normalized size = 1.88

$$\frac{a^3pr\log\left(bx+a\right)}{3\,b}-\frac{1}{9}\left(b^2pr+b^2qr-3\,b^2r\log\left(f\right)-3\,b^2\right)x^3-\frac{\left(2\,abdpr-b^2cqr+3\,abdqr-6\,abdr\log\left(f\right)-6\,abd\right)x^2}{6\,d}+\frac{1}{3}\left(b^2prx^3+3\,abprx^2+3\,a^2prx\right)\log\left(bx+a\right)+\frac{1}{3}\left(b^2qrx^3+3\,abqrx^2+3\,a^2qrx\right)\log\left(dx+c\right)-\frac{\left(a^2d^2pr+b^2c^2qr-3\,abcdqr+3\,a^2d^2qr-3\,a^2d^2r\log\left(f\right)-3\,a^2d^2\right)x}{3\,d^2}+\frac{\left(b^2c^3qr-3\,abc^2dqr+3\,a^2cd^2qr\right)\log\left(-dx-c\right)}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] 1/3*a^3*p*r*log(b*x + a)/b - 1/9*(b^2*p*r + b^2*q*r - 3*b^2*r*log(f) - 3*b^
2)*x^3 - 1/6*(2*a*b*d*p*r - b^2*c*q*r + 3*a*b*d*q*r - 6*a*b*d*r*log(f) - 6*
a*b*d)*x^2/d + 1/3*(b^2*p*r*x^3 + 3*a*b*p*r*x^2 + 3*a^2*p*r*x)*log(b*x + a)
 + 1/3*(b^2*q*r*x^3 + 3*a*b*q*r*x^2 + 3*a^2*q*r*x)*log(d*x + c) - 1/3*(a^2*
d^2*p*r + b^2*c^2*q*r - 3*a*b*c*d*q*r + 3*a^2*d^2*q*r - 3*a^2*d^2*r*log(f)
- 3*a^2*d^2)*x/d^2 + 1/3*(b^2*c^3*q*r - 3*a*b*c^2*d*q*r + 3*a^2*c*d^2*q*r)*
log(-d*x - c)/d^3
```

Mupad [B]
time = 0.47, size = 255, normalized size = 1.78

$$x\left(\frac{\left(\frac{b\,r\,\left(3\,a\,d\,p+b\,c\,p+4\,a\,d\,q\right)}{3\,d}-\frac{b\,r\,\left(p+q\right)\,\left(3\,a\,d+3\,b\,c\right)}{9\,d}\right)\,\left(3\,a\,d+3\,b\,c\right)}{3\,b\,d}-\frac{a\,r\,\left(a\,d\,p+b\,c\,p+2\,a\,d\,q\right)}{d}+\frac{a\,b\,c\,r\,\left(p+q\right)}{3\,d}\right)-x^2\,\left(\frac{b\,r\,\left(3\,a\,d\,p+b\,c\,p+4\,a\,d\,q\right)}{6\,d}-\frac{b\,r\,\left(p+q\right)\,\left(3\,a\,d+3\,b\,c\right)}{18\,d}\right)+\ln\left(e\,{\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)}^r\right)\,\left(a^2\,x+a\,b\,x^2+\frac{b^2\,x^3}{3}\right)+\frac{\ln\left(c+d\,x\right)\,\left(3\,q\,r\,a^2\,c\,d^2-3\,q\,r\,a\,b\,c^2\,d+q\,r\,b^2\,c^3\right)}{3\,d^3}-\frac{b^2\,r\,x^3\,\left(p+q\right)}{9}+\frac{a^3\,p\,r\,\ln\left(a+b\,x\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(a + b*x)^2,x)
```

```
[Out] x*((((b*r*(3*a*d*p + b*c*p + 4*a*d*q))/(3*d) - (b*r*(p + q)*(3*a*d + 3*b*c)
)/(9*d))*(3*a*d + 3*b*c))/(3*b*d) - (a*r*(a*d*p + b*c*p + 2*a*d*q))/d + (a*
b*c*r*(p + q))/(3*d)) - x^2*((b*r*(3*a*d*p + b*c*p + 4*a*d*q))/(6*d) - (b*r
*(p + q)*(3*a*d + 3*b*c))/(18*d)) + log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(a
^2*x + (b^2*x^3)/3 + a*b*x^2) + (log(c + d*x)*(b^2*c^3*q*r + 3*a^2*c*d^2*q*
r - 3*a*b*c^2*d*q*r))/(3*d^3) - (b^2*r*x^3*(p + q))/9 + (a^3*p*r*log(a + b*
x))/(3*b)
```

3.10 $\int (a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx$

Optimal. Leaf size=116

$$-\frac{1}{2}aprx+\frac{(bc-ad)qrx}{2d}-\frac{1}{4}bprx^2-\frac{qr(a+bx)^2}{4b}-\frac{(bc-ad)^2qr\log(c+dx)}{2bd^2}+\frac{(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b}$$

```
[Out] -1/2*a*p*r*x+1/2*(-a*d+b*c)*q*r*x/d-1/4*b*p*r*x^2-1/4*q*r*(b*x+a)^2/b-1/2*(
-a*d+b*c)^2*q*r*ln(d*x+c)/b/d^2+1/2*(b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^
r)/b
```

Rubi [A]
time = 0.03, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2581, 45}

$$-\frac{qr(bc-ad)^2\log(c+dx)}{2bd^2}+\frac{(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b}+\frac{qrx(bc-ad)}{2d}-\frac{qr(a+bx)^2}{4b}-\frac{1}{2}aprx-\frac{1}{4}bprx^2$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -1/2*(a*p*r*x) + ((b*c - a*d)*q*r*x)/(2*d) - (b*p*r*x^2)/4 - (q*r*(a + b*x)
^2)/(4*b) - ((b*c - a*d)^2*q*r*Log[c + d*x])/(2*b*d^2) + ((a + b*x)^2*Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \frac{(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b} - \frac{1}{2}(pr)\int (a+bx)\,dx$$

$$= -\frac{1}{2}aprx - \frac{1}{4}bprx^2 + \frac{(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b}$$

$$= -\frac{1}{2}aprx + \frac{(bc-ad)qrx}{2d} - \frac{1}{4}bprx^2 - \frac{qr(a+bx)^2}{4b} - \frac{(bc-ad)}{}$$

Mathematica [A]
time = 0.14, size = 105, normalized size = 0.91

$$\frac{a^2pr\log(a+bx)}{2b} - \frac{2c(bc-2ad)qr\log(c+dx) + dx(r(-2bcq+2ad(p+2q)+bd(p+q)x) - 2d(2a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] (a^2*p*r*Log[a + b*x])/(2*b) - (2*c*(b*c - 2*a*d)*q*r*Log[c + d*x] + d*x*(r
*(-2*b*c*q + 2*a*d*(p + 2*q) + b*d*(p + q)*x) - 2*d*(2*a + b*x)*Log[e*(f*(a
 + b*x)^p*(c + d*x)^q)^r]))/(4*d^2)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (bx+a)\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

```
[Out] int((b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

Maxima [A]
time = 0.32, size = 119, normalized size = 1.03

$$\frac{1}{2}\left(bx^2+2\,ax\right)\log\left(\left(\left(bx+a\right)^p\left(dx+c\right)^q f\right)^r e\right) + \frac{\left(\frac{2\,a^2fp\log(bx+a)}{b} - \frac{bdf(p+q)x^2+2\,(adf(p+2\,q)-bcfq)x}{d} - \frac{2\,(bc^2fq-2\,acdfq)\log(dx+c)}{d^2}\right)r}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e) + 1/4*(2*a^2*f*p*l
og(b*x + a)/b - (b*d*f*(p + q)*x^2 + 2*(a*d*f*(p + 2*q) - b*c*f*q)*x)/d - 2
*(b*c^2*f*q - 2*a*c*d*f*q)*log(d*x + c)/d^2)*r/f
```

Fricas [A]
time = 0.35, size = 195, normalized size = 1.68

$$\frac{(2\,b^2d^2-(b^2d^2p+b^2d^2q)r)x^2+2\,(2\,abd^2-(abd^2p-(b^2cd-2\,abd^2)q)r)x+2\,(b^2d^2prx^2+2\,abd^2prx+a^2d^2pr)\log\left(bx+a\right)+2\,(b^2d^2qrx^2+2\,abd^2qrx-(b^2c^2-2\,abcd)qr)\log\left(dx+c\right)+2\,(b^2d^2rx^2+2\,abd^2rx)\log\left(f\right)}{4\,bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] 1/4*((2*b^2*d^2 - (b^2*d^2*p + b^2*d^2*q)*r)*x^2 + 2*(2*a*b*d^2 - (a*b*d^2*
p - (b^2*c*d - 2*a*b*d^2)*q)*r)*x + 2*(b^2*d^2*p*r*x^2 + 2*a*b*d^2*p*r*x +
a^2*d^2*p*r)*log(b*x + a) + 2*(b^2*d^2*q*r*x^2 + 2*a*b*d^2*q*r*x - (b^2*c^2
 - 2*a*b*c*d)*q*r)*log(d*x + c) + 2*(b^2*d^2*r*x^2 + 2*a*b*d^2*r*x)*log(f))
/(b*d^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(104) = 208$.
time = 28.16, size = 325, normalized size = 2.80

$$\begin{cases} ax\log\left(e(a^pc^qf)^r\right) & \text{for}\: b=0 \wedge d=0 \\ \frac{a^2\log\left(e(c^qf(a+bx)^p)^r\right)}{2b}-\frac{aprx}{2}+ax\log\left(e(c^qf(a+bx)^p)^r\right)-\frac{bprx^2}{4}+\frac{bx^2\log\left(e(c^qf(a+bx)^p)^r\right)}{2} & \text{for}\: d=0 \\ a\left(\frac{c\log\left(e(a^pf(c+dx)^q)^r\right)}{d}-qrx+x\log\left(e(a^pf(c+dx)^q)^r\right)\right) & \text{for}\: b=0 \\ -\frac{a^2qr\log\left(\frac{c}{d}+x\right)}{2b}+\frac{a^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b}+\frac{acqr\log\left(\frac{c}{d}+x\right)}{d}-\frac{aprx}{2}-aqrx+ax\log\left(e(f(a+bx)^p\left(c+dx\right)^q)^r\right)-\frac{bc^2qr\log\left(\frac{c}{d}+x\right)}{2d^2}+\frac{bcqrx}{2d}-\frac{bprx^2}{4}-\frac{bqrx^2}{4}+\frac{bx^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Piecewise((a*x*log(e*(a**p*c**q*f)**r), Eq(b, 0) & Eq(d, 0)), (a**2*log(e*(
c**q*f*(a + b*x)**p)**r)/(2*b) - a*p*r*x/2 + a*x*log(e*(c**q*f*(a + b*x)**p
)**r) - b*p*r*x**2/4 + b*x**2*log(e*(c**q*f*(a + b*x)**p)**r)/2, Eq(d, 0)),
 (a*(c*log(e*(a**p*f*(c + d*x)**q)**r)/d - q*r*x + x*log(e*(a**p*f*(c + d*x
)**q)**r)), Eq(b, 0)), (-a**2*q*r*log(c/d + x)/(2*b) + a**2*log(e*(f*(a + b
*x)**p*(c + d*x)**q)**r)/(2*b) + a*c*q*r*log(c/d + x)/d - a*p*r*x/2 - a*q*r
*x + a*x*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r) - b*c**2*q*r*log(c/d + x)/
(2*d**2) + b*c*q*r*x/(2*d) - b*p*r*x**2/4 - b*q*r*x**2/4 + b*x**2*log(e*(f*
(a + b*x)**p*(c + d*x)**q)**r)/2, True))
```

Giac [A]
time = 4.66, size = 148, normalized size = 1.28

$$\frac{a^2pr\log\left(bx+a\right)}{2\,b}-\frac{1}{4}\,(bpr+bqr-2\,br\log\left(f\right)-2\,b)x^2+\frac{1}{2}\,(bprx^2+2\,aprx)\log\left(bx+a\right)+\frac{1}{2}\,(bqrx^2+2\,aqrx)\log\left(dx+c\right)-\frac{(adpr-bcqr+2\,adqr-2\,adr\log\left(f\right)-2\,ad)x}{2\,d}-\frac{(bc^2qr-2\,acdqr)\log\left(-dx-c\right)}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] 1/2*a^2*p*r*log(b*x + a)/b - 1/4*(b*p*r + b*q*r - 2*b*r*log(f) - 2*b)*x^2 +
 1/2*(b*p*r*x^2 + 2*a*p*r*x)*log(b*x + a) + 1/2*(b*q*r*x^2 + 2*a*q*r*x)*log
```

```
(d*x + c) - 1/2*(a*d*p*r - b*c*q*r + 2*a*d*q*r - 2*a*d*r*log(f) - 2*a*d)*x/
d - 1/2*(b*c^2*q*r - 2*a*c*d*q*r)*log(-d*x - c)/d^2
```

Mupad [B]
time = 0.40, size = 128, normalized size = 1.10

$$\ln\left(e\left(f\left(a+bx\right)^p\left(c+dx\right)^q\right)^r\right)\left(\frac{bx^2}{2}+ax\right)-x\left(\frac{r\left(2adp+bcp+3adq\right)}{2d}-\frac{r\left(p+q\right)\left(2ad+2bc\right)}{4d}\right)-\frac{\ln\left(c+dx\right)\left(bc^2qr-2acdqr\right)}{2d^2}-\frac{brx^2\left(p+q\right)}{4}+\frac{a^2pr\ln\left(a+bx\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(a + b*x),x)
```

```
[Out] log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(a*x + (b*x^2)/2) - x*((r*(2*a*d*p + b
*c*p + 3*a*d*q))/(2*d) - (r*(p + q)*(2*a*d + 2*b*c))/(4*d)) - (log(c + d*x)
*(b*c^2*q*r - 2*a*c*d*q*r))/(2*d^2) - (b*r*x^2*(p + q))/4 + (a^2*p*r*log(a
+ b*x))/(2*b)
```

3.11 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{a+bx}\,dx$

Optimal. Leaf size=107

$$-\frac{pr\log^2(a+bx)}{2b}-\frac{qr\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{b}+\frac{\log(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}-\frac{qr\,\mathrm{Li}_2\left(-\frac{d(a+bx)}{bc-ad}\right)}{b}$$

```
[Out] -1/2*p*r*ln(b*x+a)^2/b-q*r*ln(b*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/b+ln(b*x+a)*l
n(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b-q*r*polylog(2,-d*(b*x+a)/(-a*d+b*c))/b
```

Rubi [A]
time = 0.06, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2580, 2437, 2338, 2441, 2440, 2438}

$$-\frac{qr\,\mathrm{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{b}+\frac{\log(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}-\frac{qr\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{b}-\frac{pr\log^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x),x]
```

```
[Out] -1/2*(p*r*Log[a + b*x]^2)/b - (q*r*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*
d)])/b + (Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/b - (q*r*PolyL
og[2, -((d*(a + b*x))/(b*c - a*d))])/b
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2580

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/h), x] + (-Dist[b*p*(r/h), Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[d*q*(r/h), Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{a+bx}\,dx &= \frac{\log(a+bx)\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b} - (pr)\int \frac{\log(a+bx)}{a+bx}\,dx - \\
&= -\frac{qr\log(a+bx)\log \left(\frac{b(c+dx)}{bc-ad}\right)}{b} + \frac{\log(a+bx)\log \left(e\left(f(a+bx)^p(c+dx)\right.\right.}{b} \\
&= -\frac{pr\log^2(a+bx)}{2b} - \frac{qr\log(a+bx)\log \left(\frac{b(c+dx)}{bc-ad}\right)}{b} + \frac{\log(a+bx)\log \left(e(f\right.}{} \\
&= -\frac{pr\log^2(a+bx)}{2b} - \frac{qr\log(a+bx)\log \left(\frac{b(c+dx)}{bc-ad}\right)}{b} + \frac{\log(a+bx)\log \left(e(f\right.}{}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 93, normalized size = 0.87

$$
-\frac{\log(a+bx)\left(pr\log(a+bx)+2qr\log \left(\frac{b(c+dx)}{bc-ad}\right)-2\log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)+2qr\text{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)}{2b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x),x]
```

```
[Out] -1/2*(Log[a + b*x]*(p*r*Log[a + b*x] + 2*q*r*Log[(b*(c + d*x))/(b*c - a*d)]
 - 2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) + 2*q*r*PolyLog[2, (d*(a + b*x))
/(-(b*c) + a*d)])/b
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(e(f(bx+a)^p\, (dx+c)^q)^r\right)}{bx+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a),x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a),x)
```

Maxima [A]
time = 0.29, size = 165, normalized size = 1.54

$$-\frac{\left(\frac{2\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)fq}{b}-\frac{fp\log(bx+a)^2+2\,fq\log(bx+a)\log(dx+c)}{b}\right)r}{2\,f}-\frac{(fp\log(bx+a)+fq\log(dx+c))r\log(bx+a)}{bf}+\frac{\log(bx+a)\log\left(((bx+a)^p(dx+c)^qf)^r\,e\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(2*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x +
a*d)/(b*c - a*d)))*f*q/b - (f*p*log(b*x + a)^2 + 2*f*q*log(b*x + a)*log(d*x
 + c))/b)*r/f - (f*p*log(b*x + a) + f*q*log(d*x + c))*r*log(b*x + a)/(b*f)
+ log(b*x + a)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/b
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(e(f(a+bx)^p\, (c+dx)^q)^r\right)}{a+bx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(b*x+a),x)
```

```
[Out] Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)/(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a),x, algorithm="giac")
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(e\,{\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)}^r\right)}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(a + b*x),x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(a + b*x), x)
```

3.12 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^2}\,dx$

Optimal. Leaf size=95

$$-\frac{pr}{b(a+bx)}+\frac{dqr\log(a+bx)}{b(bc-ad)}-\frac{dqr\log(c+dx)}{b(bc-ad)}-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)}$$

```
[Out] -p*r/b/(b*x+a)+d*q*r*ln(b*x+a)/b/(-a*d+b*c)-d*q*r*ln(d*x+c)/b/(-a*d+b*c)-ln
(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(b*x+a)
```

Rubi [A]
time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2581, 32, 36, 31}

$$-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)}+\frac{dqr\log(a+bx)}{b(bc-ad)}-\frac{dqr\log(c+dx)}{b(bc-ad)}-\frac{pr}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^2,x]
```

```
[Out] -((p*r)/(b*(a + b*x))) + (d*q*r*Log[a + b*x])/(b*(b*c - a*d)) - (d*q*r*Log[
c + d*x])/(b*(b*c - a*d)) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(b*(a + b*
x))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
```

```
1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^2}\,dx = -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)} + (pr)\int \frac{1}{(a+bx)^2}\,dx + \frac{(dqr)\int \cdots}{b}$$

$$= -\frac{pr}{b(a+bx)} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)} + \frac{(dqr)\int \frac{1}{a+bx}\,dx}{bc-ad} - \cdots$$

$$= -\frac{pr}{b(a+bx)} + \frac{dqr\log(a+bx)}{b(bc-ad)} - \frac{dqr\log(c+dx)}{b(bc-ad)} - \cdots$$

Mathematica [A]
time = 0.04, size = 89, normalized size = 0.94

$$\frac{r\left(-\frac{p}{a+bx}+\frac{dq\log(a+bx)}{bc-ad}-\frac{dq\log(c+dx)}{bc-ad}\right)}{b}-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^2,x]
```

```
[Out] (r*(-(p/(a + b*x)) + (d*q*Log[a + b*x])/(b*c - a*d) - (d*q*Log[c + d*x])/(b
*c - a*d)))/b - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(b*(a + b*x))
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\left(bx+a\right)^{p}\left(dx+c\right)^{q}\right)^{r}\right)}{\left(bx+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^2,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^2,x)
```

Maxima [A]
time = 0.31, size = 100, normalized size = 1.05

$$\frac{\left(dfq\left(\frac{\log(bx+a)}{bc-ad}-\frac{\log(dx+c)}{bc-ad}\right)-\frac{bfp}{b^2x+ab}\right)r}{bf}-\frac{\log\left(\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)\right)}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (d*f*q*(log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d)) - b*f*p/(b^2*x
 + a*b))*r/(b*f) - log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((b*x + a)*b)
```

Fricas [A]
time = 0.36, size = 116, normalized size = 1.22

$$-\frac{(bc-ad)pr+(bc-ad)r\log(f)+bc-ad-(bdqrx+(adq-(bc-ad)p)r)\log(bx+a)+(bdqrx+bcqr)\log(dx+c)}{ab^2c-a^2bd+(b^3c-ab^2d)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -((b*c - a*d)*p*r + (b*c - a*d)*r*log(f) + b*c - a*d - (b*d*q*r*x + (a*d*q
- (b*c - a*d)*p)*r)*log(b*x + a) + (b*d*q*r*x + b*c*q*r)*log(d*x + c))/(a*b
^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1112 vs. $2(76) = 152$.
time = 141.38, size = 1112, normalized size = 11.71

$$\begin{cases} -\frac{pr}{ab+b^2x}-\frac{\log\left(e\left(c^qf(a+bx)^p\right)^r\right)}{ab+b^2x} & \text{for } d=0 \\ \tilde{\infty}\left(\frac{2\cdot 0^p\tilde{\infty}^{2p}cqr\log(c+dx)}{0^p\tilde{\infty}^pd-d}-\frac{0^p\tilde{\infty}^{2p}dqrx}{0^p\tilde{\infty}^pd-d}-\frac{2\cdot 0^p\tilde{\infty}^pcqr\log(c+dx)}{0^p\tilde{\infty}^pd-d}-\frac{0^p\tilde{\infty}^pc\log\left(e\left(0^pf(c+dx)^q\right)^r\right)}{0^p\tilde{\infty}^pd-d}+\frac{0^p\tilde{\infty}^pdqrx}{0^p\tilde{\infty}^pd-d}+\frac{0^p\tilde{\infty}^pdx\log\left(e\left(0^pf(c+dx)^q\right)^r\right)}{0^p\tilde{\infty}^pd-d}+\frac{c\log\left(e\left(0^pf(c+dx)^q\right)^r\right)}{0^p\tilde{\infty}^pd-d}-\frac{dx\log\left(e\left(0^pf(c+dx)^q\right)^r\right)}{0^p\tilde{\infty}^pd-d}\right) & \text{for } a=-bx \\ \frac{\frac{c\log\left(e\left(a^pf(c+dx)^q\right)^r\right)}{d}-qrx+x\log\left(e\left(a^pf(c+dx)^q\right)^r\right)}{a^2} & \text{for } b=0 \\ -\frac{pr}{ab+b^2x}-\frac{qr}{ab+b^2x}-\frac{\log\left(e\left(f(a+bx)^p\left(\frac{ad}{b}+dx\right)^q\right)^r\right)}{ab+b^2x} & \text{for } c=\frac{ad}{b} \\ -\frac{adqr\log\left(\frac{a}{b}+x\right)}{a^2bd-ab^2c+ab^2dx-b^3cx}+\frac{bc\log\left(e\left(f(c+dx)^q\right)^r\right)}{a^2bd-ab^2c+ab^2dx-b^3cx}-\frac{bdqrx\log\left(\frac{a}{b}+x\right)}{a^2bd-ab^2c+ab^2dx-b^3cx}+\frac{bdx\log\left(e\left(f(c+dx)^q\right)^r\right)}{a^2bd-ab^2c+ab^2dx-b^3cx} & \text{for } p=0 \\ -\frac{adp^2r}{a^2bdp-ab^2cp+ab^2dpx-b^3cpx}+\frac{adpqr\log\left(\frac{c}{d}+x\right)}{a^2bdp-ab^2cp+ab^2dpx-b^3cpx}-\frac{adp\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{a^2bdp-ab^2cp+ab^2dpx-b^3cpx}+\frac{adq^2r\log\left(\frac{c}{d}+x\right)}{a^2bdp-ab^2cp+ab^2dpx-b^3cpx}-\frac{adq\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{a^2bdp-ab^2cp+ab^2dpx-b^3cpx}+\frac{bcp^2r}{a^2bdp-ab^2cp+ab^2dpx-b^3cpx}+\frac{bcp\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{a^2bdp-ab^2cp+ab^2dpx-b^3cpx}+\frac{bdpqrx\log\left(\frac{c}{d}+x\right)}{a^2bdp-ab^2cp+ab^2dpx-b^3cpx}+\frac{bdq^2rx\log\left(\frac{c}{d}+x\right)}{a^2bdp-ab^2cp+ab^2dpx-b^3cpx}-\frac{bdqx\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{a^2bdp-ab^2cp+ab^2dpx-b^3cpx} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(b*x+a)**2,x)
```

```
[Out] Piecewise((-p*r/(a*b + b**2*x) - log(e*(c**q*f*(a + b*x)**p)**r)/(a*b + b**
2*x), Eq(d, 0)), (zoo*(2*0**p*zoo**(2*p)*c*q*r*log(c + d*x)/(0**p*zoo**p*d
- d) - 0**p*zoo**(2*p)*d*q*r*x/(0**p*zoo**p*d - d) - 2*0**p*zoo**p*c*q*r*lo
g(c + d*x)/(0**p*zoo**p*d - d) - 0**p*zoo**p*c*log(e*(0**p*f*(c + d*x)**q)*
*r)/(0**p*zoo**p*d - d) + 0**p*zoo**p*d*q*r*x/(0**p*zoo**p*d - d) + 0**p*zo
o**p*d*x*log(e*(0**p*f*(c + d*x)**q)**r)/(0**p*zoo**p*d - d) + c*log(e*(0**
p*f*(c + d*x)**q)**r)/(0**p*zoo**p*d - d) - d*x*log(e*(0**p*f*(c + d*x)**q)
**r)/(0**p*zoo**p*d - d)), Eq(a, -b*x)), ((c*log(e*(a**p*f*(c + d*x)**q)**r
)/d - q*r*x + x*log(e*(a**p*f*(c + d*x)**q)**r))/a**2, Eq(b, 0)), (-p*r/(a*
b + b**2*x) - q*r/(a*b + b**2*x) - log(e*(f*(a + b*x)**p*(a*d/b + d*x)**q)*
*r)/(a*b + b**2*x), Eq(c, a*d/b)), (-a*d*q*r*log(a/b + x)/(a**2*b*d - a*b**
2*c + a*b**2*d*x - b**3*c*x) + b*c*log(e*(f*(c + d*x)**q)**r)/(a**2*b*d - a
*b**2*c + a*b**2*d*x - b**3*c*x) - b*d*q*r*x*log(a/b + x)/(a**2*b*d - a*b**
2*c + a*b**2*d*x - b**3*c*x) + b*d*x*log(e*(f*(c + d*x)**q)**r)/(a**2*b*d -
 a*b**2*c + a*b**2*d*x - b**3*c*x), Eq(p, 0)), (-a*d*p**2*r/(a**2*b*d*p - a
```

```
*b**2*c*p + a*b**2*d*p*x - b**3*c*p*x) + a*d*p*q*r*log(c/d + x)/(a**2*b*d*p
 - a*b**2*c*p + a*b**2*d*p*x - b**3*c*p*x) - a*d*p*log(e*(f*(a + b*x)**p*(c
 + d*x)**q)**r)/(a**2*b*d*p - a*b**2*c*p + a*b**2*d*p*x - b**3*c*p*x) + a*d
*q**2*r*log(c/d + x)/(a**2*b*d*p - a*b**2*c*p + a*b**2*d*p*x - b**3*c*p*x)
- a*d*q*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)/(a**2*b*d*p - a*b**2*c*p +
a*b**2*d*p*x - b**3*c*p*x) + b*c*p**2*r/(a**2*b*d*p - a*b**2*c*p + a*b**2*d
*p*x - b**3*c*p*x) + b*c*p*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)/(a**2*b*
d*p - a*b**2*c*p + a*b**2*d*p*x - b**3*c*p*x) + b*d*p*q*r*x*log(c/d + x)/(a
**2*b*d*p - a*b**2*c*p + a*b**2*d*p*x - b**3*c*p*x) + b*d*q**2*r*x*log(c/d
+ x)/(a**2*b*d*p - a*b**2*c*p + a*b**2*d*p*x - b**3*c*p*x) - b*d*q*x*log(e*
(f*(a + b*x)**p*(c + d*x)**q)**r)/(a**2*b*d*p - a*b**2*c*p + a*b**2*d*p*x -
 b**3*c*p*x), True))
```

Giac [A]
time = 3.96, size = 112, normalized size = 1.18

$$\frac{dqr\log\left(bx+a\right)}{b^2c-abd}-\frac{dqr\log\left(dx+c\right)}{b^2c-abd}-\frac{pr\log\left(bx+a\right)}{b^2x+ab}-\frac{qr\log\left(dx+c\right)}{b^2x+ab}-\frac{pr+r\log\left(f\right)+1}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] d*q*r*log(b*x + a)/(b^2*c - a*b*d) - d*q*r*log(d*x + c)/(b^2*c - a*b*d) - p
*r*log(b*x + a)/(b^2*x + a*b) - q*r*log(d*x + c)/(b^2*x + a*b) - (p*r + r*l
og(f) + 1)/(b^2*x + a*b)
```

Mupad [B]
time = 2.16, size = 99, normalized size = 1.04

$$-\frac{\ln\left(e\left(f\left(a+b\,x\right)^p\,\left(c+d\,x\right)^q\right)^r\right)\,\left(x+\frac{a}{b}\right)}{{\left(a+b\,x\right)}^2}-\frac{p\,r}{x\,b^2+a\,b}+\frac{d\,q\,r\,\mathrm{atan}\left(\frac{b\,c\,2{}\mathrm{i}+b\,d\,x\,2{}\mathrm{i}}{a\,d-b\,c}+1{}\mathrm{i}\right)\,2{}\mathrm{i}}{b\,\left(a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(a + b*x)^2,x)
```

```
[Out] (d*q*r*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(b*(a*d - b*c)) - (p*
r)/(a*b + b^2*x) - (log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(x + a/b))/(a + b*
x)^2
```

3.13 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^3}\,dx$

Optimal. Leaf size=135

$$-\frac{pr}{4b(a+bx)^2}-\frac{dqr}{2b(bc-ad)(a+bx)}-\frac{d^2qr\log(a+bx)}{2b(bc-ad)^2}+\frac{d^2qr\log(c+dx)}{2b(bc-ad)^2}-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(a+bx)^2}$$

```
[Out] -1/4*p*r/b/(b*x+a)^2-1/2*d*q*r/b/(-a*d+b*c)/(b*x+a)-1/2*d^2*q*r*ln(b*x+a)/b
/(-a*d+b*c)^2+1/2*d^2*q*r*ln(d*x+c)/b/(-a*d+b*c)^2-1/2*ln(e*(f*(b*x+a)^p*(d
*x+c)^q)^r)/b/(b*x+a)^2
```

Rubi [A]
time = 0.05, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2581, 32, 46}

$$-\frac{d^2qr\log(a+bx)}{2b(bc-ad)^2}+\frac{d^2qr\log(c+dx)}{2b(bc-ad)^2}-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(a+bx)^2}-\frac{dqr}{2b(a+bx)(bc-ad)}-\frac{pr}{4b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^3,x]
```

```
[Out] -1/4*(p*r)/(b*(a + b*x)^2) - (d*q*r)/(2*b*(b*c - a*d)*(a + b*x)) - (d^2*q*r
*Log[a + b*x])/(2*b*(b*c - a*d)^2) + (d^2*q*r*Log[c + d*x])/(2*b*(b*c - a*d
)^2) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(2*b*(a + b*x)^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}\,dx = -\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(a+bx)^2} + \frac{1}{2}(pr)\int \frac{1}{(a+bx)^3}\,dx + \frac{(dqr)\int \frac{}{(a+}}{2}$$

$$= -\frac{pr}{4b(a+bx)^2} - \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(a+bx)^2} + \frac{(dqr)\int\left(\frac{b}{(bc-ad)(a+bx)^2}\right.}{}$$

$$= -\frac{pr}{4b(a+bx)^2} - \frac{dqr}{2b(bc-ad)(a+bx)} - \frac{d^2qr\log(a+bx)}{2b(bc-ad)^2} + \frac{d^2qr\log(c}{2b(bc-a}$$

Mathematica [A]
time = 0.18, size = 116, normalized size = 0.86

$$\frac{r\left(-\frac{p-\frac{2dq(a+bx)}{-bc+ad}}{2(a+bx)^2} - \frac{d^2q\log(a+bx)}{(bc-ad)^2} + \frac{d^2q\log(c+dx)}{(bc-ad)^2}\right) - \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^3,x]
```

```
[Out] (r*(-1/2*(p - (2*d*q*(a + b*x))/(-(b*c) + a*d))/(a + b*x)^2 - (d^2*q*Log[a
+ b*x])/(b*c - a*d)^2 + (d^2*q*Log[c + d*x])/(b*c - a*d)^2) - Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/(a + b*x)^2)/(2*b)
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\left(bx+a\right)^{p}\left(dx+c\right)^{q}\right)^{r}\right)}{\left(bx+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^3,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^3,x)
```

Maxima [A]
time = 0.28, size = 166, normalized size = 1.23

$$-\frac{\left(2\,dfq\left(\frac{d\log(bx+a)}{b^2c^2-2\,abcd+a^2d^2} - \frac{d\log(dx+c)}{b^2c^2-2\,abcd+a^2d^2} + \frac{1}{abc-a^2d+(b^2c-abd)x}\right) + \frac{bfp}{b^3x^2+2\,ab^2x+a^2b}\right)r}{4\,bf} - \frac{\log\left(\left(\left(bx+a\right)^p\left(dx+c\right)^q f\right)^r e\right)}{2\,\left(bx+a\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*d*f*q*(d*log(b*x + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - d*log(d*x +
 c)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
) + b*f*p/(b^3*x^2 + 2*a*b^2*x + a^2*b))*r/(b*f) - 1/2*log(((b*x + a)^p*(d*
x + c)^q*f)^r*e)/((b*x + a)^2*b)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(126) = 252$.
time = 0.42, size = 320, normalized size = 2.37

$$-\frac{2\,b^2c^2-4\,abcd+2\,a^2d^2+2\,(b^2cd-abd^2)qrx+2\,(b^2c^2-2\,abcd+a^2d^2)r\log\left(f\right)+((b^2c^2-2\,abcd+a^2d^2)p+2\,(abcd-a^2d^2)q)r+2\,(b^2d^2qrx^2+2\,abd^2qrx+(a^2d^2q+(b^2c^2-2\,abcd+a^2d^2)p)r)\log\left(bx+a\right)-2\,(b^2d^2qrx^2+2\,abd^2qrx-(b^2c^2-2\,abcd)qr)\log\left(dx+c\right)}{4\,(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2+(b^5c^2-2\,ab^4cd+a^2b^3d^2)x^2+2\,(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^2*c^2 - 4*a*b*c*d + 2*a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*q*r*x + 2*(
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*r*log(f) + ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*
p + 2*(a*b*c*d - a^2*d^2)*q)*r + 2*(b^2*d^2*q*r*x^2 + 2*a*b*d^2*q*r*x + (a^
2*d^2*q + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*p)*r)*log(b*x + a) - 2*(b^2*d^2*q
*r*x^2 + 2*a*b*d^2*q*r*x - (b^2*c^2 - 2*a*b*c*d)*q*r)*log(d*x + c))/(a^2*b^
3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x
^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 5.20, size = 246, normalized size = 1.82

$$-\frac{d^2qr\log\left(bx+a\right)}{2\,(b^3c^2-2\,ab^2cd+a^2bd^2)}+\frac{d^2qr\log\left(dx+c\right)}{2\,(b^3c^2-2\,ab^2cd+a^2bd^2)}-\frac{pr\log\left(bx+a\right)}{2\,(b^3x^2+2\,ab^2x+a^2b)}-\frac{qr\log\left(dx+c\right)}{2\,(b^3x^2+2\,ab^2x+a^2b)}-\frac{2\,bdqrx+bcpr-adpr+2\,adqr+2\,bcr\log\left(f\right)-2\,adr\log\left(f\right)+2\,bc-2\,ad}{4\,(b^4cx^2-ab^3dx^2+2\,ab^3cx-2\,a^2b^2dx+a^2b^2c-a^3bd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*d^2*q*r*log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + 1/2*d^2*q*r
*log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 1/2*p*r*log(b*x + a)/(b
^3*x^2 + 2*a*b^2*x + a^2*b) - 1/2*q*r*log(d*x + c)/(b^3*x^2 + 2*a*b^2*x + a
^2*b) - 1/4*(2*b*d*q*r*x + b*c*p*r - a*d*p*r + 2*a*d*q*r + 2*b*c*r*log(f) -
 2*a*d*r*log(f) + 2*b*c - 2*a*d)/(b^4*c*x^2 - a*b^3*d*x^2 + 2*a*b^3*c*x - 2
*a^2*b^2*d*x + a^2*b^2*c - a^3*b*d)
```

Mupad [B]
time = 2.39, size = 182, normalized size = 1.35

$$\frac{\frac{b\,c\,p\,r-a\,d\,p\,r+2\,a\,d\,q\,r}{2\,(a\,d-b\,c)}+\frac{b\,d\,q\,r\,x}{a\,d-b\,c}}{2\,a^2\,b+4\,a\,b^2\,x+2\,b^3\,x^2}-\frac{\ln\left(e\,\left(f\,{(a+b\,x)}^p\,{(c+d\,x)}^q\right)^r\right)\,\left(\frac{x}{2}+\frac{a}{2\,b}\right)}{{(a+b\,x)}^3}+\frac{d^2\,q\,r\,\mathrm{atanh}\left(\frac{2\,b^3\,c^2-2\,a^2\,b\,d^2}{2\,b\,{(a\,d-b\,c)}^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,{(a\,d-b\,c)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(a + b*x)^3,x)
```

```
[Out] ((b*c*p*r - a*d*p*r + 2*a*d*q*r)/(2*(a*d - b*c)) + (b*d*q*r*x)/(a*d - b*c))
/(2*a^2*b + 2*b^3*x^2 + 4*a*b^2*x) - (log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*
(x/2 + a/(2*b)))/(a + b*x)^3 + (d^2*q*r*atanh((2*b^3*c^2 - 2*a^2*b*d^2)/(2*
b*(a*d - b*c)^2) - (2*b*d*x)/(a*d - b*c)))/(b*(a*d - b*c)^2)
```

3.14 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^4} dx$

Optimal. Leaf size=164

$$-\frac{pr}{9b(a+bx)^3}-\frac{dqr}{6b(bc-ad)(a+bx)^2}+\frac{d^2qr}{3b(bc-ad)^2(a+bx)}+\frac{d^3qr\log(a+bx)}{3b(bc-ad)^3}-\frac{d^3qr\log(c+dx)}{3b(bc-ad)^3}-\frac{\log(e(f($$

```
[Out] -1/9*p*r/b/(b*x+a)^3-1/6*d*q*r/b/(-a*d+b*c)/(b*x+a)^2+1/3*d^2*q*r/b/(-a*d+b
*c)^2/(b*x+a)+1/3*d^3*q*r*ln(b*x+a)/b/(-a*d+b*c)^3-1/3*d^3*q*r*ln(d*x+c)/b/
(-a*d+b*c)^3-1/3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(b*x+a)^3
```

Rubi [A]
time = 0.06, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2581, 32, 46}

$$\frac{d^3qr\log(a+bx)}{3b(bc-ad)^3}-\frac{d^3qr\log(c+dx)}{3b(bc-ad)^3}+\frac{d^2qr}{3b(a+bx)(bc-ad)^2}-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(a+bx)^3}-\frac{dqr}{6b(a+bx)^2(bc-ad)}-\frac{pr}{9b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^4,x]
```

```
[Out] -1/9*(p*r)/(b*(a + b*x)^3) - (d*q*r)/(6*b*(b*c - a*d)*(a + b*x)^2) + (d^2*q
*r)/(3*b*(b*c - a*d)^2*(a + b*x)) + (d^3*q*r*Log[a + b*x])/(3*b*(b*c - a*d)
^3) - (d^3*q*r*Log[c + d*x])/(3*b*(b*c - a*d)^3) - Log[e*(f*(a + b*x)^p*(c
+ d*x)^q)^r]/(3*b*(a + b*x)^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
```

```
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^4}\,dx &= -\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(a+bx)^3}+\frac{1}{3}(pr)\int\frac{1}{(a+bx)^4}\,dx+\frac{(dqr)\int \frac{}{(a+}}{3}\\ &= -\frac{pr}{9b(a+bx)^3}-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(a+bx)^3}+\frac{(dqr)\int\left(\frac{b}{(bc-ad)(a+bx)}\right.}{}\\ &= -\frac{pr}{9b(a+bx)^3}-\frac{dqr}{6b(bc-ad)(a+bx)^2}+\frac{d^2qr}{3b(bc-ad)^2(a+bx)}+\frac{d^3qr}{3b(}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 141, normalized size = 0.86

$$\frac{r\left(\frac{-2p+\frac{3dq(a+bx)}{-bc+ad}+\frac{6d^2q(a+bx)^2}{(bc-ad)^2}}{6(a+bx)^3}+\frac{d^3q\log(a+bx)}{(bc-ad)^3}-\frac{d^3q\log(c+dx)}{(bc-ad)^3}\right)-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^4,x]
```

```
[Out] (r*((-2*p + (3*d*q*(a + b*x))/(-(b*c) + a*d) + (6*d^2*q*(a + b*x)^2)/(b*c -
 a*d)^2)/(6*(a + b*x)^3) + (d^3*q*Log[a + b*x])/(b*c - a*d)^3 - (d^3*q*Log[
c + d*x])/(b*c - a*d)^3) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^3
)/(3*b)
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\left(bx+a\right)^{p}\left(dx+c\right)^{q}\right)^{r}\right)}{\left(bx+a\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^4,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^4,x)
```

Maxima [A]
time = 0.29, size = 290, normalized size = 1.77

$$\frac{\left(3\left(\frac{2\,d^2\log(bx+a)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}-\frac{2\,d^2\log(dx+c)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}+\frac{2\,bdx-bc+3\,ad}{a^2b^2c^2-2\,a^3bcd+a^4d^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^2+2\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2)x}\right)dfq-\frac{2\,bfp}{b^4x^3+3\,ab^3x^2+3\,a^2b^2x+a^3b}\right)r}{18\,bf}-\frac{\log\left(\left(\left(bx+a\right)^p\left(dx+c\right)^q f\right)^r e\right)}{3\,\left(bx+a\right)^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/18*(3*(2*d^2*log(b*x + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3) - 2*d^2*log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3) + (2*b*d*x - b*c + 3*a*d)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^
2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d
^2)*x))*d*f*q - 2*b*f*p/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b))*r/(b
*f) - 1/3*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((b*x + a)^3*b)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 575 vs. $2(153) = 306$.
time = 0.38, size = 575, normalized size = 3.51

$$-\frac{6\,b^3c^3-18\,ab^2c^2d+18\,a^2bcd^2-6\,a^3d^3-6\,(b^3cd^2-ab^2d^3)qrx^2+3\,(b^3c^2d-6\,ab^2cd^2+5\,a^2bd^3)qrx+6\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)r\log\left(f\right)+(2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)p+3\,(ab^2c^2d-4\,a^2bcd^2+3\,a^3d^3)q)r-6\,(b^3d^3qrx^3+3\,ab^2d^3qrx^2+3\,a^2bd^3qrx+(a^3d^3q-(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)p)r)\log\left(bx+a\right)+6\,(b^3d^3qrx^3+3\,ab^2d^3qrx^2+3\,a^2bd^3qrx+(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2)qr)\log\left(dx+c\right)}{18\,(a^3b^4c^3-3\,a^4b^3c^2d+3\,a^5b^2cd^2-a^6bd^3+(b^7c^3-3\,ab^6c^2d+3\,a^2b^5cd^2-a^3b^4d^3)x^3+3\,(ab^6c^3-3\,a^2b^5c^2d+3\,a^3b^4cd^2-a^4b^3d^3)x^2+3\,(a^2b^5c^3-3\,a^3b^4c^2d+3\,a^4b^3cd^2-a^5b^2d^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/18*(6*b^3*c^3 - 18*a*b^2*c^2*d + 18*a^2*b*c*d^2 - 6*a^3*d^3 - 6*(b^3*c*d
^2 - a*b^2*d^3)*q*r*x^2 + 3*(b^3*c^2*d - 6*a*b^2*c*d^2 + 5*a^2*b*d^3)*q*r*x
 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*r*log(f) + (2*(b^3
*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*p + 3*(a*b^2*c^2*d - 4*a^2*
b*c*d^2 + 3*a^3*d^3)*q)*r - 6*(b^3*d^3*q*r*x^3 + 3*a*b^2*d^3*q*r*x^2 + 3*a^
2*b*d^3*q*r*x + (a^3*d^3*q - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3
*d^3)*p)*r)*log(b*x + a) + 6*(b^3*d^3*q*r*x^3 + 3*a*b^2*d^3*q*r*x^2 + 3*a^2
*b*d^3*q*r*x + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2)*q*r)*log(d*x + c))
/(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3 + (b^7*c^3 -
3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^3 + 3*(a*b^6*c^3 - 3*a^2*b
^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*
c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(153) = 306$.
time = 4.50, size = 469, normalized size = 2.86

$$\frac{d^3qr\log\left(bx+a\right)}{3\,(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)}-\frac{d^3qr\log\left(dx+c\right)}{3\,(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)}-\frac{pr\log\left(bx+a\right)}{3\,(b^4x^3+3\,ab^3x^2+3\,a^2b^2x+a^3b)}-\frac{qr\log\left(dx+c\right)}{3\,(b^4x^3+3\,ab^3x^2+3\,a^2b^2x+a^3b)}+\frac{6\,b^2d^2qrx^2-3\,b^2cdqrx+15\,abd^2qrx-2\,b^2c^2pr+4\,abcdpr-2\,a^2d^2pr-3\,abcdqr+9\,a^2d^2qr-6\,b^2c^2r\log\left(f\right)+12\,abcdr\log\left(f\right)-6\,a^2d^2r\log\left(f\right)-6\,b^2c^2+12\,abcd-6\,a^2d^2}{18\,(b^6c^2x^3-2\,ab^5cdx^3+a^2b^4d^2x^3+3\,ab^5c^2x^2-6\,a^2b^4cdx^2+3\,a^3b^3d^2x^2+3\,a^2b^4c^2x-6\,a^3b^3cdx+3\,a^4b^2d^2x+a^3b^3c^2-2\,a^4b^2cd+a^5bd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/3*d^3*q*r*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b
*d^3) - 1/3*d^3*q*r*log(d*x + c)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2
 - a^3*b*d^3) - 1/3*p*r*log(b*x + a)/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x +
 a^3*b) - 1/3*q*r*log(d*x + c)/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b
) + 1/18*(6*b^2*d^2*q*r*x^2 - 3*b^2*c*d*q*r*x + 15*a*b*d^2*q*r*x - 2*b^2*c^
2*p*r + 4*a*b*c*d*p*r - 2*a^2*d^2*p*r - 3*a*b*c*d*q*r + 9*a^2*d^2*q*r - 6*b
^2*c^2*r*log(f) + 12*a*b*c*d*r*log(f) - 6*a^2*d^2*r*log(f) - 6*b^2*c^2 + 12
*a*b*c*d - 6*a^2*d^2)/(b^6*c^2*x^3 - 2*a*b^5*c*d*x^3 + a^2*b^4*d^2*x^3 + 3*
a*b^5*c^2*x^2 - 6*a^2*b^4*c*d*x^2 + 3*a^3*b^3*d^2*x^2 + 3*a^2*b^4*c^2*x - 6
*a^3*b^3*c*d*x + 3*a^4*b^2*d^2*x + a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)
```

Mupad [B]
time = 2.87, size = 346, normalized size = 2.11

$$\frac{\frac{x\,\left(5\,a\,b\,d^2\,q\,r-b^2\,c\,d\,q\,r\right)}{2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}-\frac{2\,a^2\,d^2\,p\,r+2\,b^2\,c^2\,p\,r-9\,a^2\,d^2\,q\,r-4\,a\,b\,c\,d\,p\,r+3\,a\,b\,c\,d\,q\,r}{6\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}+\frac{b^2\,d^2\,q\,r\,x^2}{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}}{3\,a^3\,b+9\,a^2\,b^2\,x+9\,a\,b^3\,x^2+3\,b^4\,x^3}-\frac{\ln\left(e\,{\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)}^r\right)\,\left(\frac{x}{3}+\frac{a}{3\,b}\right)}{{\left(a+b\,x\right)}^4}-\frac{2\,d^3\,q\,r\,\mathrm{atanh}\left(\frac{3\,a^3\,b\,d^3-3\,a^2\,b^2\,c\,d^2-3\,a\,b^3\,c^2\,d+3\,b^4\,c^3}{3\,b\,{\left(a\,d-b\,c\right)}^3}+\frac{2\,b\,d\,x\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{{\left(a\,d-b\,c\right)}^3}\right)}{3\,b\,{\left(a\,d-b\,c\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(a + b*x)^4,x)
```

```
[Out] ((x*(5*a*b*d^2*q*r - b^2*c*d*q*r))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (2
*a^2*d^2*p*r + 2*b^2*c^2*p*r - 9*a^2*d^2*q*r - 4*a*b*c*d*p*r + 3*a*b*c*d*q*
r)/(6*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b^2*d^2*q*r*x^2)/(a^2*d^2 + b^2*c
^2 - 2*a*b*c*d))/(3*a^3*b + 3*b^4*x^3 + 9*a^2*b^2*x + 9*a*b^3*x^2) - (log(e
*(f*(a + b*x)^p*(c + d*x)^q)^r)*(x/3 + a/(3*b)))/(a + b*x)^4 - (2*d^3*q*r*a
tanh((3*b^4*c^3 + 3*a^3*b*d^3 - 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d)/(3*b*(a*d
- b*c)^3) + (2*b*d*x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(a*d - b*c)^3))/(3*b*
(a*d - b*c)^3)
```

3.15 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^5} \, dx$

Optimal. Leaf size=193

$$-\frac{pr}{16b(a+bx)^4}-\frac{dqr}{12b(bc-ad)(a+bx)^3}+\frac{d^2qr}{8b(bc-ad)^2(a+bx)^2}-\frac{d^3qr}{4b(bc-ad)^3(a+bx)}-\frac{d^4qr\log(a+bx)}{4b(bc-ad)^4}+$$

```
[Out] -1/16*p*r/b/(b*x+a)^4-1/12*d*q*r/b/(-a*d+b*c)/(b*x+a)^3+1/8*d^2*q*r/b/(-a*d
+b*c)^2/(b*x+a)^2-1/4*d^3*q*r/b/(-a*d+b*c)^3/(b*x+a)-1/4*d^4*q*r*ln(b*x+a)/
b/(-a*d+b*c)^4+1/4*d^4*q*r*ln(d*x+c)/b/(-a*d+b*c)^4-1/4*ln(e*(f*(b*x+a)^p*(
d*x+c)^q)^r)/b/(b*x+a)^4
```

Rubi [A]

time = 0.08, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2581, 32, 46}

$$-\frac{d^4qr\log(a+bx)}{4b(bc-ad)^4}+\frac{d^4qr\log(c+dx)}{4b(bc-ad)^4}-\frac{d^3qr}{4b(a+bx)(bc-ad)^3}+\frac{d^2qr}{8b(a+bx)^2(bc-ad)^2}-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b(a+bx)^4}-\frac{dqr}{12b(a+bx)^3(bc-ad)}-\frac{pr}{16b(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^5,x]
```

```
[Out] -1/16*(p*r)/(b*(a + b*x)^4) - (d*q*r)/(12*b*(b*c - a*d)*(a + b*x)^3) + (d^2
*q*r)/(8*b*(b*c - a*d)^2*(a + b*x)^2) - (d^3*q*r)/(4*b*(b*c - a*d)^3*(a + b
*x)) - (d^4*q*r*Log[a + b*x])/(4*b*(b*c - a*d)^4) + (d^4*q*r*Log[c + d*x])/
(4*b*(b*c - a*d)^4) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(4*b*(a + b*x)^4
)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
```

```
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^5}\,dx = -\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b(a+bx)^4} + \frac{1}{4}(pr)\int \frac{1}{(a+bx)^5}\,dx + \frac{(dqr)\int \cdots}{4\cdots}$$

$$= -\frac{pr}{16b(a+bx)^4} - \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b(a+bx)^4} + \frac{(dqr)\int \left(\frac{b}{(bc-ad)(a+bx\cdots}\cdots\right.}{\cdots}$$

$$= -\frac{pr}{16b(a+bx)^4} - \frac{dqr}{12b(bc-ad)(a+bx)^3} + \frac{d^2qr}{8b(bc-ad)^2(a+bx)^2} - \frac{\cdots}{4b\cdots}$$

Mathematica [A]
time = 0.25, size = 164, normalized size = 0.85

$$\frac{r\left(\frac{-3p+\frac{4dq(a+bx)}{-bc+ad}+\frac{6d^2q(a+bx)^2}{(bc-ad)^2}-\frac{12d^3q(a+bx)^3}{(bc-ad)^3}}{12(a+bx)^4}-\frac{d^4q\log(a+bx)}{(bc-ad)^4}+\frac{d^4q\log(c+dx)}{(bc-ad)^4}\right)-\frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^4}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^5,x]
```

```
[Out] (r*((-3*p + (4*d*q*(a + b*x))/(-(b*c) + a*d) + (6*d^2*q*(a + b*x)^2)/(b*c -
 a*d)^2 - (12*d^3*q*(a + b*x)^3)/(b*c - a*d)^3)/(12*(a + b*x)^4) - (d^4*q*L
og[a + b*x])/(b*c - a*d)^4 + (d^4*q*Log[c + d*x])/(b*c - a*d)^4) - Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^4)/(4*b)
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\left(bx+a\right)^{p}\left(dx+c\right)^{q}\right)^{r}\right)}{\left(bx+a\right)^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^5,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^5,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 460 vs. 2(180) = 360.
time = 0.31, size = 460, normalized size = 2.38

$$-\frac{\left(2\left(\frac{6\,d^3\log(bx+a)}{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}-\frac{6\,d^3\log(dx+c)}{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}+\frac{6\,b^2d^2x^2+2\,b^2c^2-7\,abcd+11\,a^2d^2-3\left(b^2cd-5\,abd^2\right)x}{a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3+\left(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3\right)x^3+3\left(ab^5c^3-3\,a^2b^4c^2d+3\,a^3b^3cd^2-a^4b^2d^3\right)x^2+3\left(a^2b^4c^3-3\,a^3b^3c^2d+3\,a^4b^2cd^2-a^5bd^3\right)x}\right)dfq+\frac{3\,bfp}{b^5x^4+4\,ab^4x^3+6\,a^2b^3x^2+4\,a^3b^2x+a^4b}\right)r}{48\,bf}-\frac{\log\left(\left(\left(bx+a\right)^p\left(dx+c\right)^q f\right)^r e\right)}{4\left(bx+a\right)^4 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/48*(2*(6*d^3*log(b*x + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
 4*a^3*b*c*d^3 + a^4*d^4) - 6*d^3*log(d*x + c)/(b^4*c^4 - 4*a*b^3*c^3*d + 6
*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) + (6*b^2*d^2*x^2 + 2*b^2*c^2 -
7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/(a^3*b^3*c^3 - 3*a^4*b^
2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*
d^2 - a^3*b^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 -
 a^4*b^2*d^3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^
5*b*d^3)*x))*d*f*q + 3*b*f*p/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3
*b^2*x + a^4*b))*r/(b*f) - 1/4*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((b*x +
 a)^4*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 858 vs. $2(180) = 360$.
time = 0.38, size = 858, normalized size = 4.45

$$-\frac{12 b^4 c^4 - 48 a b^3 c^3 d + 72 a^2 b^2 c^2 d^2 - 48 a^3 b c d^3 + 12 a^4 d^4 + 12 (b^4 c d^3 - a b^3 d^4) q r x^3 - 6 (b^4 c^2 d^2 - 8 a b^3 c d^3 + 7 a^2 b^2 d^4) q r x^2 + 4 (b^4 c^3 d - 6 a b^3 c^2 d^2 + 18 a^2 b^2 c d^3 - 13 a^3 b d^4) q r x + 12 (b^4 c^4 - 4 a b^3 c^3 d + 6 a^2 b^2 c^2 d^2 - 4 a^3 b c d^3 + a^4 d^4) r \log(f) + (3 (b^4 c^4 - 4 a b^3 c^3 d + 6 a^2 b^2 c^2 d^2 - 4 a^3 b c d^3 + a^4 d^4) p + 2 (2 a b^3 c^3 d - 9 a^2 b^2 c^2 d^2 + 18 a^3 b c d^3 - 11 a^4 d^4) q) r + 12 (b^4 d^4 q r x^4 + 4 a b^3 d^4 q r x^3 + 6 a^2 b^2 d^4 q r x^2 + 4 a^3 b d^4 q r x + (a^4 d^4 q + (b^4 c^4 - 4 a b^3 c^3 d + 6 a^2 b^2 c^2 d^2 - 4 a^3 b c d^3 + a^4 d^4) p) r) \log(b x + a) - 12 (b^4 d^4 q r x^4 + 4 a b^3 d^4 q r x^3 + 6 a^2 b^2 d^4 q r x^2 + 4 a^3 b d^4 q r x - (b^4 c^4 - 4 a b^3 c^3 d + 6 a^2 b^2 c^2 d^2 - 4 a^3 b c d^3) q r) \log(d x + c)}{48 (a^4 b^5 c^4 - 4 a^5 b^4 c^3 d + 6 a^6 b^3 c^2 d^2 - 4 a^7 b^2 c d^3 + a^8 b d^4 + (b^9 c^4 - 4 a b^8 c^3 d + 6 a^2 b^7 c^2 d^2 - 4 a^3 b^6 c d^3 + a^4 b^5 d^4) x^4 + 4 (a b^8 c^4 - 4 a^2 b^7 c^3 d + 6 a^3 b^6 c^2 d^2 - 4 a^4 b^5 c d^3 + a^5 b^4 d^4) x^3 + 6 (a^2 b^7 c^4 - 4 a^3 b^6 c^3 d + 6 a^4 b^5 c^2 d^2 - 4 a^5 b^4 c d^3 + a^6 b^3 d^4) x^2 + 4 (a^3 b^6 c^4 - 4 a^4 b^5 c^3 d + 6 a^5 b^4 c^2 d^2 - 4 a^6 b^3 c d^3 + a^7 b^2 d^4) x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/48*(12*b^4*c^4 - 48*a*b^3*c^3*d + 72*a^2*b^2*c^2*d^2 - 48*a^3*b*c*d^3 +
12*a^4*d^4 + 12*(b^4*c*d^3 - a*b^3*d^4)*q*r*x^3 - 6*(b^4*c^2*d^2 - 8*a*b^3*
c*d^3 + 7*a^2*b^2*d^4)*q*r*x^2 + 4*(b^4*c^3*d - 6*a*b^3*c^2*d^2 + 18*a^2*b^
2*c*d^3 - 13*a^3*b*d^4)*q*r*x + 12*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2
*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*r*log(f) + (3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*
a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*p + 2*(2*a*b^3*c^3*d - 9*a^2*b^2
*c^2*d^2 + 18*a^3*b*c*d^3 - 11*a^4*d^4)*q)*r + 12*(b^4*d^4*q*r*x^4 + 4*a*b^
3*d^4*q*r*x^3 + 6*a^2*b^2*d^4*q*r*x^2 + 4*a^3*b*d^4*q*r*x + (a^4*d^4*q + (b
^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*p)*r)
*log(b*x + a) - 12*(b^4*d^4*q*r*x^4 + 4*a*b^3*d^4*q*r*x^3 + 6*a^2*b^2*d^4*q
*r*x^2 + 4*a^3*b*d^4*q*r*x - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
 4*a^3*b*c*d^3)*q*r)*log(d*x + c))/(a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b
^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4 + (b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2
*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^
7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*x^3 + 6*(a^2*b
^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^
4)*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c
*d^3 + a^7*b^2*d^4)*x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(b*x+a)**5,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 748 vs. 2(180) = 360.
time = 4.77, size = 748, normalized size = 3.88

$$-\frac{d^4qr\log(bx+a)}{4\,(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4)}+\frac{d^4qr\log(dx+c)}{4\,(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4)}-\frac{pr\log(bx+a)}{4\,(b^5x^4+4\,ab^4x^3+6\,a^2b^3x^2+4\,a^3b^2x+a^4b)}-\frac{qr\log(dx+c)}{4\,(b^5x^4+4\,ab^4x^3+6\,a^2b^3x^2+4\,a^3b^2x+a^4b)}-\frac{12\,b^3d^3qrx^3-6\,b^3cd^2qrx^2+42\,ab^2d^3qrx^2+4\,b^3c^2dqrx-20\,ab^2cd^2qrx+52\,a^2bd^3qrx+3\,b^3c^3pr-9\,ab^2c^2dpr+9\,a^2bcd^2pr-3\,a^3d^3pr+4\,ab^2c^2dqr-14\,a^2bcd^2qr+22\,a^3d^3qr+12\,b^3c^3r\log(f)-36\,ab^2c^2dr\log(f)+36\,a^2bcd^2r\log(f)-12\,a^3d^3r\log(f)+12\,b^3c^3-36\,ab^2c^2d+36\,a^2bcd^2-12\,a^3d^3}{48\,(b^8c^3x^4-3\,ab^7c^2dx^4+3\,a^2b^6cd^2x^4-a^3b^5d^3x^4+4\,ab^7c^3x^3-12\,a^2b^6c^2dx^3+12\,a^3b^5cd^2x^3-4\,a^4b^4d^3x^3+6\,a^2b^6c^3x^2-18\,a^3b^5c^2dx^2+18\,a^4b^4cd^2x^2-6\,a^5b^3d^3x^2+4\,a^3b^5c^3x-12\,a^4b^4c^2dx+12\,a^5b^3cd^2x-4\,a^6b^2d^3x+a^4b^4c^3-3\,a^5b^3c^2d+3\,a^6b^2cd^2-a^7bd^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/4*d^4*q*r*log(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*
a^3*b^2*c*d^3 + a^4*b*d^4) + 1/4*d^4*q*r*log(d*x + c)/(b^5*c^4 - 4*a*b^4*c^
3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 1/4*p*r*log(b*x +
a)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b) - 1/4*q*r*
log(d*x + c)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)
- 1/48*(12*b^3*d^3*q*r*x^3 - 6*b^3*c*d^2*q*r*x^2 + 42*a*b^2*d^3*q*r*x^2 + 4
*b^3*c^2*d*q*r*x - 20*a*b^2*c*d^2*q*r*x + 52*a^2*b*d^3*q*r*x + 3*b^3*c^3*p*
r - 9*a*b^2*c^2*d*p*r + 9*a^2*b*c*d^2*p*r - 3*a^3*d^3*p*r + 4*a*b^2*c^2*d*q
*r - 14*a^2*b*c*d^2*q*r + 22*a^3*d^3*q*r + 12*b^3*c^3*r*log(f) - 36*a*b^2*c
^2*d*r*log(f) + 36*a^2*b*c*d^2*r*log(f) - 12*a^3*d^3*r*log(f) + 12*b^3*c^3
- 36*a*b^2*c^2*d + 36*a^2*b*c*d^2 - 12*a^3*d^3)/(b^8*c^3*x^4 - 3*a*b^7*c^2*
d*x^4 + 3*a^2*b^6*c*d^2*x^4 - a^3*b^5*d^3*x^4 + 4*a*b^7*c^3*x^3 - 12*a^2*b^
6*c^2*d*x^3 + 12*a^3*b^5*c*d^2*x^3 - 4*a^4*b^4*d^3*x^3 + 6*a^2*b^6*c^3*x^2
- 18*a^3*b^5*c^2*d*x^2 + 18*a^4*b^4*c*d^2*x^2 - 6*a^5*b^3*d^3*x^2 + 4*a^3*b
^5*c^3*x - 12*a^4*b^4*c^2*d*x + 12*a^5*b^3*c*d^2*x - 4*a^6*b^2*d^3*x + a^4*
b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)
```

Mupad [B]
time = 3.71, size = 526, normalized size = 2.73

$$\frac{\frac{3\,b^3\,c^3\,p\,r-3\,a^3\,d^3\,p\,r+22\,a^3\,d^3\,q\,r-9\,a\,b^2\,c^2\,d\,p\,r+9\,a^2\,b\,c\,d^2\,p\,r+4\,a\,b^2\,c^2\,d\,q\,r-14\,a^2\,b\,c\,d^2\,q\,r}{12\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}+\frac{x\,(13\,q\,r\,a^2\,b\,d^3-5\,q\,r\,a\,b^2\,c\,d^2+q\,r\,b^3\,c^2\,d)}{3\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}+\frac{d\,x^2\,(7\,a\,b^2\,d^2\,q\,r-b^3\,c\,d\,q\,r)}{2\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}+\frac{b^3\,d^3\,q\,r\,x^3}{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}}{4\,a^4\,b+16\,a^3\,b^2\,x+24\,a^2\,b^3\,x^2+16\,a\,b^4\,x^3+4\,b^5\,x^4}-\frac{\ln\left(e\,{\left(f\,{(a+b\,x)}^p\,{(c+d\,x)}^q\right)}^r\right)\,\left(\frac{x}{4}+\frac{a}{4\,b}\right)}{{(a+b\,x)}^5}+\frac{d^4\,q\,r\,\mathrm{atanh}\left(\frac{-4\,a^4\,b\,d^4+8\,a^3\,b^2\,c\,d^3-8\,a\,b^4\,c^3\,d+4\,b^5\,c^4}{4\,b\,{(a\,d-b\,c)}^4}-\frac{2\,b\,d\,x\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}{{(a\,d-b\,c)}^4}\right)}{2\,b\,{(a\,d-b\,c)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(a + b*x)^5,x)
```

```
[Out] ((3*b^3*c^3*p*r - 3*a^3*d^3*p*r + 22*a^3*d^3*q*r - 9*a*b^2*c^2*d*p*r + 9*a^
2*b*c*d^2*p*r + 4*a*b^2*c^2*d*q*r - 14*a^2*b*c*d^2*q*r)/(12*(a^3*d^3 - b^3*
c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (x*(13*a^2*b*d^3*q*r + b^3*c^2*d*q*
r - 5*a*b^2*c*d^2*q*r))/(3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d
^2)) + (d*x^2*(7*a*b^2*d^2*q*r - b^3*c*d*q*r))/(2*(a^3*d^3 - b^3*c^3 + 3*a*
b^2*c^2*d - 3*a^2*b*c*d^2)) + (b^3*d^3*q*r*x^3)/(a^3*d^3 - b^3*c^3 + 3*a*b^
```

```
2*c^2*d - 3*a^2*b*c*d^2))/(4*a^4*b + 4*b^5*x^4 + 16*a^3*b^2*x + 16*a*b^4*x^
3 + 24*a^2*b^3*x^2) - (log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(x/4 + a/(4*b))
)/(a + b*x)^5 + (d^4*q*r*atanh((4*b^5*c^4 - 4*a^4*b*d^4 + 8*a^3*b^2*c*d^3 -
 8*a*b^4*c^3*d)/(4*b*(a*d - b*c)^4) - (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b^2
*c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^4))/(2*b*(a*d - b*c)^4)
```

3.16 $\int (a+bx)^4 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$

Optimal. Leaf size=920

$$-\frac{a(bc-ad)^3pqr^2x}{5d^3}+\frac{2(bc-ad)^4pqr^2x}{25d^4}+\frac{77(bc-ad)^4q^2r^2x}{150d^4}+\frac{2(bc-ad)^4q(p+q)r^2x}{5d^4}-\frac{b(bc-ad)^3pqr^2x^2}{10d^3}-(b$$

```
[Out] -77/300*(-a*d+b*c)^3*q^2*r^2*(b*x+a)^2/b/d^3+47/450*(-a*d+b*c)^2*q^2*r^2*(b
*x+a)^3/b/d^2-9/200*(-a*d+b*c)*q^2*r^2*(b*x+a)^4/b/d+2/25*(-a*d+b*c)^4*p*q*
r^2*x/d^4+2/5*(-a*d+b*c)^4*q*(p+q)*r^2*x/d^4+1/5*(b*x+a)^5*ln(e*(f*(b*x+a)^
p*(d*x+c)^q)^r)^2/b-1/5*a*(-a*d+b*c)^3*p*q*r^2*x/d^3-1/10*b*(-a*d+b*c)^3*p*
q*r^2*x^2/d^3-1/25*(-a*d+b*c)^3*p*q*r^2*(b*x+a)^2/b/d^3+16/225*(-a*d+b*c)^2
*p*q*r^2*(b*x+a)^3/b/d^2-9/200*(-a*d+b*c)*p*q*r^2*(b*x+a)^4/b/d-2/5*(-a*d+b
*c)^5*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c))/b/d^5+2/125*p^2*r^2*(b*x+a)^5
/b+2/125*q^2*r^2*(b*x+a)^5/b-2/25*p*r*(b*x+a)^5*ln(e*(f*(b*x+a)^p*(d*x+c)^q
)^r)/b-2/25*q*r*(b*x+a)^5*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b+77/150*(-a*d+b*
c)^4*q^2*r^2*x/d^4+4/125*p*q*r^2*(b*x+a)^5/b-2/25*(-a*d+b*c)^5*p*q*r^2*ln(d
*x+c)/b/d^5-2/5*(-a*d+b*c)^4*q*r*(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/
d^4+1/5*(-a*d+b*c)^3*q*r*(b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^3-2/
15*(-a*d+b*c)^2*q*r*(b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^2+1/10*(-
a*d+b*c)*q*r*(b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d+2/5*(-a*d+b*c)^5
*q*r*ln(d*x+c)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^5-137/150*(-a*d+b*c)^5*q
^2*r^2*ln(d*x+c)/b/d^5-1/5*(-a*d+b*c)^5*q^2*r^2*ln(d*x+c)^2/b/d^5-2/5*(-a*d
+b*c)^5*p*q*r^2*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/b/d^5
```

Rubi [A]
time = 0.61, antiderivative size = 920, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 14, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {2584, 2581, 32, 45, 2594, 2579, 31, 8, 2580, 2441, 2440, 2438, 2437, 2338}

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] -1/5*(a*(b*c - a*d)^3*p*q*r^2*x)/d^3 + (2*(b*c - a*d)^4*p*q*r^2*x)/(25*d^4)
 + (77*(b*c - a*d)^4*q^2*r^2*x)/(150*d^4) + (2*(b*c - a*d)^4*q*(p + q)*r^2*
x)/(5*d^4) - (b*(b*c - a*d)^3*p*q*r^2*x^2)/(10*d^3) - ((b*c - a*d)^3*p*q*r^
2*(a + b*x)^2)/(25*b*d^3) - (77*(b*c - a*d)^3*q^2*r^2*(a + b*x)^2)/(300*b*d
^3) + (16*(b*c - a*d)^2*p*q*r^2*(a + b*x)^3)/(225*b*d^2) + (47*(b*c - a*d)^
2*q^2*r^2*(a + b*x)^3)/(450*b*d^2) - (9*(b*c - a*d)*p*q*r^2*(a + b*x)^4)/(2
00*b*d) - (9*(b*c - a*d)*q^2*r^2*(a + b*x)^4)/(200*b*d) + (2*p^2*r^2*(a + b
*x)^5)/(125*b) + (4*p*q*r^2*(a + b*x)^5)/(125*b) + (2*q^2*r^2*(a + b*x)^5)/
(125*b) - (2*(b*c - a*d)^5*p*q*r^2*Log[c + d*x])/(25*b*d^5) - (137*(b*c - a
*d)^5*q^2*r^2*Log[c + d*x])/(150*b*d^5) - (2*(b*c - a*d)^5*p*q*r^2*Log[-((d
```

```
*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(5*b*d^5) - ((b*c - a*d)^5*q^2*r^2*
Log[c + d*x]^2)/(5*b*d^5) - (2*(b*c - a*d)^4*q*r*(a + b*x)*Log[e*(f*(a + b*
x)^p*(c + d*x)^q)^r])/(5*b*d^4) + ((b*c - a*d)^3*q*r*(a + b*x)^2*Log[e*(f*(
a + b*x)^p*(c + d*x)^q)^r])/(5*b*d^3) - (2*(b*c - a*d)^2*q*r*(a + b*x)^3*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(15*b*d^2) + ((b*c - a*d)*q*r*(a + b*x)
^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(10*b*d) - (2*p*r*(a + b*x)^5*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(25*b) - (2*q*r*(a + b*x)^5*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/(25*b) + (2*(b*c - a*d)^5*q*r*Log[c + d*x]*Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r])/(5*b*d^5) + ((a + b*x)^5*Log[e*(f*(a + b*x)^
p*(c + d*x)^q)^r]^2)/(5*b) - (2*(b*c - a*d)^5*p*q*r^2*PolyLog[2, (b*(c + d*
x))/(b*c - a*d)])/(5*b*d^5)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2579

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[(a + b*x)*(Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s/b), x] + (Dist[q*r*s*((b*c - a*d)/b), Int[Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]^(s - 1)/(c + d*x), x], x] - Dist[r*s*(p + q), Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^(s - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p, q,
r, s}, x] && NeQ[b*c - a*d, 0] && NeQ[p + q, 0] && IGtQ[s, 0] && LtQ[s, 4]
```

Rule 2580

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/h), x] + (-Dist[b*p*(r/h), Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[d*q*(r/h), Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 2584

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
```

```
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Dist[b*p*r*(
s/(h*(m + 1))), Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1)/(a + b*x)), x], x] - Dist[d*q*r*(s/(h*(m + 1))), Int[(g + h*x)^(m
+ 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2594

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rubi steps

$$\int (a+bx)^4 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx = \frac{(a+bx)^5 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5b} - \frac{1}{5}(2pr) \int (a+b$$

$$= -\frac{2pr(a+bx)^5 \log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{25b} + \frac{(a+bx)^5 \mathrm{lo}}{}$$

$$= \frac{2p^2r^2(a+bx)^5}{125b} - \frac{2pr(a+bx)^5 \log\left(e(f(a+bx)^p(c+dx)^q)\right.}{25b}$$

$$= \frac{2(bc-ad)^4pqr^2x}{25d^4} - \frac{(bc-ad)^3pqr^2(a+bx)^2}{25bd^3} + \frac{2(bc-ad)}{7}$$

$$= -\frac{a(bc-ad)^3pqr^2x}{5d^3} + \frac{2(bc-ad)^4pqr^2x}{25d^4} + \frac{2(bc-ad)^4q(p}{5d^4}$$

$$= -\frac{a(bc-ad)^3pqr^2x}{5d^3} + \frac{2(bc-ad)^4pqr^2x}{25d^4} + \frac{77(bc-ad)^4q^2r}{150d^4}$$

$$= -\frac{a(bc-ad)^3pqr^2x}{5d^3} + \frac{2(bc-ad)^4pqr^2x}{25d^4} + \frac{77(bc-ad)^4q^2r}{150d^4}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 2508 vs. $2(920) = 1840$.
time = 1.73, size = 2508, normalized size = 2.73

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (2*a^5*p*q*r^2)/b + (2*a*b^3*c^4*p*q*r^2)/(5*d^4) - (2*a^2*b^2*c^3*p*q*r^2)
/d^3 + (4*a^3*b*c^2*p*q*r^2)/d^2 - (4*a^4*c*p*q*r^2)/d + (2*a^4*p^2*r^2*x)/
25 + (197*a^4*p*q*r^2*x)/150 + (12*b^4*c^4*p*q*r^2*x)/(25*d^4) - (11*a*b^3*
c^3*p*q*r^2*x)/(5*d^3) + (59*a^2*b^2*c^2*p*q*r^2*x)/(15*d^2) - (101*a^3*b*c
*p*q*r^2*x)/(30*d) + 2*a^4*q^2*r^2*x + (137*b^4*c^4*q^2*r^2*x)/(150*d^4) -
(25*a*b^3*c^3*q^2*r^2*x)/(6*d^3) + (22*a^2*b^2*c^2*q^2*r^2*x)/(3*d^2) - (6*
a^3*b*c*q^2*r^2*x)/d + (4*a^3*b*p^2*r^2*x^2)/25 + (283*a^3*b*p*q*r^2*x^2)/3
00 - (7*b^4*c^3*p*q*r^2*x^2)/(50*d^3) + (19*a*b^3*c^2*p*q*r^2*x^2)/(30*d^2)
 - (67*a^2*b^2*c*p*q*r^2*x^2)/(60*d) + a^3*b*q^2*r^2*x^2 - (77*b^4*c^3*q^2*
r^2*x^2)/(300*d^3) + (13*a*b^3*c^2*q^2*r^2*x^2)/(12*d^2) - (5*a^2*b^2*c*q^2
*r^2*x^2)/(3*d) + (4*a^2*b^2*p^2*r^2*x^3)/25 + (257*a^2*b^2*p*q*r^2*x^3)/45
0 + (16*b^4*c^2*p*q*r^2*x^3)/(225*d^2) - (29*a*b^3*c*p*q*r^2*x^3)/(90*d) +
(4*a^2*b^2*q^2*r^2*x^3)/9 + (47*b^4*c^2*q^2*r^2*x^3)/(450*d^2) - (7*a*b^3*c
*q^2*r^2*x^3)/(18*d) + (2*a*b^3*p^2*r^2*x^4)/25 + (41*a*b^3*p*q*r^2*x^4)/20
0 - (9*b^4*c*p*q*r^2*x^4)/(200*d) + (a*b^3*q^2*r^2*x^4)/8 - (9*b^4*c*q^2*r^
2*x^4)/(200*d) + (2*b^4*p^2*r^2*x^5)/125 + (4*b^4*p*q*r^2*x^5)/125 + (2*b^4
*q^2*r^2*x^5)/125 - (a^5*p^2*r^2*Log[a + b*x]^2)/(5*b) + (2*a^5*p*q*r^2*Log
[c + d*x])/b - (2*b^4*c^5*p*q*r^2*Log[c + d*x])/(25*d^5) + (2*a*b^3*c^4*p*q
*r^2*Log[c + d*x])/(5*d^4) - (4*a^2*b^2*c^3*p*q*r^2*Log[c + d*x])/(5*d^3) +
 (4*a^3*b*c^2*p*q*r^2*Log[c + d*x])/(5*d^2) - (2*a^4*c*p*q*r^2*Log[c + d*x]
)/(5*d) - (137*b^4*c^5*q^2*r^2*Log[c + d*x])/(150*d^5) + (25*a*b^3*c^4*q^2*
r^2*Log[c + d*x])/(6*d^4) - (22*a^2*b^2*c^3*q^2*r^2*Log[c + d*x])/(3*d^3) +
 (6*a^3*b*c^2*q^2*r^2*Log[c + d*x])/d^2 - (2*a^4*c*q^2*r^2*Log[c + d*x])/d
- (b^4*c^5*q^2*r^2*Log[c + d*x]^2)/(5*d^5) + (a*b^3*c^4*q^2*r^2*Log[c + d*x
]^2)/d^4 - (2*a^2*b^2*c^3*q^2*r^2*Log[c + d*x]^2)/d^3 + (2*a^3*b*c^2*q^2*r^
2*Log[c + d*x]^2)/d^2 - (a^4*c*q^2*r^2*Log[c + d*x]^2)/d - (2*a^5*p*r*Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r])/b - (2*a^4*p*r*x*Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r])/5 - 2*a^4*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - (2*b^4*
c^4*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(5*d^4) + (2*a*b^3*c^3*q*r*
x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^3 - (4*a^2*b^2*c^2*q*r*x*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r])/d^2 + (4*a^3*b*c*q*r*x*Log[e*(f*(a + b*x)^p*(
c + d*x)^q)^r])/d - (4*a^3*b*p*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/
5 - 2*a^3*b*q*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (b^4*c^3*q*r*x^2
*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(5*d^3) - (a*b^3*c^2*q*r*x^2*Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r])/d^2 + (2*a^2*b^2*c*q*r*x^2*Log[e*(f*(a + b*x
)^p*(c + d*x)^q)^r])/d - (4*a^2*b^2*p*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r])/5 - (4*a^2*b^2*q*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/3 - (2*
b^4*c^2*q*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(15*d^2) + (2*a*b^3*c
*q*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*d) - (2*a*b^3*p*r*x^4*Log
[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/5 - (a*b^3*q*r*x^4*Log[e*(f*(a + b*x)^p*
(c + d*x)^q)^r])/2 + (b^4*c*q*r*x^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(
10*d) - (2*b^4*p*r*x^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/25 - (2*b^4*q*
r*x^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/25 + (2*b^4*c^5*q*r*Log[c + d*x
```

```
]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(5*d^5) - (2*a*b^3*c^4*q*r*Log[c +
d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^4 + (4*a^2*b^2*c^3*q*r*Log[c +
 d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^3 - (4*a^3*b*c^2*q*r*Log[c +
d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^2 + (2*a^4*c*q*r*Log[c + d*x]*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d + a^4*x*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^2 + 2*a^3*b*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + 2*a^2*b^
2*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + a*b^3*x^4*Log[e*(f*(a + b*x)
^p*(c + d*x)^q)^r]^2 + (b^4*x^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/5 +
 (p*r*Log[a + b*x]*(a*d*(a^4*d^4*(288*p - 137*q) - 60*b^4*c^4*q + 270*a*b^3
*c^3*d*q - 470*a^2*b^2*c^2*d^2*q + 385*a^3*b*c*d^3*q)*r - 60*b*c*(b^4*c^4 -
 5*a*b^3*c^3*d + 10*a^2*b^2*c^2*d^2 - 10*a^3*b*c*d^3 + 5*a^4*d^4)*q*r*Log[c
 + d*x] + 60*(b*c - a*d)^5*q*r*Log[(b*(c + d*x))/(b*c - a*d)] + 60*a^5*d^5*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(150*b*d^5) + (2*(b*c - a*d)^5*p*q*r
^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*d^5)
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \left(bx+a\right)^{4} \ln \left(e {\left(f \left(bx+a\right)^{p} \left(dx+c\right)^{q}\right)}^{r}\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

```
[Out] int((b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

Maxima [A]
time = 0.34, size = 1423, normalized size = 1.55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima
")
```

```
[Out] 1/5*(b^4*x^5 + 5*a*b^3*x^4 + 10*a^2*b^2*x^3 + 10*a^3*b*x^2 + 5*a^4*x)*log((
(b*x + a)^p*(d*x + c)^q*f)^r*e)^2 + 1/150*(60*a^5*f*p*log(b*x + a)/b - (12*
b^4*d^4*f*(p + q)*x^5 + 15*(a*b^3*d^4*f*(4*p + 5*q) - b^4*c*d^3*f*q)*x^4 +
20*(2*a^2*b^2*d^4*f*(3*p + 5*q) + b^4*c^2*d^2*f*q - 5*a*b^3*c*d^3*f*q)*x^3
+ 30*(2*a^3*b*d^4*f*(2*p + 5*q) - b^4*c^3*d*f*q + 5*a*b^3*c^2*d^2*f*q - 10*
a^2*b^2*c*d^3*f*q)*x^2 + 60*(a^4*d^4*f*(p + 5*q) + b^4*c^4*f*q - 5*a*b^3*c^
3*d*f*q + 10*a^2*b^2*c^2*d^2*f*q - 10*a^3*b*c*d^3*f*q)*x)/d^4 + 60*(b^4*c^5
*f*q - 5*a*b^3*c^4*d*f*q + 10*a^2*b^2*c^3*d^2*f*q - 10*a^3*b*c^2*d^3*f*q +
5*a^4*c*d^4*f*q)*log(d*x + c)/d^5)*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/f
 - 1/9000*r^2*(60*((12*p*q + 137*q^2)*b^4*c^5*f^2 - 5*(12*p*q + 125*q^2)*a*
b^3*c^4*d*f^2 + 20*(6*p*q + 55*q^2)*a^2*b^2*c^3*d^2*f^2 - 60*(2*p*q + 15*q^
```

```
2)*a^3*b*c^2*d^3*f^2 + 60*(p*q + 5*q^2)*a^4*c*d^4*f^2)*log(d*x + c)/d^5 - 3
600*(b^5*c^5*f^2*p*q - 5*a*b^4*c^4*d*f^2*p*q + 10*a^2*b^3*c^3*d^2*f^2*p*q -
 10*a^3*b^2*c^2*d^3*f^2*p*q + 5*a^4*b*c*d^4*f^2*p*q - a^5*d^5*f^2*p*q)*(log
(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c -
a*d)))/(b*d^5) - (144*(p^2 + 2*p*q + q^2)*b^5*d^5*f^2*x^5 - 1800*a^5*d^5*f^
2*p^2*log(b*x + a)^2 - 45*(9*(p*q + q^2)*b^5*c*d^4*f^2 - (16*p^2 + 41*p*q +
 25*q^2)*a*b^4*d^5*f^2)*x^4 + 20*((32*p*q + 47*q^2)*b^5*c^2*d^3*f^2 - 5*(29
*p*q + 35*q^2)*a*b^4*c*d^4*f^2 + (72*p^2 + 257*p*q + 200*q^2)*a^2*b^3*d^5*f
^2)*x^3 - 30*(7*(6*p*q + 11*q^2)*b^5*c^3*d^2*f^2 - 5*(38*p*q + 65*q^2)*a*b^
4*c^2*d^3*f^2 + 5*(67*p*q + 100*q^2)*a^2*b^3*c*d^4*f^2 - (48*p^2 + 283*p*q
+ 300*q^2)*a^3*b^2*d^5*f^2)*x^2 - 3600*(b^5*c^5*f^2*p*q - 5*a*b^4*c^4*d*f^2
*p*q + 10*a^2*b^3*c^3*d^2*f^2*p*q - 10*a^3*b^2*c^2*d^3*f^2*p*q + 5*a^4*b*c*
d^4*f^2*p*q)*log(b*x + a)*log(d*x + c) - 1800*(b^5*c^5*f^2*q^2 - 5*a*b^4*c^
4*d*f^2*q^2 + 10*a^2*b^3*c^3*d^2*f^2*q^2 - 10*a^3*b^2*c^2*d^3*f^2*q^2 + 5*a
^4*b*c*d^4*f^2*q^2)*log(d*x + c)^2 + 60*((72*p*q + 137*q^2)*b^5*c^4*d*f^2 -
 5*(66*p*q + 125*q^2)*a*b^4*c^3*d^2*f^2 + 10*(59*p*q + 110*q^2)*a^2*b^3*c^2
*d^3*f^2 - 5*(101*p*q + 180*q^2)*a^3*b^2*c*d^4*f^2 + (12*p^2 + 197*p*q + 30
0*q^2)*a^4*b*d^5*f^2)*x - 60*(60*a*b^4*c^4*d*f^2*p*q - 270*a^2*b^3*c^3*d^2*
f^2*p*q + 470*a^3*b^2*c^2*d^3*f^2*p*q - 385*a^4*b*c*d^4*f^2*p*q + (12*p^2 +
 137*p*q)*a^5*d^5*f^2)*log(b*x + a))/(b*d^5))/f^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas
")
```

```
[Out] integral((b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*log(((b*
x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2\,(a+b\,x)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(a + b*x)^4,x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(a + b*x)^4, x)
```

3.17 $\int (a+bx)^3 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx$

Optimal. Leaf size=805

$$\frac{a(bc-ad)^2pqr^2x}{4d^2}-\frac{(bc-ad)^3pqr^2x}{8d^3}-\frac{13(bc-ad)^3q^2r^2x}{24d^3}-\frac{(bc-ad)^3q(p+q)r^2x}{2d^3}+\frac{b(bc-ad)^2pqr^2x^2}{8d^2}+\frac{(bc-}{}$$

```
[Out] 1/4*a*(-a*d+b*c)^2*p*q*r^2*x/d^2-1/8*(-a*d+b*c)^3*p*q*r^2*x/d^3-13/24*(-a*d
+b*c)^3*q^2*r^2*x/d^3-1/2*(-a*d+b*c)^3*q*(p+q)*r^2*x/d^3+1/8*b*(-a*d+b*c)^2
*p*q*r^2*x^2/d^2+1/16*(-a*d+b*c)^2*p*q*r^2*(b*x+a)^2/b/d^2+13/48*(-a*d+b*c)
^2*q^2*r^2*(b*x+a)^2/b/d^2-7/72*(-a*d+b*c)*p*q*r^2*(b*x+a)^3/b/d-7/72*(-a*d
+b*c)*q^2*r^2*(b*x+a)^3/b/d+1/32*p^2*r^2*(b*x+a)^4/b+1/16*p*q*r^2*(b*x+a)^4
/b+1/32*q^2*r^2*(b*x+a)^4/b+1/8*(-a*d+b*c)^4*p*q*r^2*ln(d*x+c)/b/d^4+25/24*
(-a*d+b*c)^4*q^2*r^2*ln(d*x+c)/b/d^4+1/2*(-a*d+b*c)^4*p*q*r^2*ln(-d*(b*x+a)
/(-a*d+b*c))*ln(d*x+c)/b/d^4+1/4*(-a*d+b*c)^4*q^2*r^2*ln(d*x+c)^2/b/d^4+1/2
*(-a*d+b*c)^3*q*r*(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^3-1/4*(-a*d+b
*c)^2*q*r*(b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^2+1/6*(-a*d+b*c)*q*
r*(b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d-1/8*p*r*(b*x+a)^4*ln(e*(f*(
b*x+a)^p*(d*x+c)^q)^r)/b-1/8*q*r*(b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/
b-1/2*(-a*d+b*c)^4*q*r*ln(d*x+c)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^4+1/4*
(b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/b+1/2*(-a*d+b*c)^4*p*q*r^2*poly
log(2,b*(d*x+c)/(-a*d+b*c))/b/d^4
```

Rubi [A]
time = 0.46, antiderivative size = 805, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 14, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {2584, 2581, 32, 45, 2594, 2579, 31, 8, 2580, 2441, 2440, 2438, 2437, 2338}

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (a*(b*c - a*d)^2*p*q*r^2*x)/(4*d^2) - ((b*c - a*d)^3*p*q*r^2*x)/(8*d^3) - (
13*(b*c - a*d)^3*q^2*r^2*x)/(24*d^3) - ((b*c - a*d)^3*q*(p + q)*r^2*x)/(2*d
^3) + (b*(b*c - a*d)^2*p*q*r^2*x^2)/(8*d^2) + ((b*c - a*d)^2*p*q*r^2*(a + b
*x)^2)/(16*b*d^2) + (13*(b*c - a*d)^2*q^2*r^2*(a + b*x)^2)/(48*b*d^2) - (7*
(b*c - a*d)*p*q*r^2*(a + b*x)^3)/(72*b*d) - (7*(b*c - a*d)*q^2*r^2*(a + b*x
)^3)/(72*b*d) + (p^2*r^2*(a + b*x)^4)/(32*b) + (p*q*r^2*(a + b*x)^4)/(16*b)
 + (q^2*r^2*(a + b*x)^4)/(32*b) + ((b*c - a*d)^4*p*q*r^2*Log[c + d*x])/(8*b
*d^4) + (25*(b*c - a*d)^4*q^2*r^2*Log[c + d*x])/(24*b*d^4) + ((b*c - a*d)^4
*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(2*b*d^4) + ((b*c
- a*d)^4*q^2*r^2*Log[c + d*x]^2)/(4*b*d^4) + ((b*c - a*d)^3*q*r*(a + b*x)*L
og[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b*d^3) - ((b*c - a*d)^2*q*r*(a + b*
x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*b*d^2) + ((b*c - a*d)*q*r*(a
```

```
+ b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(6*b*d) - (p*r*(a + b*x)^4*L
og[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(8*b) - (q*r*(a + b*x)^4*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/(8*b) - ((b*c - a*d)^4*q*r*Log[c + d*x]*Log[e*(f*(
a + b*x)^p*(c + d*x)^q)^r])/(2*b*d^4) + ((a + b*x)^4*Log[e*(f*(a + b*x)^p*(
c + d*x)^q)^r]^2)/(4*b) + ((b*c - a*d)^4*p*q*r^2*PolyLog[2, (b*(c + d*x))/(
b*c - a*d)])/(2*b*d^4)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2579

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[(a + b*x)*(Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s/b), x] + (Dist[q*r*s*((b*c - a*d)/b), Int[Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]^(s - 1)/(c + d*x), x], x] - Dist[r*s*(p + q), Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^(s - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p, q,
r, s}, x] && NeQ[b*c - a*d, 0] && NeQ[p + q, 0] && IGtQ[s, 0] && LtQ[s, 4]
```

Rule 2580

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/h), x] + (-Dist[b*p*(r/h), Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[d*q*(r/h), Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 2584

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Dist[b*p*r*(
s/(h*(m + 1))), Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1)/(a + b*x)), x], x] - Dist[d*q*r*(s/(h*(m + 1))), Int[(g + h*x)^(m
+ 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /; F
```

```
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2594

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rubi steps

$$\int (a+bx)^3 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \frac{(a+bx)^4\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b} - \frac{1}{2}(pr)\int (a+bx$$

$$= -\frac{pr(a+bx)^4\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{8b} + \frac{(a+bx)^4\log$$

$$= \frac{p^2r^2(a+bx)^4}{32b} - \frac{pr(a+bx)^4\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{8b}$$

$$= -\frac{(bc-ad)^3pqr^2x}{8d^3} + \frac{(bc-ad)^2pqr^2(a+bx)^2}{16bd^2} - \frac{(bc-ad)p}{2}$$

$$= \frac{a(bc-ad)^2pqr^2x}{4d^2} - \frac{(bc-ad)^3pqr^2x}{8d^3} - \frac{(bc-ad)^3q(p+q)}{2d^3}$$

$$= \frac{a(bc-ad)^2pqr^2x}{4d^2} - \frac{(bc-ad)^3pqr^2x}{8d^3} - \frac{13(bc-ad)^3q^2r^2x}{24d^3}$$

$$= \frac{a(bc-ad)^2pqr^2x}{4d^2} - \frac{(bc-ad)^3pqr^2x}{8d^3} - \frac{13(bc-ad)^3q^2r^2x}{24d^3}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1853 vs. $2(805) = 1610$.
time = 1.25, size = 1853, normalized size = 2.30

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
[Out] (2*a^4*p*q*r^2)/b - (a*b^2*c^3*p*q*r^2)/(2*d^3) + (2*a^2*b*c^2*p*q*r^2)/d^2
 - (3*a^3*c*p*q*r^2)/d + (a^3*p^2*r^2*x)/8 + (37*a^3*p*q*r^2*x)/24 - (5*b^3
```

```
*c^3*p*q*r^2*x)/(8*d^3) + (9*a*b^2*c^2*p*q*r^2*x)/(4*d^2) - (35*a^2*b*c*p*q
*r^2*x)/(12*d) + 2*a^3*q^2*r^2*x - (25*b^3*c^3*q^2*r^2*x)/(24*d^3) + (11*a*
b^2*c^2*q^2*r^2*x)/(3*d^2) - (9*a^2*b*c*q^2*r^2*x)/(2*d) + (3*a^2*b*p^2*r^2
*x^2)/16 + (41*a^2*b*p*q*r^2*x^2)/48 + (3*b^3*c^2*p*q*r^2*x^2)/(16*d^2) - (
2*a*b^2*c*p*q*r^2*x^2)/(3*d) + (3*a^2*b*q^2*r^2*x^2)/4 + (13*b^3*c^2*q^2*r^
2*x^2)/(48*d^2) - (5*a*b^2*c*q^2*r^2*x^2)/(6*d) + (a*b^2*p^2*r^2*x^3)/8 + (
25*a*b^2*p*q*r^2*x^3)/72 - (7*b^3*c*p*q*r^2*x^3)/(72*d) + (2*a*b^2*q^2*r^2*
x^3)/9 - (7*b^3*c*q^2*r^2*x^3)/(72*d) + (b^3*p^2*r^2*x^4)/32 + (b^3*p*q*r^2
*x^4)/16 + (b^3*q^2*r^2*x^4)/32 - (a^4*p^2*r^2*Log[a + b*x]^2)/(4*b) + (2*a
^4*p*q*r^2*Log[c + d*x])/b + (b^3*c^4*p*q*r^2*Log[c + d*x])/(8*d^4) - (a*b^
2*c^3*p*q*r^2*Log[c + d*x])/(2*d^3) + (3*a^2*b*c^2*p*q*r^2*Log[c + d*x])/(4
*d^2) - (a^3*c*p*q*r^2*Log[c + d*x])/(2*d) + (25*b^3*c^4*q^2*r^2*Log[c + d*
x])/(24*d^4) - (11*a*b^2*c^3*q^2*r^2*Log[c + d*x])/(3*d^3) + (9*a^2*b*c^2*q
^2*r^2*Log[c + d*x])/(2*d^2) - (2*a^3*c*q^2*r^2*Log[c + d*x])/d + (b^3*c^4*
q^2*r^2*Log[c + d*x]^2)/(4*d^4) - (a*b^2*c^3*q^2*r^2*Log[c + d*x]^2)/d^3 +
(3*a^2*b*c^2*q^2*r^2*Log[c + d*x]^2)/(2*d^2) - (a^3*c*q^2*r^2*Log[c + d*x]^
2)/d - (2*a^4*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/b - (a^3*p*r*x*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/2 - 2*a^3*q*r*x*Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r] + (b^3*c^3*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*d^3)
- (2*a*b^2*c^2*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^2 + (3*a^2*b*c
*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d - (3*a^2*b*p*r*x^2*Log[e*(f*
(a + b*x)^p*(c + d*x)^q)^r])/4 - (3*a^2*b*q*r*x^2*Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r])/2 - (b^3*c^2*q*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*
d^2) + (a*b^2*c*q*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d - (a*b^2*p*
r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/2 - (2*a*b^2*q*r*x^3*Log[e*(f*(
a + b*x)^p*(c + d*x)^q)^r])/3 + (b^3*c*q*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*
x)^q)^r])/(6*d) - (b^3*p*r*x^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/8 - (b
^3*q*r*x^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/8 - (b^3*c^4*q*r*Log[c + d
*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*d^4) + (2*a*b^2*c^3*q*r*Log[c
+ d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^3 - (3*a^2*b*c^2*q*r*Log[c +
 d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^2 + (2*a^3*c*q*r*Log[c + d*x]
*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d + a^3*x*Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^2 + (3*a^2*b*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/2 + a*b
^2*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + (b^3*x^4*Log[e*(f*(a + b*x)
^p*(c + d*x)^q)^r]^2)/4 + (p*r*Log[a + b*x]*(a*d*(5*a^3*d^3*(9*p - 5*q) + 1
2*b^3*c^3*q - 42*a*b^2*c^2*d*q + 52*a^2*b*c*d^2*q)*r + 12*b*c*(b^3*c^3 - 4*
a*b^2*c^2*d + 6*a^2*b*c*d^2 - 4*a^3*d^3)*q*r*Log[c + d*x] - 12*(b*c - a*d)^
4*q*r*Log[(b*(c + d*x))/(b*c - a*d)] + 12*a^4*d^4*Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]))/(24*b*d^4) - ((b*c - a*d)^4*p*q*r^2*PolyLog[2, (d*(a + b*x))/
(-(b*c) + a*d)])/(2*b*d^4)
```

Maple [F]

time = 0.21, size = 0, normalized size = 0.00

$$\int (bx+a)^3 \ln\left(e(f(bx+a)^p (dx+c)^q)^r\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

```
[Out] int((b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

Maxima [A]
time = 0.31, size = 1073, normalized size = 1.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima
")
```

```
[Out] 1/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)*log(((b*x + a)^p*(d*x +
 c)^q*f)^r*e)^2 + 1/24*(12*a^4*f*p*log(b*x + a)/b - (3*b^3*d^3*f*(p + q)*x^
4 + 4*(a*b^2*d^3*f*(3*p + 4*q) - b^3*c*d^2*f*q)*x^3 + 6*(3*a^2*b*d^3*f*(p +
 2*q) + b^3*c^2*d*f*q - 4*a*b^2*c*d^2*f*q)*x^2 + 12*(a^3*d^3*f*(p + 4*q) -
b^3*c^3*f*q + 4*a*b^2*c^2*d*f*q - 6*a^2*b*c*d^2*f*q)*x)/d^3 - 12*(b^3*c^4*f
*q - 4*a*b^2*c^3*d*f*q + 6*a^2*b*c^2*d^2*f*q - 4*a^3*c*d^3*f*q)*log(d*x + c
)/d^4)*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/f + 1/288*r^2*(12*((3*p*q + 2
5*q^2)*b^3*c^4*f^2 - 4*(3*p*q + 22*q^2)*a*b^2*c^3*d*f^2 + 18*(p*q + 6*q^2)*
a^2*b*c^2*d^2*f^2 - 12*(p*q + 4*q^2)*a^3*c*d^3*f^2)*log(d*x + c)/d^4 - 144*
(b^4*c^4*f^2*p*q - 4*a*b^3*c^3*d*f^2*p*q + 6*a^2*b^2*c^2*d^2*f^2*p*q - 4*a^
3*b*c*d^3*f^2*p*q + a^4*d^4*f^2*p*q)*(log(b*x + a)*log((b*d*x + a*d)/(b*c -
 a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/(b*d^4) + (9*(p^2 + 2*p*q +
 q^2)*b^4*d^4*f^2*x^4 - 72*a^4*d^4*f^2*p^2*log(b*x + a)^2 - 4*(7*(p*q + q^2
)*b^4*c*d^3*f^2 - (9*p^2 + 25*p*q + 16*q^2)*a*b^3*d^4*f^2)*x^3 + 6*((9*p*q
+ 13*q^2)*b^4*c^2*d^2*f^2 - 8*(4*p*q + 5*q^2)*a*b^3*c*d^3*f^2 + (9*p^2 + 41
*p*q + 36*q^2)*a^2*b^2*d^4*f^2)*x^2 + 144*(b^4*c^4*f^2*p*q - 4*a*b^3*c^3*d*
f^2*p*q + 6*a^2*b^2*c^2*d^2*f^2*p*q - 4*a^3*b*c*d^3*f^2*p*q)*log(b*x + a)*l
og(d*x + c) + 72*(b^4*c^4*f^2*q^2 - 4*a*b^3*c^3*d*f^2*q^2 + 6*a^2*b^2*c^2*d
^2*f^2*q^2 - 4*a^3*b*c*d^3*f^2*q^2)*log(d*x + c)^2 - 12*(5*(3*p*q + 5*q^2)*
b^4*c^3*d*f^2 - 2*(27*p*q + 44*q^2)*a*b^3*c^2*d^2*f^2 + 2*(35*p*q + 54*q^2)
*a^2*b^2*c*d^3*f^2 - (3*p^2 + 37*p*q + 48*q^2)*a^3*b*d^4*f^2)*x + 12*(12*a*
b^3*c^3*d*f^2*p*q - 42*a^2*b^2*c^2*d^2*f^2*p*q + 52*a^3*b*c*d^3*f^2*p*q - (
3*p^2 + 25*p*q)*a^4*d^4*f^2)*log(b*x + a))/(b*d^4))/f^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas
")
```

```
[Out] integral((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*log(((b*x + a)^p*(d*x +
c)^q*f)^r*e)^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^3*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(e\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)^r\right)^2\,{\left(a+b\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(a + b*x)^3,x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(a + b*x)^3, x)
```

3.18 $\int (a+bx)^2 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$

Optimal. Leaf size=686

$$-\frac{a(bc-ad)pqr^2x}{3d}+\frac{2(bc-ad)^2pqr^2x}{9d^2}+\frac{5(bc-ad)^2q^2r^2x}{9d^2}+\frac{2(bc-ad)^2q(p+q)r^2x}{3d^2}-\frac{b(bc-ad)pqr^2x^2}{6d}-\frac{(bc}{}$$

```
[Out] -1/3*a*(-a*d+b*c)*p*q*r^2*x/d+2/9*(-a*d+b*c)^2*p*q*r^2*x/d^2+5/9*(-a*d+b*c)
^2*q^2*r^2*x/d^2+2/3*(-a*d+b*c)^2*q*(p+q)*r^2*x/d^2-1/6*b*(-a*d+b*c)*p*q*r^
2*x^2/d-1/9*(-a*d+b*c)*p*q*r^2*(b*x+a)^2/b/d-5/18*(-a*d+b*c)*q^2*r^2*(b*x+a
)^2/b/d+2/27*p^2*r^2*(b*x+a)^3/b+4/27*p*q*r^2*(b*x+a)^3/b+2/27*q^2*r^2*(b*x
+a)^3/b-2/9*(-a*d+b*c)^3*p*q*r^2*ln(d*x+c)/b/d^3-11/9*(-a*d+b*c)^3*q^2*r^2*
ln(d*x+c)/b/d^3-2/3*(-a*d+b*c)^3*p*q*r^2*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c
)/b/d^3-1/3*(-a*d+b*c)^3*q^2*r^2*ln(d*x+c)^2/b/d^3-2/3*(-a*d+b*c)^2*q*r*(b*
x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^2+1/3*(-a*d+b*c)*q*r*(b*x+a)^2*ln(
e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d-2/9*p*r*(b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c
)^q)^r)/b-2/9*q*r*(b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b+2/3*(-a*d+b*c
)^3*q*r*ln(d*x+c)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d^3+1/3*(b*x+a)^3*ln(e*
(f*(b*x+a)^p*(d*x+c)^q)^r)^2/b-2/3*(-a*d+b*c)^3*p*q*r^2*polylog(2,b*(d*x+c)
/(-a*d+b*c))/b/d^3
```

Rubi [A]

time = 0.38, antiderivative size = 686, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 14, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {2584, 2581, 32, 45, 2594, 2579, 31, 8, 2580, 2441, 2440, 2438, 2437, 2338}

$$-\frac{2pqr^2(bc-ad)^3\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{3bd^3}+\frac{2qr(bc-ad)^3\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3bd^3}-\frac{2pqr^2(bc-ad)^3\log(c+dx)}{9bd^3}-\frac{2pqr^2(bc-ad)^3\log(c+dx)\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{3bd^3}-\frac{q^2r^2(bc-ad)^3\log^2(c+dx)}{3bd^3}-\frac{11q^2r^2(bc-ad)^3\log(c+dx)}{9bd^3}-\frac{2qr(a+bx)(bc-ad)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3bd^2}+\frac{2pqr^2x(bc-ad)^2}{9d^2}+\frac{2qr^2x(p+q)(bc-ad)^2}{3d^2}+\frac{5q^2r^2x(bc-ad)^2}{9d^2}+\frac{(a+bx)^3\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b}+\frac{qr(a+bx)^2(bc-ad)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3bd}-\frac{2pr(a+bx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{9b}-\frac{2qr(a+bx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{9b}-\frac{bpqr^2x^2(bc-ad)}{6d}-\frac{pqr^2(a+bx)^2(bc-ad)}{9bd}-\frac{apqr^2x(bc-ad)}{3d}-\frac{5q^2r^2(a+bx)^2(bc-ad)}{18bd}+\frac{2p^2r^2(a+bx)^3}{27b}+\frac{4pqr^2(a+bx)^3}{27b}+\frac{2q^2r^2(a+bx)^3}{27b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] -1/3*(a*(b*c - a*d)*p*q*r^2*x)/d + (2*(b*c - a*d)^2*p*q*r^2*x)/(9*d^2) + (5
*(b*c - a*d)^2*q^2*r^2*x)/(9*d^2) + (2*(b*c - a*d)^2*q*(p + q)*r^2*x)/(3*d^
2) - (b*(b*c - a*d)*p*q*r^2*x^2)/(6*d) - ((b*c - a*d)*p*q*r^2*(a + b*x)^2)/
(9*b*d) - (5*(b*c - a*d)*q^2*r^2*(a + b*x)^2)/(18*b*d) + (2*p^2*r^2*(a + b*
x)^3)/(27*b) + (4*p*q*r^2*(a + b*x)^3)/(27*b) + (2*q^2*r^2*(a + b*x)^3)/(27
*b) - (2*(b*c - a*d)^3*p*q*r^2*Log[c + d*x])/(9*b*d^3) - (11*(b*c - a*d)^3*
q^2*r^2*Log[c + d*x])/(9*b*d^3) - (2*(b*c - a*d)^3*p*q*r^2*Log[-((d*(a + b*
x))/(b*c - a*d))]*Log[c + d*x])/(3*b*d^3) - ((b*c - a*d)^3*q^2*r^2*Log[c +
d*x]^2)/(3*b*d^3) - (2*(b*c - a*d)^2*q*r*(a + b*x)*Log[e*(f*(a + b*x)^p*(c
+ d*x)^q)^r])/(3*b*d^2) + ((b*c - a*d)*q*r*(a + b*x)^2*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r])/(3*b*d) - (2*p*r*(a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*
x)^q)^r])/(9*b) - (2*q*r*(a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/
(9*b) + (2*(b*c - a*d)^3*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)
^r])/(3*b*d^3) + ((a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/(3*b)
```

```
 - (2*(b*c - a*d)^3*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(3*b*d^3
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2579

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[(a + b*x)*(Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s/b), x] + (Dist[q*r*s*((b*c - a*d)/b), Int[Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]^(s - 1)/(c + d*x), x], x] - Dist[r*s*(p + q), Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^(s - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p, q,
r, s}, x] && NeQ[b*c - a*d, 0] && NeQ[p + q, 0] && IGtQ[s, 0] && LtQ[s, 4]
```

Rule 2580

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/h), x] + (-Dist[b*p*(r/h), Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[d*q*(r/h), Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 2584

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Dist[b*p*r*(
s/(h*(m + 1))), Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1)/(a + b*x)), x], x] - Dist[d*q*r*(s/(h*(m + 1))), Int[(g + h*x)^(m
+ 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2594

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rubi steps

$$\int (a+bx)^2 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \frac{(a+bx)^3\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b} - \frac{1}{3}(2pr)\int (a+bx$$

$$= -\frac{2pr(a+bx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{9b} + \frac{(a+bx)^3\log$$

$$= \frac{2p^2r^2(a+bx)^3}{27b} - \frac{2pr(a+bx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{9b}$$

$$= \frac{2(bc-ad)^2pqr^2x}{9d^2} - \frac{(bc-ad)pqr^2(a+bx)^2}{9bd} + \frac{2p^2r^2(a+bx}{27b}$$

$$= -\frac{a(bc-ad)pqr^2x}{3d} + \frac{2(bc-ad)^2pqr^2x}{9d^2} + \frac{2(bc-ad)^2q(p+}{3d^2}$$

$$= -\frac{a(bc-ad)pqr^2x}{3d} + \frac{2(bc-ad)^2pqr^2x}{9d^2} + \frac{5(bc-ad)^2q^2r^2x}{9d^2}$$

$$= -\frac{a(bc-ad)pqr^2x}{3d} + \frac{2(bc-ad)^2pqr^2x}{9d^2} + \frac{5(bc-ad)^2q^2r^2x}{9d^2}$$

Mathematica [**A**]
time = 0.73, size = 1211, normalized size = 1.77

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] ((108*a^3*p*q*r^2)/b + (36*a*b*c^2*p*q*r^2)/d^2 - (108*a^2*c*p*q*r^2)/d + 1
2*a^2*p^2*r^2*x + 102*a^2*p*q*r^2*x + (48*b^2*c^2*p*q*r^2*x)/d^2 - (126*a*b
*c*p*q*r^2*x)/d + 108*a^2*q^2*r^2*x + (66*b^2*c^2*q^2*r^2*x)/d^2 - (162*a*b
*c*q^2*r^2*x)/d + 12*a*b*p^2*r^2*x^2 + 39*a*b*p*q*r^2*x^2 - (15*b^2*c*p*q*r
^2*x^2)/d + 27*a*b*q^2*r^2*x^2 - (15*b^2*c*q^2*r^2*x^2)/d + 4*b^2*p^2*r^2*x
^3 + 8*b^2*p*q*r^2*x^3 + 4*b^2*q^2*r^2*x^3 - (18*a^3*p^2*r^2*Log[a + b*x]^2
)/b + (108*a^3*p*q*r^2*Log[c + d*x])/b - (12*b^2*c^3*p*q*r^2*Log[c + d*x])/
```

```
d^3 + (36*a*b*c^2*p*q*r^2*Log[c + d*x])/d^2 - (36*a^2*c*p*q*r^2*Log[c + d*x
])/d - (66*b^2*c^3*q^2*r^2*Log[c + d*x])/d^3 + (162*a*b*c^2*q^2*r^2*Log[c +
 d*x])/d^2 - (108*a^2*c*q^2*r^2*Log[c + d*x])/d - (18*b^2*c^3*q^2*r^2*Log[c
 + d*x]^2)/d^3 + (54*a*b*c^2*q^2*r^2*Log[c + d*x]^2)/d^2 - (54*a^2*c*q^2*r^
2*Log[c + d*x]^2)/d - (108*a^3*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/b
- 36*a^2*p*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 108*a^2*q*r*x*Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r] - (36*b^2*c^2*q*r*x*Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r])/d^2 + (108*a*b*c*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d
 - 36*a*b*p*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 54*a*b*q*r*x^2*Log
[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (18*b^2*c*q*r*x^2*Log[e*(f*(a + b*x)^p*
(c + d*x)^q)^r])/d - 12*b^2*p*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] -
12*b^2*q*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (36*b^2*c^3*q*r*Log[c
 + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^3 - (108*a*b*c^2*q*r*Log[c
+ d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^2 + (108*a^2*c*q*r*Log[c + d
*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d + 54*a^2*x*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^2 + 54*a*b*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + 18
*b^2*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + (6*p*r*Log[a + b*x]*(a*d*
(a^2*d^2*(16*p - 11*q) - 6*b^2*c^2*q + 15*a*b*c*d*q)*r - 6*b*c*(b^2*c^2 - 3
*a*b*c*d + 3*a^2*d^2)*q*r*Log[c + d*x] + 6*(b*c - a*d)^3*q*r*Log[(b*(c + d*
x))/(b*c - a*d)] + 6*a^3*d^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(b*d^3)
 + (36*(b*c - a*d)^3*p*q*r^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*d
^3))/54
```

Maple [**F**]
time = 0.19, size = 0, normalized size = 0.00

$$\int (bx+a)^2 \ln\left(e(f(bx+a)^p\,(dx+c)^q)^r\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

```
[Out] int((b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

Maxima [**A**]
time = 0.31, size = 771, normalized size = 1.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima
")
```

```
[Out] 1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2
+ 1/9*(6*a^3*f*p*log(b*x + a)/b - (2*b^2*d^2*f*(p + q)*x^3 + 3*(a*b*d^2*f*(
2*p + 3*q) - b^2*c*d*f*q)*x^2 + 6*(a^2*d^2*f*(p + 3*q) + b^2*c^2*f*q - 3*a*
```

```
b*c*d*f*q)*x)/d^2 + 6*(b^2*c^3*f*q - 3*a*b*c^2*d*f*q + 3*a^2*c*d^2*f*q)*log
(d*x + c)/d^3)*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/f - 1/54*r^2*(6*((2*p
*q + 11*q^2)*b^2*c^3*f^2 - 3*(2*p*q + 9*q^2)*a*b*c^2*d*f^2 + 6*(p*q + 3*q^2
)*a^2*c*d^2*f^2)*log(d*x + c)/d^3 - 36*(b^3*c^3*f^2*p*q - 3*a*b^2*c^2*d*f^2
*p*q + 3*a^2*b*c*d^2*f^2*p*q - a^3*d^3*f^2*p*q)*(log(b*x + a)*log((b*d*x +
a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/(b*d^3) - (4*(p^
2 + 2*p*q + q^2)*b^3*d^3*f^2*x^3 - 18*a^3*d^3*f^2*p^2*log(b*x + a)^2 - 3*(5
*(p*q + q^2)*b^3*c*d^2*f^2 - (4*p^2 + 13*p*q + 9*q^2)*a*b^2*d^3*f^2)*x^2 -
36*(b^3*c^3*f^2*p*q - 3*a*b^2*c^2*d*f^2*p*q + 3*a^2*b*c*d^2*f^2*p*q)*log(b*
x + a)*log(d*x + c) - 18*(b^3*c^3*f^2*q^2 - 3*a*b^2*c^2*d*f^2*q^2 + 3*a^2*b
*c*d^2*f^2*q^2)*log(d*x + c)^2 + 6*((8*p*q + 11*q^2)*b^3*c^2*d*f^2 - 3*(7*p
*q + 9*q^2)*a*b^2*c*d^2*f^2 + (2*p^2 + 17*p*q + 18*q^2)*a^2*b*d^3*f^2)*x -
6*(6*a*b^2*c^2*d*f^2*p*q - 15*a^2*b*c*d^2*f^2*p*q + (2*p^2 + 11*p*q)*a^3*d^
3*f^2)*log(b*x + a))/(b*d^3))/f^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2,
x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^2*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln \left(e \left(f \left(a+b x\right)^p \left(c+d x\right)^q\right)^r\right)^2 \left(a+b x\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(a + b*x)^2,x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(a + b*x)^2, x)
```

3.19 $\int (a+bx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$

Optimal. Leaf size=540

$$\frac{1}{2}ap^2r^2x+\frac{1}{2}apqr^2x-\frac{(bc-ad)pqr^2x}{2d}-\frac{(bc-ad)q^2r^2x}{2d}-\frac{(bc-ad)q(p+q)r^2x}{d}+\frac{1}{4}bp^2r^2x^2+\frac{1}{4}bpqr^2x^2+\frac{pqr^2(a-}{4b}$$

```
[Out] 1/2*a*p^2*r^2*x+1/2*a*p*q*r^2*x-1/2*(-a*d+b*c)*p*q*r^2*x/d-1/2*(-a*d+b*c)*q
^2*r^2*x/d-(-a*d+b*c)*q*(p+q)*r^2*x/d+1/4*b*p^2*r^2*x^2+1/4*b*p*q*r^2*x^2+1
/4*p*q*r^2*(b*x+a)^2/b+1/4*q^2*r^2*(b*x+a)^2/b+1/2*(-a*d+b*c)^2*p*q*r^2*ln(
d*x+c)/b/d^2+3/2*(-a*d+b*c)^2*q^2*r^2*ln(d*x+c)/b/d^2+(-a*d+b*c)^2*p*q*r^2*
ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/b/d^2+1/2*(-a*d+b*c)^2*q^2*r^2*ln(d*x+c
)^2/b/d^2+(-a*d+b*c)*q*r*(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d-1/2*p*
r*(b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b-1/2*q*r*(b*x+a)^2*ln(e*(f*(b*
x+a)^p*(d*x+c)^q)^r)/b-(-a*d+b*c)^2*q*r*ln(d*x+c)*ln(e*(f*(b*x+a)^p*(d*x+c)
^q)^r)/b/d^2+1/2*(b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/b+(-a*d+b*c)^2
*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c))/b/d^2
```

Rubi [A]
time = 0.27, antiderivative size = 540, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$, Rules used = {2584, 2581, 45, 2594, 2579, 31, 8, 2580, 2441, 2440, 2438, 2437, 2338}

$$\frac{pqr^2(bc-ad)^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bd^2}-\frac{qr(bc-ad)^2\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{bd^2}+\frac{pqr^2(bc-ad)^2\log(c+dx)}{2bd^2}+\frac{pqr^2(bc-ad)^2\log(c+dx)\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{bd^2}+\frac{q^2r^2(bc-ad)^2\log^2(c+dx)}{2bd^2}+\frac{3q^2r^2(bc-ad)^2\log(c+dx)}{2bd^2}+\frac{(a+bx)^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b}+\frac{qr(a+bx)(bc-ad)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{bd}-\frac{pr(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b}-\frac{qr(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b}-\frac{pqr^2x(bc-ad)}{2d}-\frac{qr^2x(p+q)(bc-ad)}{d}-\frac{q^2r^2x(bc-ad)}{2d}+\frac{pqr^2(a+bx)^2}{4b}+\frac{q^2r^2(a+bx)^2}{4b}+\frac{1}{2}ap^2r^2x+\frac{1}{2}apqr^2x+\frac{1}{4}bp^2r^2x^2+\frac{1}{4}bpqr^2x^2$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (a*p^2*r^2*x)/2 + (a*p*q*r^2*x)/2 - ((b*c - a*d)*p*q*r^2*x)/(2*d) - ((b*c -
 a*d)*q^2*r^2*x)/(2*d) - ((b*c - a*d)*q*(p + q)*r^2*x)/d + (b*p^2*r^2*x^2)/
4 + (b*p*q*r^2*x^2)/4 + (p*q*r^2*(a + b*x)^2)/(4*b) + (q^2*r^2*(a + b*x)^2)
/(4*b) + ((b*c - a*d)^2*p*q*r^2*Log[c + d*x])/(2*b*d^2) + (3*(b*c - a*d)^2*
q^2*r^2*Log[c + d*x])/(2*b*d^2) + ((b*c - a*d)^2*p*q*r^2*Log[-((d*(a + b*x)
)/(b*c - a*d))]*Log[c + d*x])/(b*d^2) + ((b*c - a*d)^2*q^2*r^2*Log[c + d*x]
^2)/(2*b*d^2) + ((b*c - a*d)*q*r*(a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q
)^r])/(b*d) - (p*r*(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b)
- (q*r*(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b) - ((b*c - a*
d)^2*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*d^2) + ((a +
 b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/(2*b) + ((b*c - a*d)^2*p*q*
r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b*d^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2579

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[(a + b*x)*(Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s/b), x] + (Dist[q*r*s*((b*c - a*d)/b), Int[Log[e*(f*(a + b*x)^p*(c +
```

```
 d*x)^q)^r]^(s - 1)/(c + d*x), x], x] - Dist[r*s*(p + q), Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^(s - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p, q,
r, s}, x] && NeQ[b*c - a*d, 0] && NeQ[p + q, 0] && IGtQ[s, 0] && LtQ[s, 4]
```

Rule 2580

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/h), x] + (-Dist[b*p*(r/h), Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[d*q*(r/h), Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 2584

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Dist[b*p*r*(
s/(h*(m + 1))), Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1)/(a + b*x)), x], x] - Dist[d*q*r*(s/(h*(m + 1))), Int[(g + h*x)^(m
+ 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2594

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rubi steps

$$\int (a+bx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \frac{(a+bx)^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b} - (pr)\int (a+bx)\,\mathrm{l}$$

$$= -\frac{pr(a+bx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b} + \frac{(a+bx)^2\log^2}{}$$

$$= \frac{1}{2}ap^2r^2x + \frac{1}{4}bp^2r^2x^2 - \frac{pr(a+bx)^2\log\left(e(f(a+bx)^p(c+dx\right.}{2b}$$

$$= \frac{1}{2}ap^2r^2x - \frac{(bc-ad)pqr^2x}{2d} + \frac{1}{4}bp^2r^2x^2 + \frac{pqr^2(a+bx)^2}{4b} + $$

$$= \frac{1}{2}ap^2r^2x + \frac{1}{2}apqr^2x - \frac{(bc-ad)pqr^2x}{2d} - \frac{(bc-ad)q(p+q)r}{d}$$

$$= \frac{1}{2}ap^2r^2x + \frac{1}{2}apqr^2x - \frac{(bc-ad)pqr^2x}{2d} - \frac{(bc-ad)q^2r^2x}{2d} - $$

$$= \frac{1}{2}ap^2r^2x + \frac{1}{2}apqr^2x - \frac{(bc-ad)pqr^2x}{2d} - \frac{(bc-ad)q^2r^2x}{2d} - $$

Mathematica [A]
time = 0.41, size = 781, normalized size = 1.45

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (-4*a*b*c*d*p*q*r^2 + 8*a^2*d^2*p*q*r^2 + 2*a*b*d^2*p^2*r^2*x - 6*b^2*c*d*p
*q*r^2*x + 10*a*b*d^2*p*q*r^2*x - 6*b^2*c*d*q^2*r^2*x + 8*a*b*d^2*q^2*r^2*x
 + b^2*d^2*p^2*r^2*x^2 + 2*b^2*d^2*p*q*r^2*x^2 + b^2*d^2*q^2*r^2*x^2 - 2*a^
2*d^2*p^2*r^2*Log[a + b*x]^2 + 2*b^2*c^2*p*q*r^2*Log[c + d*x] - 4*a*b*c*d*p
*q*r^2*Log[c + d*x] + 8*a^2*d^2*p*q*r^2*Log[c + d*x] + 6*b^2*c^2*q^2*r^2*Lo
g[c + d*x] - 8*a*b*c*d*q^2*r^2*Log[c + d*x] + 2*b^2*c^2*q^2*r^2*Log[c + d*x
]^2 - 4*a*b*c*d*q^2*r^2*Log[c + d*x]^2 - 8*a^2*d^2*p*r*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r] - 4*a*b*d^2*p*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 4
*b^2*c*d*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 8*a*b*d^2*q*r*x*Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*b^2*d^2*p*r*x^2*Log[e*(f*(a + b*x)^p*(c
 + d*x)^q)^r] - 2*b^2*d^2*q*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 4*
b^2*c^2*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 8*a*b*c*d*q
*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 4*a*b*d^2*x*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^2 + 2*b^2*d^2*x^2*Log[e*(f*(a + b*x)^p*(c + d*
```

```
x)^q)^r]^2 + 2*p*r*Log[a + b*x]*(2*b*c*(b*c - 2*a*d)*q*r*Log[c + d*x] - 2*(
b*c - a*d)^2*q*r*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*(3*a*d*(p - q)*r + 2*
b*c*q*r + 2*a*d*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])) - 4*(b*c - a*d)^2*p*
q*r^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(4*b*d^2)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int (bx+a)\ln \left(e(f(bx+a)^p\,(dx+c)^q)^r\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

```
[Out] int((b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

Maxima [A]
time = 0.34, size = 506, normalized size = 0.94

$$\frac{1}{2}\left(bx^2+2\,ax\right)\log\left(\left(((bx+a)^p(dx+c)^qf\right)^r e\right)^2+\frac{\left(\frac{2\,a^2fp\log(bx+a)}{b}-\frac{bdf(p+q)x^2+2\,(adf(p+2\,q)-bcfq)x}{d}-\frac{2\,(bc^2fq-2\,acdfq)\log(dx+c)}{d^2}\right)r\log\left(\left(((bx+a)^p(dx+c)^qf\right)^r e\right)}{2\,f}+\frac{r^2\left(\frac{2\,((pq+3\,q^2)bc^2f^2-2\,(pq+2\,q^2)acdf^2)\log(dx+c)}{d^2}-\frac{4\,(b^2c^2f^2pq-2\,abcdf^2pq+a^2d^2f^2pq)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)}{bd^2}-\frac{2\,a^2d^2f^2p^2\log(bx+a)^2-(p^2+2\,pq+q^2)b^2d^2f^2x^2-4\,(b^2c^2f^2pq-2\,abcdf^2pq)\log(bx+a)\log(dx+c)-2\,(b^2c^2f^2q^2-2\,abcdf^2q^2)\log(dx+c)^2+2\,(3\,(pq+q^2)b^2cdf^2-(p^2+5\,pq+4\,q^2)abd^2f^2)x-2\,(2\,abcdf^2pq-(p^2+3\,pq)a^2d^2f^2)\log(bx+a)}{bd^2}\right)}{4\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2 + 1/2*(2*a^2*f*p
*log(b*x + a)/b - (b*d*f*(p + q)*x^2 + 2*(a*d*f*(p + 2*q) - b*c*f*q)*x)/d -
 2*(b*c^2*f*q - 2*a*c*d*f*q)*log(d*x + c)/d^2)*r*log(((b*x + a)^p*(d*x + c)
^q*f)^r*e)/f + 1/4*r^2*(2*((p*q + 3*q^2)*b*c^2*f^2 - 2*(p*q + 2*q^2)*a*c*d*
f^2)*log(d*x + c)/d^2 - 4*(b^2*c^2*f^2*p*q - 2*a*b*c*d*f^2*p*q + a^2*d^2*f^
2*p*q)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a
*d)/(b*c - a*d)))/(b*d^2) - (2*a^2*d^2*f^2*p^2*log(b*x + a)^2 - (p^2 + 2*p*
q + q^2)*b^2*d^2*f^2*x^2 - 4*(b^2*c^2*f^2*p*q - 2*a*b*c*d*f^2*p*q)*log(b*x
+ a)*log(d*x + c) - 2*(b^2*c^2*f^2*q^2 - 2*a*b*c*d*f^2*q^2)*log(d*x + c)^2
+ 2*(3*(p*q + q^2)*b^2*c*d*f^2 - (p^2 + 5*p*q + 4*q^2)*a*b*d^2*f^2)*x - 2*(
2*a*b*c*d*f^2*p*q - (p^2 + 3*p*q)*a^2*d^2*f^2)*log(b*x + a))/(b*d^2))/f^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)\log\left(e(f(a+bx)^p\,(c+dx)^q)^r\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

```
[Out] Integral((a + b*x)*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(e\,{\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)}^r\right)^2\,\left(a+b\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(a + b*x),x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(a + b*x), x)
```

3.20 $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{a+bx}\,dx$

Optimal. Leaf size=431

$$\frac{\log^3\left((a+bx)^{pr}\right)}{3bpr}-\frac{q\log^2\left((a+bx)^{pr}\right)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{bp}+\frac{\log^2\left((a+bx)^{pr}\right)\log\left((c+dx)^{qr}\right)}{bpr}+\frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log^2\left((c\right.}{b}$$

```
[Out] 1/3*ln((b*x+a)^(p*r))^3/b/p/r-q*ln((b*x+a)^(p*r))^2*ln(b*(d*x+c)/(-a*d+b*c)
)/b/p+ln((b*x+a)^(p*r))^2*ln((d*x+c)^(q*r))/b/p/r+ln(-d*(b*x+a)/(-a*d+b*c))
*ln((d*x+c)^(q*r))^2/b-2*q*r*ln((b*x+a)^(p*r))*polylog(2,-d*(b*x+a)/(-a*d+b
*c))/b+2*q*r*ln((d*x+c)^(q*r))*polylog(2,b*(d*x+c)/(-a*d+b*c))/b-1/4*(ln((b
*x+a)^(p*r))+ln((d*x+c)^(q*r))-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))*((ln((b*x+a
)^(p*r))-ln((d*x+c)^(q*r))+ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))^2/b/p/r+8*ln(-d
*(b*x+a)/(-a*d+b*c))*ln((d*x+c)^(q*r))/b+8*q*r*polylog(2,b*(d*x+c)/(-a*d+b*
c))/b)+2*p*q*r^2*polylog(3,-d*(b*x+a)/(-a*d+b*c))/b-2*q^2*r^2*polylog(3,b*(
d*x+c)/(-a*d+b*c))/b
```

Rubi [A]

time = 0.34, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.484$, Rules used = {2582, 6874, 2437, 2339, 30, 2481, 2422, 2354, 2421, 6724, 2443, 2441, 2440, 2438, 6818}

$$-\frac{1}{4}\left(-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)+\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right)\right)\left(8\left(\frac{qr\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b}+\frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log\left((c+dx)^{qr}\right)}{b}\right)+\frac{\left(\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)+\log\left((a+bx)^{pr}\right)-\log\left((c+dx)^{qr}\right)\right)^2}{bpr}\right)+\frac{2pqr^2\text{PolyLog}\left(3,-\frac{d(a+bx)}{bc-ad}\right)}{b}-\frac{2qr\log\left((a+bx)^{pr}\right)\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{b}-\frac{2q^2r^2\text{PolyLog}\left(3,\frac{b(c+dx)}{bc-ad}\right)}{b}+\frac{2qr\log\left((c+dx)^{qr}\right)\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b}-\frac{q\log\left(\frac{b(c+dx)}{bc-ad}\right)\log^2\left((a+bx)^{pr}\right)}{bp}+\frac{\log^2\left((a+bx)^{pr}\right)\log\left((c+dx)^{qr}\right)}{bpr}+\frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log^2\left((c+dx)^{qr}\right)}{b}+\frac{\log^3\left((a+bx)^{pr}\right)}{3bpr}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x),x]
```

```
[Out] Log[(a + b*x)^(p*r)]^3/(3*b*p*r) - (q*Log[(a + b*x)^(p*r)]^2*Log[(b*(c + d*
x))/(b*c - a*d)])/(b*p) + (Log[(a + b*x)^(p*r)]^2*Log[(c + d*x)^(q*r)])/(b*
p*r) + (Log[-((d*(a + b*x))/(b*c - a*d))]*Log[(c + d*x)^(q*r)]^2)/b - (2*q*
r*Log[(a + b*x)^(p*r)]*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/b + (2*q*r
*Log[(c + d*x)^(q*r)]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/b - ((Log[(a +
 b*x)^(p*r)] + Log[(c + d*x)^(q*r)] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])
*((Log[(a + b*x)^(p*r)] - Log[(c + d*x)^(q*r)] + Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r])^2/(b*p*r) + 8*((Log[-((d*(a + b*x))/(b*c - a*d))]*Log[(c + d*x)
^(q*r)])/b + (q*r*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/b)))/4 + (2*p*q*r^
2*PolyLog[3, -((d*(a + b*x))/(b*c - a*d))])/b - (2*q^2*r^2*PolyLog[3, (b*(c
 + d*x))/(b*c - a*d)])/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2582

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^2/((g_.) + (h_.)*(x_)), x_Symbol] :> Int[(Log[(a + b*x)^(p*r)] + Lo
g[(c + d*x)^(q*r)])^2/(g + h*x), x] + Simp[(Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r] - Log[(a + b*x)^(p*r)] - Log[(c + d*x)^(q*r)])*(2*Int[Log[(c + d*x)^(
q*r)]/(g + h*x), x] + Int[(Log[(a + b*x)^(p*r)] - Log[(c + d*x)^(q*r)] + Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(g + h*x), x]), x] /; FreeQ[{a, b, c, d
, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0] && EqQ[b*g - a*h, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6818

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{a+bx}\,dx = \int \frac{\left(\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right)\right)^2}{a+bx}\,dx - (\log\left((a+bx)^{pr}\right)+\log$$

$$= -(\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)$$

$$= 2\int \frac{\log\left((a+bx)^{pr}\right)\log\left((c+dx)^{qr}\right)}{a+bx}\,dx - (\log\left((a+bx)^{pr}\right)+\log((c$$

$$= \frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log^2\left((c+dx)^{qr}\right)}{b} - (\log\left((a+bx)^{pr}\right)+\log((c+dx)^{q$$

$$= \frac{\log^2\left((a+bx)^{pr}\right)\log\left((c+dx)^{qr}\right)}{bpr} + \frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log^2\left((c+dx)^{qr}\right)}{b}$$

$$= \frac{\log^3\left((a+bx)^{pr}\right)}{3bpr} - \frac{q\log^2\left((a+bx)^{pr}\right)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{bp} + \frac{\log^2((a+bx)}{}$$

$$= \frac{\log^3\left((a+bx)^{pr}\right)}{3bpr} - \frac{q\log^2\left((a+bx)^{pr}\right)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{bp} + \frac{\log^2((a+bx)}{}$$

$$= \frac{\log^3\left((a+bx)^{pr}\right)}{3bpr} - \frac{q\log^2\left((a+bx)^{pr}\right)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{bp} + \frac{\log^2((a+bx)}{}$$

Mathematica [A]
time = 0.12, size = 460, normalized size = 1.07

$$\frac{p^2r^2\log^3(a+bx)+6pqr^2\log^2(a+bx)\log(c+dx)-6pqr^2\log(a+bx)\log\left(\frac{d(a+bx)}{-bc+ad}\right)\log(c+dx)+3q^2r^2\log(a+bx)\log^2(c+dx)-3q^2r^2\log\left(\frac{d(a+bx)}{-bc+ad}\right)\log^2(c+dx)-3pqr^2\log^2(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)-3pr\log^2(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)-6qr\log(a+bx)\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+6qr\log\left(\frac{d(a+bx)}{-bc+ad}\right)\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+3\log(a+bx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)-6pqr^2\log(a+bx)\mathrm{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)+6qr\left(-pr\log(a+bx)+\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)\mathrm{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)+6pqr^2\mathrm{Li}_3\left(\frac{d(a+bx)}{-bc+ad}\right)-6q^2r^2\mathrm{Li}_3\left(\frac{b(c+dx)}{bc-ad}\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x),x]
```

```
[Out] (p^2*r^2*Log[a + b*x]^3 + 6*p*q*r^2*Log[a + b*x]^2*Log[c + d*x] - 6*p*q*r^2
*Log[a + b*x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] + 3*q^2*r^2*Lo
g[a + b*x]*Log[c + d*x]^2 - 3*q^2*r^2*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log
[c + d*x]^2 - 3*p*q*r^2*Log[a + b*x]^2*Log[(b*(c + d*x))/(b*c - a*d)] - 3*p
*r*Log[a + b*x]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 6*q*r*Log[a + b*x]
*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 6*q*r*Log[(d*(a + b*x)
)/(-(b*c) + a*d)]*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 3*Log
[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 6*p*q*r^2*Log[a + b*x]*P
olyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 6*q*r*(-(p*r*Log[a + b*x]) + Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] + 6
*p*q*r^2*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] - 6*q^2*r^2*PolyLog[3, (b
*(c + d*x))/(b*c - a*d)])/(3*b)
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(e\left(f\left(bx+a\right)^{p} \left(dx+c\right)^{q}\right)^{r}\right)^{2}}{bx+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a),x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] log(b*x + a)*log(((d*x + c)^q)^r)^2/b + integrate(((r^2*log(f)^2 + 2*r*log(
f) + 1)*b*d*x + (r^2*log(f)^2 + 2*r*log(f) + 1)*b*c + (b*d*x + b*c)*log(((b
*x + a)^p)^r)^2 + 2*((r*log(f) + 1)*b*d*x + (r*log(f) + 1)*b*c)*log(((b*x +
 a)^p)^r) + 2*((r*log(f) + 1)*b*d*x + (r*log(f) + 1)*b*c - (b*d*q*r*x + a*d
*q*r)*log(b*x + a) + (b*d*x + b*c)*log(((b*x + a)^p)^r))*log(((d*x + c)^q)^
r))/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a),x, algorithm="fricas")
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(e(f(a+bx)^p\,(c+dx)^q)^r\right)^2}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(b*x+a),x)
[Out] Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2/(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a),x, algorithm="giac")
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\,(f\,(a+b\,x)^p\,(c+d\,x)^q)^r\right)^2}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x),x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x), x)
```

3.21 $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^2}\,dx$

Optimal. Leaf size=465

$$-\frac{2p^2r^2}{b(a+bx)}+\frac{2dpqr^2\log(a+bx)}{b(bc-ad)}-\frac{dpqr^2\log^2(a+bx)}{b(bc-ad)}-\frac{2dpqr^2\log(c+dx)}{b(bc-ad)}+\frac{2dpqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx}{b(bc-ad)}$$

```
[Out] -2*p^2*r^2/b/(b*x+a)+2*d*p*q*r^2*ln(b*x+a)/b/(-a*d+b*c)-d*p*q*r^2*ln(b*x+a)
^2/b/(-a*d+b*c)-2*d*p*q*r^2*ln(d*x+c)/b/(-a*d+b*c)+2*d*p*q*r^2*ln(-d*(b*x+a
)/(-a*d+b*c))*ln(d*x+c)/b/(-a*d+b*c)+d*q^2*r^2*ln(d*x+c)^2/b/(-a*d+b*c)-2*d
*q^2*r^2*ln(b*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/b/(-a*d+b*c)-2*p*r*ln(e*(f*(b*x
+a)^p*(d*x+c)^q)^r)/b/(b*x+a)+2*d*q*r*ln(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q
)^r)/b/(-a*d+b*c)-2*d*q*r*ln(d*x+c)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(-a*d
+b*c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/b/(b*x+a)-2*d*q^2*r^2*polylog(2,-d*
(b*x+a)/(-a*d+b*c))/b/(-a*d+b*c)+2*d*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c)
)/b/(-a*d+b*c)
```

Rubi [A]

time = 0.27, antiderivative size = 465, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {2584, 2581, 32, 36, 31, 2594, 2580, 2437, 2338, 2441, 2440, 2438}

$$\frac{2dpqr^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)}-\frac{2dq^2r^2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{b(bc-ad)}-\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)}+\frac{2dqr\log(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(bc-ad)}-\frac{2dqr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(bc-ad)}-\frac{2pr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)}-\frac{dpqr^2\log^2(a+bx)}{b(bc-ad)}+\frac{2dpqr^2\log(a+bx)}{b(bc-ad)}-\frac{2dpqr^2\log(c+dx)}{b(bc-ad)}+\frac{2dpqr^2\log(c+dx)\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{b(bc-ad)}+\frac{dq^2r^2\log^2(c+dx)}{b(bc-ad)}-\frac{2dq^2r^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)}-\frac{2p^2r^2}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^2,x]
```

```
[Out] (-2*p^2*r^2)/(b*(a + b*x)) + (2*d*p*q*r^2*Log[a + b*x])/(b*(b*c - a*d)) - (
d*p*q*r^2*Log[a + b*x]^2)/(b*(b*c - a*d)) - (2*d*p*q*r^2*Log[c + d*x])/(b*(
b*c - a*d)) + (2*d*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/
(b*(b*c - a*d)) + (d*q^2*r^2*Log[c + d*x]^2)/(b*(b*c - a*d)) - (2*d*q^2*r^2
*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)) - (2*p*r*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*(a + b*x)) + (2*d*q*r*Log[a + b*x]*Log
[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*(b*c - a*d)) - (2*d*q*r*Log[c + d*x]*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*(b*c - a*d)) - Log[e*(f*(a + b*x)^
p*(c + d*x)^q)^r]^2/(b*(a + b*x)) - (2*d*q^2*r^2*PolyLog[2, -((d*(a + b*x))
/(b*c - a*d))])/(b*(b*c - a*d)) + (2*d*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*
c - a*d)])/(b*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2580

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/h), x] + (-Dist[b*p*(r/h), Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[d*q*(r/h), Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
```

```
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 2584

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Dist[b*p*r*(
s/(h*(m + 1))), Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1)/(a + b*x)), x], x] - Dist[d*q*r*(s/(h*(m + 1))), Int[(g + h*x)^(m
+ 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2594

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^2}\,dx &= -\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)} + (2pr)\int \frac{\log\left(e(f(a+bx)^p(c+dx\right.}{(a+bx)^2} \\
&= -\frac{2pr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)} - \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right.}{b(a+bx)} \\
&= -\frac{2p^2r^2}{b(a+bx)} - \frac{2pr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b(a+bx)} - \frac{\log^2\left(e(f(a+bx\right.}{b(a+} \\
&= -\frac{2p^2r^2}{b(a+bx)} + \frac{2dpqr^2\log(a+bx)}{b(bc-ad)} - \frac{2dpqr^2\log(c+dx)}{b(bc-ad)} - \frac{2pr\log(e}{} \\
&= -\frac{2p^2r^2}{b(a+bx)} + \frac{2dpqr^2\log(a+bx)}{b(bc-ad)} - \frac{2dpqr^2\log(c+dx)}{b(bc-ad)} + \frac{2dpqr^2\,\mathrm{lo}}{} \\
&= -\frac{2p^2r^2}{b(a+bx)} + \frac{2dpqr^2\log(a+bx)}{b(bc-ad)} - \frac{dpqr^2\log^2(a+bx)}{b(bc-ad)} - \frac{2dpqr^2\,\mathrm{lo}}{b(bc-} \\
&= -\frac{2p^2r^2}{b(a+bx)} + \frac{2dpqr^2\log(a+bx)}{b(bc-ad)} - \frac{dpqr^2\log^2(a+bx)}{b(bc-ad)} - \frac{2dpqr^2\,\mathrm{lo}}{b(bc-}
\end{aligned}$$

<u>Mathematica</u> [A]
time = 0.45, size = 411, normalized size = 0.88

$$\frac{-2bcp^2r^2+2adp^2r^2-dpqr^2(a+bx)\log^2(a+bx)-2adpqr^2\log(c+dx)-2bdpqr^2x\log(c+dx)+adq^2r^2\log^2(c+dx)+bdq^2r^2x\log^2(c+dx)-2bcpr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+2adpr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)-2adqr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)-2bdqrx\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)-bc\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)+ad\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)+2dqr(a+bx)\log(a+bx)\left(pr+pr\log(c+dx)-(p+q)r\log\left(\frac{b(c+dx)}{bc-ad}\right)+\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\right)-2dq(p+q)r^2(a+bx)\mathrm{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)}{b(bc-ad)(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^2,x]
```

```
[Out] (-2*b*c*p^2*r^2 + 2*a*d*p^2*r^2 - d*p*q*r^2*(a + b*x)*Log[a + b*x]^2 - 2*a*
d*p*q*r^2*Log[c + d*x] - 2*b*d*p*q*r^2*x*Log[c + d*x] + a*d*q^2*r^2*Log[c +
 d*x]^2 + b*d*q^2*r^2*x*Log[c + d*x]^2 - 2*b*c*p*r*Log[e*(f*(a + b*x)^p*(c
+ d*x)^q)^r] + 2*a*d*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*a*d*q*r*L
og[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*b*d*q*r*x*Log[c + d*x]
*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - b*c*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^2 + a*d*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + 2*d*q*r*(a + b*x)*Lo
g[a + b*x]*(p*r + p*r*Log[c + d*x] - (p + q)*r*Log[(b*(c + d*x))/(b*c - a*d
)] + Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) - 2*d*q*(p + q)*r^2*(a + b*x)*Po
lyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(b*c - a*d)*(a + b*x))
```

<u>Maple</u> [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f(bx+a)^p\,(dx+c)^q\right)^r\right)^2}{(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^2,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^2,x)
```

Maxima [A]
time = 0.31, size = 394, normalized size = 0.85

$$\frac{2\left(\frac{dfq\log(bx+a)}{bc-ad}-\frac{dfq\log(dx+c)}{bc-ad}-\frac{fp}{bx+a}\right)r\log\left(\left(\left(bx+a\right)^{p}\left(dx+c\right)^{q}f\right)^{r}e\right)}{bf}-\frac{\left(\frac{2\,df^{2}pq\log(dx+c)}{bc-ad}+\frac{2\left(pq+q^{2}\right)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)df^{2}}{bc-ad}+\frac{2\,bcf^{2}p^{2}-2\,adf^{2}p^{2}+\left(bdf^{2}pqx+adf^{2}pq\right)\log(bx+a)^{2}-2\left(bdf^{2}pqx+adf^{2}pq\right)\log(bx+a)\log(dx+c)-\left(bdf^{2}q^{2}x+adf^{2}q^{2}\right)\log(dx+c)^{2}-2\left(bdf^{2}pqx+adf^{2}pq\right)\log(bx+a)}{abc-a^{2}d+\left(b^{2}c-abd\right)x}\right)r^{2}}{bf^{2}}-\frac{\log\left(\left(\left(bx+a\right)^{p}\left(dx+c\right)^{q}f\right)^{r}e\right)^{2}}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^2,x, algorithm="maxima
")
```

```
[Out] 2*(d*f*q*log(b*x + a)/(b*c - a*d) - d*f*q*log(d*x + c)/(b*c - a*d) - f*p/(b
*x + a))*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(b*f) - (2*d*f^2*p*q*log(d*
x + c)/(b*c - a*d) + 2*(p*q + q^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a
*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*d*f^2/(b*c - a*d) + (2*b*c*f^
2*p^2 - 2*a*d*f^2*p^2 + (b*d*f^2*p*q*x + a*d*f^2*p*q)*log(b*x + a)^2 - 2*(b
*d*f^2*p*q*x + a*d*f^2*p*q)*log(b*x + a)*log(d*x + c) - (b*d*f^2*q^2*x + a*
d*f^2*q^2)*log(d*x + c)^2 - 2*(b*d*f^2*p*q*x + a*d*f^2*p*q)*log(b*x + a))/(
a*b*c - a^2*d + (b^2*c - a*b*d)*x))*r^2/(b*f^2) - log(((b*x + a)^p*(d*x + c
)^q*f)^r*e)^2/((b*x + a)*b)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^2,x, algorithm="fricas
")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b^2*x^2 + 2*a*b*x + a^2),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(e\left(f\left(a+bx\right)^{p}\left(c+dx\right)^{q}\right)^{r}\right)^{2}}{\left(a+bx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(b*x+a)**2,x)
```

```
[Out] Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2/(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^2,x, algorithm="giac")
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln \left(e\,{\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)}^r\right)^2}{{\left(a+b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x)^2,x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x)^2, x)
```

3.22 $\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^3}\,dx$

Optimal. Leaf size=632

$$-\frac{p^2r^2}{4b(a+bx)^2}-\frac{3dpqr^2}{2b(bc-ad)(a+bx)}-\frac{d^2pqr^2\log(a+bx)}{2b(bc-ad)^2}+\frac{d^2q^2r^2\log(a+bx)}{b(bc-ad)^2}+\frac{d^2pqr^2\log^2(a+bx)}{2b(bc-ad)^2}+\frac{d^2pqr^2\,\mathrm{lo}}{2b(bc}$$

```
[Out] -1/4*p^2*r^2/b/(b*x+a)^2-3/2*d*p*q*r^2/b/(-a*d+b*c)/(b*x+a)-1/2*d^2*p*q*r^2
*ln(b*x+a)/b/(-a*d+b*c)^2+d^2*q^2*r^2*ln(b*x+a)/b/(-a*d+b*c)^2+1/2*d^2*p*q*
r^2*ln(b*x+a)^2/b/(-a*d+b*c)^2+1/2*d^2*p*q*r^2*ln(d*x+c)/b/(-a*d+b*c)^2-d^2
*q^2*r^2*ln(d*x+c)/b/(-a*d+b*c)^2-d^2*p*q*r^2*ln(-d*(b*x+a)/(-a*d+b*c))*ln(
d*x+c)/b/(-a*d+b*c)^2-1/2*d^2*q^2*r^2*ln(d*x+c)^2/b/(-a*d+b*c)^2+d^2*q^2*r^
2*ln(b*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/b/(-a*d+b*c)^2-1/2*p*r*ln(e*(f*(b*x+a)
^p*(d*x+c)^q)^r)/b/(b*x+a)^2-d*q*r*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(-a*d+
b*c)/(b*x+a)-d^2*q*r*ln(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(-a*d+b*c)
^2+d^2*q*r*ln(d*x+c)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(-a*d+b*c)^2-1/2*ln(
e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/b/(b*x+a)^2+d^2*q^2*r^2*polylog(2,-d*(b*x+a)
/(-a*d+b*c))/b/(-a*d+b*c)^2-d^2*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c))/b/(
-a*d+b*c)^2
```

Rubi [A]
time = 0.34, antiderivative size = 632, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 13, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {2584, 2581, 32, 46, 2594, 36, 31, 2580, 2437, 2338, 2441, 2440, 2438}

$$-\frac{d^2pqr^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)^2}+\frac{d^2q^2r^2\,\mathrm{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{b(bc-ad)^2}-\frac{d^2qr\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(bc-ad)^2}+\frac{d^2qr\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(bc-ad)^2}+\frac{d^2pqr^2\log^2(a+bx)}{2b(bc-ad)^2}-\frac{d^2pqr^2\log(a+bx)}{2b(bc-ad)^2}+\frac{d^2pqr^2\log(c+dx)}{2b(bc-ad)^2}-\frac{d^2pqr^2\log(c+dx)\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{b(bc-ad)^2}-\frac{d^2q^2r^2\log^2(c+dx)}{2b(bc-ad)^2}+\frac{d^2q^2r^2\log(a+bx)}{b(bc-ad)^2}-\frac{d^2q^2r^2\log(c+dx)}{b(bc-ad)^2}+\frac{d^2q^2r^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)^2}-\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2}-\frac{dqr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)(bc-ad)}-\frac{pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2}-\frac{3dpqr^2}{2b(a+bx)(bc-ad)}-\frac{p^2r^2}{4b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^3,x]
```

```
[Out] -1/4*(p^2*r^2)/(b*(a + b*x)^2) - (3*d*p*q*r^2)/(2*b*(b*c - a*d)*(a + b*x))
- (d^2*p*q*r^2*Log[a + b*x])/(2*b*(b*c - a*d)^2) + (d^2*q^2*r^2*Log[a + b*x
])/(b*(b*c - a*d)^2) + (d^2*p*q*r^2*Log[a + b*x]^2)/(2*b*(b*c - a*d)^2) + (
d^2*p*q*r^2*Log[c + d*x])/(2*b*(b*c - a*d)^2) - (d^2*q^2*r^2*Log[c + d*x])/
(b*(b*c - a*d)^2) - (d^2*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c +
d*x])/(b*(b*c - a*d)^2) - (d^2*q^2*r^2*Log[c + d*x]^2)/(2*b*(b*c - a*d)^2)
+ (d^2*q^2*r^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)^
2) - (p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b*(a + b*x)^2) - (d*q*r*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*(b*c - a*d)*(a + b*x)) - (d^2*q*r*
Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*(b*c - a*d)^2) + (d^2
*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*(b*c - a*d)^2) -
 Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(2*b*(a + b*x)^2) + (d^2*q^2*r^2*Po
lyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(b*(b*c - a*d)^2) - (d^2*p*q*r^2*Po
lyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2580

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/h), x] + (-Dist[b*p*(r/h), Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[d*q*(r/h), Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 2584

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Dist[b*p*r*(
s/(h*(m + 1))), Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1)/(a + b*x)), x], x] - Dist[d*q*r*(s/(h*(m + 1))), Int[(g + h*x)^(m
+ 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2594

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rubi steps

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^3}\,dx = -\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(a+bx)^2} + (pr)\int \frac{\log\left(e(f(a+bx)^p(c+dx\right.}{(a+bx)^3}$$

$$= -\frac{pr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(a+bx)^2} - \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(a+bx)^2}$$

$$= -\frac{p^2r^2}{4b(a+bx)^2} - \frac{pr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b(a+bx)^2} - \frac{\log^2\left(e(f(a+bx\right.}{2b(a+}$$

$$= -\frac{p^2r^2}{4b(a+bx)^2} - \frac{dpqr^2}{2b(bc-ad)(a+bx)} - \frac{d^2pqr^2\log(a+bx)}{2b(bc-ad)^2} + \frac{d^2pqr}{2b(}$$

$$= -\frac{p^2r^2}{4b(a+bx)^2} - \frac{3dpqr^2}{2b(bc-ad)(a+bx)} - \frac{d^2pqr^2\log(a+bx)}{2b(bc-ad)^2} + \frac{d^2pqr}{2b(}$$

$$= -\frac{p^2r^2}{4b(a+bx)^2} - \frac{3dpqr^2}{2b(bc-ad)(a+bx)} - \frac{d^2pqr^2\log(a+bx)}{2b(bc-ad)^2} + \frac{d^2q^2r^2}{b(b}$$

$$= -\frac{p^2r^2}{4b(a+bx)^2} - \frac{3dpqr^2}{2b(bc-ad)(a+bx)} - \frac{d^2pqr^2\log(a+bx)}{2b(bc-ad)^2} + \frac{d^2q^2r^2}{b(b}$$

<u>Mathematica</u> [A]
time = 0.81, size = 872, normalized size = 1.38

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^3,x]
```

```
[Out] -1/4*(b^2*c^2*p^2*r^2 - 2*a*b*c*d*p^2*r^2 + a^2*d^2*p^2*r^2 + 6*a*b*c*d*p*q
*r^2 - 6*a^2*d^2*p*q*r^2 + 6*b^2*c*d*p*q*r^2*x - 6*a*b*d^2*p*q*r^2*x - 2*d^
2*p*q*r^2*(a + b*x)^2*Log[a + b*x]^2 - 2*a^2*d^2*p*q*r^2*Log[c + d*x] + 4*a
^2*d^2*q^2*r^2*Log[c + d*x] - 4*a*b*d^2*p*q*r^2*x*Log[c + d*x] + 8*a*b*d^2*
q^2*r^2*x*Log[c + d*x] - 2*b^2*d^2*p*q*r^2*x^2*Log[c + d*x] + 4*b^2*d^2*q^2
*r^2*x^2*Log[c + d*x] + 2*a^2*d^2*q^2*r^2*Log[c + d*x]^2 + 4*a*b*d^2*q^2*r^
2*x*Log[c + d*x]^2 + 2*b^2*d^2*q^2*r^2*x^2*Log[c + d*x]^2 - 2*d^2*q*r*(a +
b*x)^2*Log[a + b*x]*(-(p*r) + 2*q*r - 2*p*r*Log[c + d*x] + 2*(p + q)*r*Log[
(b*(c + d*x))/(b*c - a*d)] - 2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) + 2*b^
2*c^2*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 4*a*b*c*d*p*r*Log[e*(f*(a
+ b*x)^p*(c + d*x)^q)^r] + 2*a^2*d^2*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^
r] + 4*a*b*c*d*q*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 4*a^2*d^2*q*r*Log
[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 4*b^2*c*d*q*r*x*Log[e*(f*(a + b*x)^p*(c
```

```
 + d*x)^q)^r] - 4*a*b*d^2*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 4*a^
2*d^2*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 8*a*b*d^2*q*r
*x*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 4*b^2*d^2*q*r*x^2*Lo
g[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 2*b^2*c^2*Log[e*(f*(a + b
*x)^p*(c + d*x)^q)^r]^2 - 4*a*b*c*d*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2
+ 2*a^2*d^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 4*d^2*q*(p + q)*r^2*(a
 + b*x)^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(b*c - a*d)^2*(a + b
*x)^2)
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(e\left(f (bx+a)^{p} (dx+c)^{q}\right)^{r}\right)^{2}}{(bx+a)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^3,x)

[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^3,x)
```

Maxima [A]
time = 0.35, size = 757, normalized size = 1.20

$$-\frac{\left(\frac{2\,d^{2} f q \log(bx+a)}{b^{2} c^{2}-2\,abcd+a^{2} d^{2}}-\frac{2\,d^{2} f q \log(dx+c)}{b^{2} c^{2}-2\,abcd+a^{2} d^{2}}+\frac{2\,bdfqx-adf(p-2\,q)+bcfp}{a^{2}bc-a^{3}d+(b^{3}c-ab^{2}d)x^{2}+2\,(ab^{2}c-a^{2}bd)x}\right) r \log\left(\left((bx+a)^{p}(dx+c)^{q} f\right)^{r} e\right)}{2\,bf}+\frac{\left(\frac{4\,(pq+q^{2})\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)d^{2} f^{2}}{b^{2} c^{2}-2\,abcd+a^{2} d^{2}}+\frac{2\,(pq-2\,q^{2})d^{2} f^{2} \log(dx+c)}{b^{2} c^{2}-2\,abcd+a^{2} d^{2}}-\frac{b^{2} c^{2} f^{2} p^{2}-2\,(p^{2}-3\,pq)abcdf^{2}+(p^{2}-6\,pq)a^{2} d^{2} f^{2}-2\,(b^{2} d^{2} f^{2} pqx^{2}+2\,abd^{2} f^{2} pqx+a^{2} d^{2} f^{2} pq)\log(bx+a)^{2}+4\,(b^{2} d^{2} f^{2} pqx^{2}+2\,abd^{2} f^{2} pqx+a^{2} d^{2} f^{2} pq)\log(bx+a)\log(dx+c)+2\,(b^{2} d^{2} f^{2} q^{2} x^{2}+2\,abd^{2} f^{2} q^{2} x+a^{2} d^{2} f^{2} q^{2})\log(dx+c)^{2}+6\,(b^{2} cdf^{2} pq-abd^{2} f^{2} pq)x+2\,((pq-2\,q^{2})b^{2} d^{2} f^{2} x^{2}+2\,(pq-2\,q^{2})abd^{2} f^{2} x+(pq-2\,q^{2})a^{2} d^{2} f^{2})\log(bx+a)}{a^{2} b^{2} c^{2}-2\,a^{3} bcd+a^{4} d^{2}+(b^{4} c^{2}-2\,ab^{3} cd+a^{2} b^{2} d^{2})x^{2}+2\,(ab^{3} c^{2}-2\,a^{2} b^{2} cd+a^{3} bd^{2})x}\right) r^{2}}{4\,bf^{2}}-\frac{\log\left(\left((bx+a)^{p}(dx+c)^{q} f\right)^{r} e\right)^{2}}{2\,(bx+a)^{2} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^3,x, algorithm="maxima
")

[Out] -1/2*(2*d^2*f*q*log(b*x + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 2*d^2*f*q*lo
g(d*x + c)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + (2*b*d*f*q*x - a*d*f*(p - 2*q)
 + b*c*f*p)/(a^2*b*c - a^3*d + (b^3*c - a*b^2*d)*x^2 + 2*(a*b^2*c - a^2*b*d
)*x))*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(b*f) + 1/4*(4*(p*q + q^2)*(lo
g(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c -
 a*d)))*d^2*f^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 2*(p*q - 2*q^2)*d^2*f^2*l
og(d*x + c)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - (b^2*c^2*f^2*p^2 - 2*(p^2 - 3
*p*q)*a*b*c*d*f^2 + (p^2 - 6*p*q)*a^2*d^2*f^2 - 2*(b^2*d^2*f^2*p*q*x^2 + 2*
a*b*d^2*f^2*p*q*x + a^2*d^2*f^2*p*q)*log(b*x + a)^2 + 4*(b^2*d^2*f^2*p*q*x^
2 + 2*a*b*d^2*f^2*p*q*x + a^2*d^2*f^2*p*q)*log(b*x + a)*log(d*x + c) + 2*(b
^2*d^2*f^2*q^2*x^2 + 2*a*b*d^2*f^2*q^2*x + a^2*d^2*f^2*q^2)*log(d*x + c)^2
+ 6*(b^2*c*d*f^2*p*q - a*b*d^2*f^2*p*q)*x + 2*((p*q - 2*q^2)*b^2*d^2*f^2*x^
2 + 2*(p*q - 2*q^2)*a*b*d^2*f^2*x + (p*q - 2*q^2)*a^2*d^2*f^2)*log(b*x + a)
)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d
^2)*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x))*r^2/(b*f^2) - 1/2*l
og(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/((b*x + a)^2*b)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^3,x, algorithm="fricas
")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b^3*x^3 + 3*a*b^2*x^2 + 3*
a^2*b*x + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(e(f(a+bx)^p\,(c+dx)^q)^r\right)^2}{(a+bx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(b*x+a)**3,x)
```

```
[Out] Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2/(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\,{\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)}^r\right)^2}{{\left(a+b\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x)^3,x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x)^3, x)
```

3.23 $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^4}\,dx$

Optimal. Leaf size=764

$$-\frac{2p^2r^2}{27b(a+bx)^3}-\frac{5dpqr^2}{18b(bc-ad)(a+bx)^2}+\frac{8d^2pqr^2}{9b(bc-ad)^2(a+bx)}-\frac{d^2q^2r^2}{3b(bc-ad)^2(a+bx)}+\frac{2d^3pqr^2\log(a+bx)}{9b(bc-ad)^3}-$$

```
[Out] -2/27*p^2*r^2/b/(b*x+a)^3-5/18*d*p*q*r^2/b/(-a*d+b*c)/(b*x+a)^2+8/9*d^2*p*q
*r^2/b/(-a*d+b*c)^2/(b*x+a)-1/3*d^2*q^2*r^2/b/(-a*d+b*c)^2/(b*x+a)+2/9*d^3*
p*q*r^2*ln(b*x+a)/b/(-a*d+b*c)^3-d^3*q^2*r^2*ln(b*x+a)/b/(-a*d+b*c)^3-1/3*d
^3*p*q*r^2*ln(b*x+a)^2/b/(-a*d+b*c)^3-2/9*d^3*p*q*r^2*ln(d*x+c)/b/(-a*d+b*c
)^3+d^3*q^2*r^2*ln(d*x+c)/b/(-a*d+b*c)^3+2/3*d^3*p*q*r^2*ln(-d*(b*x+a)/(-a*
d+b*c))*ln(d*x+c)/b/(-a*d+b*c)^3+1/3*d^3*q^2*r^2*ln(d*x+c)^2/b/(-a*d+b*c)^3
-2/3*d^3*q^2*r^2*ln(b*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/b/(-a*d+b*c)^3-2/9*p*r*
ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(b*x+a)^3-1/3*d*q*r*ln(e*(f*(b*x+a)^p*(d*
x+c)^q)^r)/b/(-a*d+b*c)/(b*x+a)^2+2/3*d^2*q*r*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^
r)/b/(-a*d+b*c)^2/(b*x+a)+2/3*d^3*q*r*ln(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q
)^r)/b/(-a*d+b*c)^3-2/3*d^3*q*r*ln(d*x+c)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b
/(-a*d+b*c)^3-1/3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/b/(b*x+a)^3-2/3*d^3*q^2
*r^2*polylog(2,-d*(b*x+a)/(-a*d+b*c))/b/(-a*d+b*c)^3+2/3*d^3*p*q*r^2*polylo
g(2,b*(d*x+c)/(-a*d+b*c))/b/(-a*d+b*c)^3
```

Rubi [A]

time = 0.44, antiderivative size = 764, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 13, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {2584, 2581, 32, 46, 2594, 36, 31, 2580, 2437, 2338, 2441, 2440, 2438}

$$\frac{2d^3pqr^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{3b(bc-ad)^3}-\frac{2d^3q^2r^2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{3b(bc-ad)^3}+\frac{2d^3qr\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(bc-ad)^3}-\frac{2d^3qr\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(bc-ad)^3}-\frac{d^3pqr^2\log^2(a+bx)}{3b(bc-ad)^3}+\frac{2d^3pqr^2\log(a+bx)}{9b(bc-ad)^3}-\frac{2d^3pqr^2\log(c+dx)}{9b(bc-ad)^3}+\frac{2d^3pqr^2\log(c+dx)\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{3b(bc-ad)^3}+\frac{d^3q^2r^2\log^2(c+dx)}{3b(bc-ad)^3}-\frac{d^3q^2r^2\log(a+bx)}{b(bc-ad)^3}+\frac{d^3q^2r^2\log(c+dx)}{b(bc-ad)^3}-\frac{2d^3q^2r^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{3b(bc-ad)^3}+\frac{2d^2qr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(a+bx)(bc-ad)^2}+\frac{8d^2pqr^2}{9b(a+bx)(bc-ad)^2}-\frac{d^2q^2r^2}{3b(a+bx)(bc-ad)^2}-\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(a+bx)^3}-\frac{dqr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(a+bx)^2(bc-ad)}-\frac{2pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{9b(a+bx)^3}-\frac{5dpqr^2}{18b(a+bx)^2(bc-ad)}-\frac{2p^2r^2}{27b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^4,x]
```

```
[Out] (-2*p^2*r^2)/(27*b*(a + b*x)^3) - (5*d*p*q*r^2)/(18*b*(b*c - a*d)*(a + b*x)
^2) + (8*d^2*p*q*r^2)/(9*b*(b*c - a*d)^2*(a + b*x)) - (d^2*q^2*r^2)/(3*b*(b
*c - a*d)^2*(a + b*x)) + (2*d^3*p*q*r^2*Log[a + b*x])/(9*b*(b*c - a*d)^3) -
 (d^3*q^2*r^2*Log[a + b*x])/(b*(b*c - a*d)^3) - (d^3*p*q*r^2*Log[a + b*x]^2
)/(3*b*(b*c - a*d)^3) - (2*d^3*p*q*r^2*Log[c + d*x])/(9*b*(b*c - a*d)^3) +
(d^3*q^2*r^2*Log[c + d*x])/(b*(b*c - a*d)^3) + (2*d^3*p*q*r^2*Log[-((d*(a +
 b*x))/(b*c - a*d))]*Log[c + d*x])/(3*b*(b*c - a*d)^3) + (d^3*q^2*r^2*Log[c
 + d*x]^2)/(3*b*(b*c - a*d)^3) - (2*d^3*q^2*r^2*Log[a + b*x]*Log[(b*(c + d*
x))/(b*c - a*d)])/(3*b*(b*c - a*d)^3) - (2*p*r*Log[e*(f*(a + b*x)^p*(c + d*
x)^q)^r])/(9*b*(a + b*x)^3) - (d*q*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/
(3*b*(b*c - a*d)*(a + b*x)^2) + (2*d^2*q*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q
)^r])/(3*b*(b*c - a*d)^2*(a + b*x)) + (2*d^3*q*r*Log[a + b*x]*Log[e*(f*(a +
```

```
b*x)^p*(c + d*x)^q)^r])/(3*b*(b*c - a*d)^3) - (2*d^3*q*r*Log[c + d*x]*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*b*(b*c - a*d)^3) - Log[e*(f*(a + b*x)^
p*(c + d*x)^q)^r]^2/(3*b*(a + b*x)^3) - (2*d^3*q^2*r^2*PolyLog[2, -((d*(a +
 b*x))/(b*c - a*d))])/(3*b*(b*c - a*d)^3) + (2*d^3*p*q*r^2*PolyLog[2, (b*(c
 + d*x))/(b*c - a*d)])/(3*b*(b*c - a*d)^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2580

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/h), x] + (-Dist[b*p*(r/h), Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[d*q*(r/h), Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 2584

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Dist[b*p*r*(
s/(h*(m + 1))), Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1)/(a + b*x)), x], x] - Dist[d*q*r*(s/(h*(m + 1))), Int[(g + h*x)^(m
+ 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2594

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
```

```
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rubi steps

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^4}\,dx = -\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(a+bx)^3}+\frac{1}{3}(2pr)\int \frac{\log\left(e(f(a+bx)^p(c+\right.}{(a+bx)^4}$$

$$= -\frac{2pr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{9b(a+bx)^3}-\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3b(a+bx)^3}$$

$$= -\frac{2p^2r^2}{27b(a+bx)^3}-\frac{2pr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{9b(a+bx)^3}-\frac{\log^2\left(e(f(a+\right.}{3b(a}$$

$$= -\frac{2p^2r^2}{27b(a+bx)^3}-\frac{dpqr^2}{9b(bc-ad)(a+bx)^2}+\frac{2d^2pqr^2}{9b(bc-ad)^2(a+bx)}+\frac{2d}{}$$

$$= -\frac{2p^2r^2}{27b(a+bx)^3}-\frac{5dpqr^2}{18b(bc-ad)(a+bx)^2}+\frac{8d^2pqr^2}{9b(bc-ad)^2(a+bx)}+$$

$$= -\frac{2p^2r^2}{27b(a+bx)^3}-\frac{5dpqr^2}{18b(bc-ad)(a+bx)^2}+\frac{8d^2pqr^2}{9b(bc-ad)^2(a+bx)}-$$

$$= -\frac{2p^2r^2}{27b(a+bx)^3}-\frac{5dpqr^2}{18b(bc-ad)(a+bx)^2}+\frac{8d^2pqr^2}{9b(bc-ad)^2(a+bx)}-$$

Mathematica [A]
time = 1.16, size = 1407, normalized size = 1.84

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^4,x]

[Out] -1/54*(4*b^3*c^3*p^2*r^2 - 12*a*b^2*c^2*d*p^2*r^2 + 12*a^2*b*c*d^2*p^2*r^2
- 4*a^3*d^3*p^2*r^2 + 15*a*b^2*c^2*d*p*q*r^2 - 78*a^2*b*c*d^2*p*q*r^2 + 63*
a^3*d^3*p*q*r^2 + 18*a^2*b*c*d^2*q^2*r^2 - 18*a^3*d^3*q^2*r^2 + 15*b^3*c^2*
d*p*q*r^2*x - 126*a*b^2*c*d^2*p*q*r^2*x + 111*a^2*b*d^3*p*q*r^2*x + 36*a*b^
2*c*d^2*q^2*r^2*x - 36*a^2*b*d^3*q^2*r^2*x - 48*b^3*c*d^2*p*q*r^2*x^2 + 48*
a*b^2*d^3*p*q*r^2*x^2 + 18*b^3*c*d^2*q^2*r^2*x^2 - 18*a*b^2*d^3*q^2*r^2*x^2
 + 18*d^3*p*q*r^2*(a + b*x)^3*Log[a + b*x]^2 + 12*a^3*d^3*p*q*r^2*Log[c + d
*x] - 54*a^3*d^3*q^2*r^2*Log[c + d*x] + 36*a^2*b*d^3*p*q*r^2*x*Log[c + d*x]
 - 162*a^2*b*d^3*q^2*r^2*x*Log[c + d*x] + 36*a*b^2*d^3*p*q*r^2*x^2*Log[c +
```

```
d*x] - 162*a*b^2*d^3*q^2*r^2*x^2*Log[c + d*x] + 12*b^3*d^3*p*q*r^2*x^3*Log[
c + d*x] - 54*b^3*d^3*q^2*r^2*x^3*Log[c + d*x] - 18*a^3*d^3*q^2*r^2*Log[c +
 d*x]^2 - 54*a^2*b*d^3*q^2*r^2*x*Log[c + d*x]^2 - 54*a*b^2*d^3*q^2*r^2*x^2*
Log[c + d*x]^2 - 18*b^3*d^3*q^2*r^2*x^3*Log[c + d*x]^2 + 12*b^3*c^3*p*r*Log
[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 36*a*b^2*c^2*d*p*r*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r] + 36*a^2*b*c*d^2*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
- 12*a^3*d^3*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 18*a*b^2*c^2*d*q*r*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 72*a^2*b*c*d^2*q*r*Log[e*(f*(a + b*x
)^p*(c + d*x)^q)^r] + 54*a^3*d^3*q*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] +
 18*b^3*c^2*d*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 108*a*b^2*c*d^2*
q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 90*a^2*b*d^3*q*r*x*Log[e*(f*(a
 + b*x)^p*(c + d*x)^q)^r] - 36*b^3*c*d^2*q*r*x^2*Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r] + 36*a*b^2*d^3*q*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 36
*a^3*d^3*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 108*a^2*b*
d^3*q*r*x*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 108*a*b^2*d^3
*q*r*x^2*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 36*b^3*d^3*q*r
*x^3*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 18*b^3*c^3*Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 54*a*b^2*c^2*d*Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^2 + 54*a^2*b*c*d^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 18*a
^3*d^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 6*d^3*q*r*(a + b*x)^3*Log[a
 + b*x]*(2*p*r - 9*q*r + 6*p*r*Log[c + d*x] - 6*(p + q)*r*Log[(b*(c + d*x))
/(b*c - a*d)] + 6*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) + 36*d^3*q*(p + q)*
r^2*(a + b*x)^3*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(b*c - a*d)^3*
(a + b*x)^3)
```

Maple [**F**]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(e(f(bx+a)^{p} (dx+c)^{q})^{r}\right)^{2}}{(bx+a)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^4,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^4,x)
```

Maxima [**A**]
time = 0.40, size = 1254, normalized size = 1.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^4,x, algorithm="maxima
")
```

```
[Out] 1/9*(6*d^3*f*q*log(b*x + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3) - 6*d^3*f*q*log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3) + (6*b^2*d^2*f*q*x^2 + a*b*c*d*f*(4*p - 3*q) - a^2*d^2*f*(2*p - 9*q)
 - 2*b^2*c^2*f*p - 3*(b^2*c*d*f*q - 5*a*b*d^2*f*q)*x)/(a^3*b^2*c^2 - 2*a^4*
b*c*d + a^5*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^3 + 3*(a*b^4*c^2
- 2*a^2*b^3*c*d + a^3*b^2*d^2)*x^2 + 3*(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b
*d^2)*x))*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(b*f) - 1/54*(36*(p*q + q^
2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/
(b*c - a*d)))*d^3*f^2/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) +
 6*(2*p*q - 9*q^2)*d^3*f^2*log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*
c*d^2 - a^3*d^3) + (4*b^3*c^3*f^2*p^2 - 3*(4*p^2 - 5*p*q)*a*b^2*c^2*d*f^2 +
 6*(2*p^2 - 13*p*q + 3*q^2)*a^2*b*c*d^2*f^2 - (4*p^2 - 63*p*q + 18*q^2)*a^3
*d^3*f^2 - 6*((8*p*q - 3*q^2)*b^3*c*d^2*f^2 - (8*p*q - 3*q^2)*a*b^2*d^3*f^2
)*x^2 + 18*(b^3*d^3*f^2*p*q*x^3 + 3*a*b^2*d^3*f^2*p*q*x^2 + 3*a^2*b*d^3*f^2
*p*q*x + a^3*d^3*f^2*p*q)*log(b*x + a)^2 - 36*(b^3*d^3*f^2*p*q*x^3 + 3*a*b^
2*d^3*f^2*p*q*x^2 + 3*a^2*b*d^3*f^2*p*q*x + a^3*d^3*f^2*p*q)*log(b*x + a)*l
og(d*x + c) - 18*(b^3*d^3*f^2*q^2*x^3 + 3*a*b^2*d^3*f^2*q^2*x^2 + 3*a^2*b*d
^3*f^2*q^2*x + a^3*d^3*f^2*q^2)*log(d*x + c)^2 + 3*(5*b^3*c^2*d*f^2*p*q - 6
*(7*p*q - 2*q^2)*a*b^2*c*d^2*f^2 + (37*p*q - 12*q^2)*a^2*b*d^3*f^2)*x - 6*(
(2*p*q - 9*q^2)*b^3*d^3*f^2*x^3 + 3*(2*p*q - 9*q^2)*a*b^2*d^3*f^2*x^2 + 3*(
2*p*q - 9*q^2)*a^2*b*d^3*f^2*x + (2*p*q - 9*q^2)*a^3*d^3*f^2)*log(b*x + a))
/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*
b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c
^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*
d + 3*a^4*b^2*c*d^2 - a^5*b*d^3)*x))*r^2/(b*f^2) - 1/3*log(((b*x + a)^p*(d*
x + c)^q*f)^r*e)^2/((b*x + a)^3*b)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^4,x, algorithm="fricas
")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b^4*x^4 + 4*a*b^3*x^3 + 6*
a^2*b^2*x^2 + 4*a^3*b*x + a^4), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(e(f(a+bx)^p\,(c+dx)^q)^r\right)^2}{(a+bx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(b*x+a)**4,x)
```

```
[Out] Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2/(a + b*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)^r\right)^2}{{\left(a+b\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x)^4,x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x)^4, x)
```

3.24 $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)^5}\,dx$

Optimal. Leaf size=884

$$-\frac{p^2r^2}{32b(a+bx)^4}-\frac{7dpqr^2}{72b(bc-ad)(a+bx)^3}+\frac{3d^2pqr^2}{16b(bc-ad)^2(a+bx)^2}-\frac{d^2q^2r^2}{12b(bc-ad)^2(a+bx)^2}-\frac{5d^3pqr^2}{8b(bc-ad)^3(a+\cdots}$$

```
[Out] -1/32*p^2*r^2/b/(b*x+a)^4-7/72*d*p*q*r^2/b/(-a*d+b*c)/(b*x+a)^3+3/16*d^2*p*
q*r^2/b/(-a*d+b*c)^2/(b*x+a)^2-1/12*d^2*q^2*r^2/b/(-a*d+b*c)^2/(b*x+a)^2-5/
8*d^3*p*q*r^2/b/(-a*d+b*c)^3/(b*x+a)+5/12*d^3*q^2*r^2/b/(-a*d+b*c)^3/(b*x+a
)-1/8*d^4*p*q*r^2*ln(b*x+a)/b/(-a*d+b*c)^4+11/12*d^4*q^2*r^2*ln(b*x+a)/b/(-
a*d+b*c)^4+1/4*d^4*p*q*r^2*ln(b*x+a)^2/b/(-a*d+b*c)^4+1/8*d^4*p*q*r^2*ln(d*
x+c)/b/(-a*d+b*c)^4-11/12*d^4*q^2*r^2*ln(d*x+c)/b/(-a*d+b*c)^4-1/2*d^4*p*q*
r^2*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/b/(-a*d+b*c)^4-1/4*d^4*q^2*r^2*ln(d
*x+c)^2/b/(-a*d+b*c)^4+1/2*d^4*q^2*r^2*ln(b*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/b
/(-a*d+b*c)^4-1/8*p*r*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(b*x+a)^4-1/6*d*q*r
*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/(-a*d+b*c)/(b*x+a)^3+1/4*d^2*q*r*ln(e*(f
*(b*x+a)^p*(d*x+c)^q)^r)/b/(-a*d+b*c)^2/(b*x+a)^2-1/2*d^3*q*r*ln(e*(f*(b*x+
a)^p*(d*x+c)^q)^r)/b/(-a*d+b*c)^3/(b*x+a)-1/2*d^4*q*r*ln(b*x+a)*ln(e*(f*(b*
x+a)^p*(d*x+c)^q)^r)/b/(-a*d+b*c)^4+1/2*d^4*q*r*ln(d*x+c)*ln(e*(f*(b*x+a)^p
*(d*x+c)^q)^r)/b/(-a*d+b*c)^4-1/4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/b/(b*x+
a)^4+1/2*d^4*q^2*r^2*polylog(2,-d*(b*x+a)/(-a*d+b*c))/b/(-a*d+b*c)^4-1/2*d^
4*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c))/b/(-a*d+b*c)^4
```

Rubi [A]
time = 0.54, antiderivative size = 884, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 13, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {2584, 2581, 32, 46, 2594, 36, 31, 2580, 2437, 2338, 2441, 2440, 2438}

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^5,x]
```

```
[Out] -1/32*(p^2*r^2)/(b*(a + b*x)^4) - (7*d*p*q*r^2)/(72*b*(b*c - a*d)*(a + b*x)
^3) + (3*d^2*p*q*r^2)/(16*b*(b*c - a*d)^2*(a + b*x)^2) - (d^2*q^2*r^2)/(12*
b*(b*c - a*d)^2*(a + b*x)^2) - (5*d^3*p*q*r^2)/(8*b*(b*c - a*d)^3*(a + b*x)
) + (5*d^3*q^2*r^2)/(12*b*(b*c - a*d)^3*(a + b*x)) - (d^4*p*q*r^2*Log[a + b
*x])/(8*b*(b*c - a*d)^4) + (11*d^4*q^2*r^2*Log[a + b*x])/(12*b*(b*c - a*d)^
4) + (d^4*p*q*r^2*Log[a + b*x]^2)/(4*b*(b*c - a*d)^4) + (d^4*p*q*r^2*Log[c
+ d*x])/(8*b*(b*c - a*d)^4) - (11*d^4*q^2*r^2*Log[c + d*x])/(12*b*(b*c - a*
d)^4) - (d^4*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(2*b*(
b*c - a*d)^4) - (d^4*q^2*r^2*Log[c + d*x]^2)/(4*b*(b*c - a*d)^4) + (d^4*q^2
*r^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(2*b*(b*c - a*d)^4) - (p*
```

```
r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(8*b*(a + b*x)^4) - (d*q*r*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r])/(6*b*(b*c - a*d)*(a + b*x)^3) + (d^2*q*r*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*b*(b*c - a*d)^2*(a + b*x)^2) - (d^3*q*
r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b*(b*c - a*d)^3*(a + b*x)) - (d^
4*q*r*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b*(b*c - a*d)^4
) + (d^4*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b*(b*c -
 a*d)^4) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(4*b*(a + b*x)^4) + (d^4*
q^2*r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(2*b*(b*c - a*d)^4) - (d^
4*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(2*b*(b*c - a*d)^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2580

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/h), x] + (-Dist[b*p*(r/h), Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[d*q*(r/h), Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 2584

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Dist[b*p*r*(
s/(h*(m + 1))), Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1)/(a + b*x)), x], x] - Dist[d*q*r*(s/(h*(m + 1))), Int[(g + h*x)^(m
+ 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2594

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rubi steps

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(a+bx)^5}\,dx = -\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b(a+bx)^4} + \frac{1}{2}(pr)\int \frac{\log\left(e(f(a+bx)^p(c+dx\right.}{(a+bx)^5}$$

$$= -\frac{pr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{8b(a+bx)^4} - \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4b(a+bx)^4} +$$

$$= -\frac{p^2r^2}{32b(a+bx)^4} - \frac{pr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{8b(a+bx)^4} - \frac{\log^2\left(e(f(a+bx\right.}{4b(a+}$$

$$= -\frac{p^2r^2}{32b(a+bx)^4} - \frac{dpqr^2}{24b(bc-ad)(a+bx)^3} + \frac{d^2pqr^2}{16b(bc-ad)^2(a+bx)^2} -$$

$$= -\frac{p^2r^2}{32b(a+bx)^4} - \frac{7dpqr^2}{72b(bc-ad)(a+bx)^3} + \frac{3d^2pqr^2}{16b(bc-ad)^2(a+bx)^2} -$$

$$= -\frac{p^2r^2}{32b(a+bx)^4} - \frac{7dpqr^2}{72b(bc-ad)(a+bx)^3} + \frac{3d^2pqr^2}{16b(bc-ad)^2(a+bx)^2} -$$

$$= -\frac{p^2r^2}{32b(a+bx)^4} - \frac{7dpqr^2}{72b(bc-ad)(a+bx)^3} + \frac{3d^2pqr^2}{16b(bc-ad)^2(a+bx)^2} -$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2003 vs. $2(884) = 1768$.
time = 1.78, size = 2003, normalized size = 2.27

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^5,x]
```

```
[Out] (-9*b^4*c^4*p^2*r^2 + 36*a*b^3*c^3*d*p^2*r^2 - 54*a^2*b^2*c^2*d^2*p^2*r^2 +
 36*a^3*b*c*d^3*p^2*r^2 - 9*a^4*d^4*p^2*r^2 - 28*a*b^3*c^3*d*p*q*r^2 + 138*
a^2*b^2*c^2*d^2*p*q*r^2 - 372*a^3*b*c*d^3*p*q*r^2 + 262*a^4*d^4*p*q*r^2 - 2
4*a^2*b^2*c^2*d^2*q^2*r^2 + 168*a^3*b*c*d^3*q^2*r^2 - 144*a^4*d^4*q^2*r^2 -
 28*b^4*c^3*d*p*q*r^2*x + 192*a*b^3*c^2*d^2*p*q*r^2*x - 840*a^2*b^2*c*d^3*p
```

*q*r^2*x + 676*a^3*b*d^4*p*q*r^2*x - 48*a*b^3*c^2*d^2*q^2*r^2*x + 456*a^2*b
^2*c*d^3*q^2*r^2*x - 408*a^3*b*d^4*q^2*r^2*x + 54*b^4*c^2*d^2*p*q*r^2*x^2 -
648*a*b^3*c*d^3*p*q*r^2*x^2 + 594*a^2*b^2*d^4*p*q*r^2*x^2 - 24*b^4*c^2*d^2
*q^2*r^2*x^2 + 408*a*b^3*c*d^3*q^2*r^2*x^2 - 384*a^2*b^2*d^4*q^2*r^2*x^2 -
180*b^4*c*d^3*p*q*r^2*x^3 + 180*a*b^3*d^4*p*q*r^2*x^3 + 120*b^4*c*d^3*q^2*r
^2*x^3 - 120*a*b^3*d^4*q^2*r^2*x^3 + 72*d^4*p*q*r^2*(a + b*x)^4*Log[a + b*x
]^2 + 36*a^4*d^4*p*q*r^2*Log[c + d*x] - 264*a^4*d^4*q^2*r^2*Log[c + d*x] +
144*a^3*b*d^4*p*q*r^2*x*Log[c + d*x] - 1056*a^3*b*d^4*q^2*r^2*x*Log[c + d*x
] + 216*a^2*b^2*d^4*p*q*r^2*x^2*Log[c + d*x] - 1584*a^2*b^2*d^4*q^2*r^2*x^2
*Log[c + d*x] + 144*a*b^3*d^4*p*q*r^2*x^3*Log[c + d*x] - 1056*a*b^3*d^4*q^2
*r^2*x^3*Log[c + d*x] + 36*b^4*d^4*p*q*r^2*x^4*Log[c + d*x] - 264*b^4*d^4*q
^2*r^2*x^4*Log[c + d*x] - 72*a^4*d^4*q^2*r^2*Log[c + d*x]^2 - 288*a^3*b*d^4
*q^2*r^2*x*Log[c + d*x]^2 - 432*a^2*b^2*d^4*q^2*r^2*x^2*Log[c + d*x]^2 - 28
8*a*b^3*d^4*q^2*r^2*x^3*Log[c + d*x]^2 - 72*b^4*d^4*q^2*r^2*x^4*Log[c + d*x
]^2 + 12*d^4*q*r*(a + b*x)^4*Log[a + b*x]*(-3*p*r + 22*q*r - 12*p*r*Log[c +
d*x] + 12*(p + q)*r*Log[(b*(c + d*x))/(b*c - a*d)] - 12*Log[e*(f*(a + b*x)
^p*(c + d*x)^q)^r]) - 36*b^4*c^4*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] +
144*a*b^3*c^3*d*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 216*a^2*b^2*c^2
*d^2*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 144*a^3*b*c*d^3*p*r*Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r] - 36*a^4*d^4*p*r*Log[e*(f*(a + b*x)^p*(c + d*
x)^q)^r] - 48*a*b^3*c^3*d*q*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 216*a^
2*b^2*c^2*d^2*q*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 432*a^3*b*c*d^3*q*
r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 264*a^4*d^4*q*r*Log[e*(f*(a + b*x)
^p*(c + d*x)^q)^r] - 48*b^4*c^3*d*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r
] + 288*a*b^3*c^2*d^2*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 864*a^2*
b^2*c*d^3*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 624*a^3*b*d^4*q*r*x*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 72*b^4*c^2*d^2*q*r*x^2*Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r] - 576*a*b^3*c*d^3*q*r*x^2*Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r] + 504*a^2*b^2*d^4*q*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
- 144*b^4*c*d^3*q*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 144*a*b^3*d^
4*q*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 144*a^4*d^4*q*r*Log[c + d*
x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 576*a^3*b*d^4*q*r*x*Log[c + d*x]*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 864*a^2*b^2*d^4*q*r*x^2*Log[c + d*x]
Log[e(f*(a + b*x)^p*(c + d*x)^q)^r] + 576*a*b^3*d^4*q*r*x^3*Log[c + d*x]*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 144*b^4*d^4*q*r*x^4*Log[c + d*x]*Log
[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 72*b^4*c^4*Log[e*(f*(a + b*x)^p*(c + d*
x)^q)^r]^2 + 288*a*b^3*c^3*d*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 432*a
^2*b^2*c^2*d^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + 288*a^3*b*c*d^3*Log
[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 72*a^4*d^4*Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^2 + 144*d^4*q*(p + q)*r^2*(a + b*x)^4*PolyLog[2, (d*(a + b*x))/(
-(b*c) + a*d)])/(288*b*(b*c - a*d)^4*(a + b*x)^4)

Maple [F]

time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(e(f(bx+a)^p (dx+c)^q)^r\right)^2}{(bx+a)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^5,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^5,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1818 vs. $2(843) = 1686$.
time = 0.47, size = 1818, normalized size = 2.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^5,x, algorithm="maxima
")
```

```
[Out] -1/24*(12*d^4*f*q*log(b*x + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
 - 4*a^3*b*c*d^3 + a^4*d^4) - 12*d^4*f*q*log(d*x + c)/(b^4*c^4 - 4*a*b^3*c^
3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) + (12*b^3*d^3*f*q*x^3 -
a*b^2*c^2*d*f*(9*p - 4*q) + a^2*b*c*d^2*f*(9*p - 14*q) - a^3*d^3*f*(3*p - 2
2*q) + 3*b^3*c^3*f*p - 6*(b^3*c*d^2*f*q - 7*a*b^2*d^3*f*q)*x^2 + 4*(b^3*c^2
*d*f*q - 5*a*b^2*c*d^2*f*q + 13*a^2*b*d^3*f*q)*x)/(a^4*b^3*c^3 - 3*a^5*b^2*
c^2*d + 3*a^6*b*c*d^2 - a^7*d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^
2 - a^3*b^4*d^3)*x^4 + 4*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a
^4*b^3*d^3)*x^3 + 6*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*
b^2*d^3)*x^2 + 4*(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d
^3)*x))*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(b*f) + 1/288*(144*(p*q + q^
2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/
(b*c - a*d)))*d^4*f^2/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4) + 12*(3*p*q - 22*q^2)*d^4*f^2*log(d*x + c)/(b^4*c^4 - 4*
a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) - (9*b^4*c^4*f^2
*p^2 - 4*(9*p^2 - 7*p*q)*a*b^3*c^3*d*f^2 + 6*(9*p^2 - 23*p*q + 4*q^2)*a^2*b
^2*c^2*d^2*f^2 - 12*(3*p^2 - 31*p*q + 14*q^2)*a^3*b*c*d^3*f^2 + (9*p^2 - 26
2*p*q + 144*q^2)*a^4*d^4*f^2 + 60*((3*p*q - 2*q^2)*b^4*c*d^3*f^2 - (3*p*q -
 2*q^2)*a*b^3*d^4*f^2)*x^3 - 6*((9*p*q - 4*q^2)*b^4*c^2*d^2*f^2 - 4*(27*p*q
 - 17*q^2)*a*b^3*c*d^3*f^2 + (99*p*q - 64*q^2)*a^2*b^2*d^4*f^2)*x^2 - 72*(b
^4*d^4*f^2*p*q*x^4 + 4*a*b^3*d^4*f^2*p*q*x^3 + 6*a^2*b^2*d^4*f^2*p*q*x^2 +
4*a^3*b*d^4*f^2*p*q*x + a^4*d^4*f^2*p*q)*log(b*x + a)^2 + 144*(b^4*d^4*f^2*
p*q*x^4 + 4*a*b^3*d^4*f^2*p*q*x^3 + 6*a^2*b^2*d^4*f^2*p*q*x^2 + 4*a^3*b*d^4
*f^2*p*q*x + a^4*d^4*f^2*p*q)*log(b*x + a)*log(d*x + c) + 72*(b^4*d^4*f^2*q
^2*x^4 + 4*a*b^3*d^4*f^2*q^2*x^3 + 6*a^2*b^2*d^4*f^2*q^2*x^2 + 4*a^3*b*d^4*
```

```
f^2*q^2*x + a^4*d^4*f^2*q^2)*log(d*x + c)^2 + 4*(7*b^4*c^3*d*f^2*p*q - 12*(
4*p*q - q^2)*a*b^3*c^2*d^2*f^2 + 6*(35*p*q - 19*q^2)*a^2*b^2*c*d^3*f^2 - (1
69*p*q - 102*q^2)*a^3*b*d^4*f^2)*x + 12*((3*p*q - 22*q^2)*b^4*d^4*f^2*x^4 +
 4*(3*p*q - 22*q^2)*a*b^3*d^4*f^2*x^3 + 6*(3*p*q - 22*q^2)*a^2*b^2*d^4*f^2*
x^2 + 4*(3*p*q - 22*q^2)*a^3*b*d^4*f^2*x + (3*p*q - 22*q^2)*a^4*d^4*f^2)*lo
g(b*x + a))/(a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*
d^3 + a^8*d^4 + (b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*
d^3 + a^4*b^4*d^4)*x^4 + 4*(a*b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2
 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4)*x^3 + 6*(a^2*b^6*c^4 - 4*a^3*b^5*c^3*d +
6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*c*d^3 + a^6*b^2*d^4)*x^2 + 4*(a^3*b^5*c^4 - 4
*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4)*x))*r^2/(
b*f^2) - 1/4*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/((b*x + a)^4*b)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^5,x, algorithm="fricas
")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b^5*x^5 + 5*a*b^4*x^4 + 10
*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x + a^5), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(e\left(f\left(a+bx\right)^{p}\left(c+dx\right)^{q}\right)^{r}\right)^{2}}{\left(a+bx\right)^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(b*x+a)**5,x)
```

```
[Out] Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2/(a + b*x)**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\left(f\left(a+b\,x\right)^p\,\left(c+d\,x\right)^q\right)^r\right)^2}{\left(a+b\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x)^5,x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(a + b*x)^5, x)
```

3.25 $\int (g+hx)^4 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) dx$

Optimal. Leaf size=334

$$-\frac{(bg-ah)^4prx}{5b^4}-\frac{(dg-ch)^4qrx}{5d^4}-\frac{(bg-ah)^3pr(g+hx)^2}{10b^3h}-\frac{(dg-ch)^3qr(g+hx)^2}{10d^3h}-\frac{(bg-ah)^2pr(g+hx)^3}{15b^2h}$$

```
[Out] -1/5*(-a*h+b*g)^4*p*r*x/b^4-1/5*(-c*h+d*g)^4*q*r*x/d^4-1/10*(-a*h+b*g)^3*p*
r*(h*x+g)^2/b^3/h-1/10*(-c*h+d*g)^3*q*r*(h*x+g)^2/d^3/h-1/15*(-a*h+b*g)^2*p
*r*(h*x+g)^3/b^2/h-1/15*(-c*h+d*g)^2*q*r*(h*x+g)^3/d^2/h-1/20*(-a*h+b*g)*p*
r*(h*x+g)^4/b/h-1/20*(-c*h+d*g)*q*r*(h*x+g)^4/d/h-1/25*p*r*(h*x+g)^5/h-1/25
*q*r*(h*x+g)^5/h-1/5*(-a*h+b*g)^5*p*r*ln(b*x+a)/b^5/h-1/5*(-c*h+d*g)^5*q*r*
ln(d*x+c)/d^5/h+1/5*(h*x+g)^5*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/h
```

Rubi [A]
time = 0.13, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2581, 45}

$$-\frac{pr(bg-ah)^5\log(a+bx)}{5b^5h}-\frac{prx(bg-ah)^4}{5b^4}-\frac{pr(g+hx)^2(bg-ah)^3}{10b^3h}-\frac{pr(g+hx)^3(bg-ah)^2}{15b^2h}+\frac{(g+hx)^5\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5h}-\frac{pr(g+hx)^4(bg-ah)}{20bh}-\frac{qr(dg-ch)^5\log(c+dx)}{5d^5h}-\frac{qrx(dg-ch)^4}{5d^4}-\frac{qr(g+hx)^2(dg-ch)^3}{10d^3h}-\frac{qr(g+hx)^3(dg-ch)^2}{15d^2h}-\frac{qr(g+hx)^4(dg-ch)}{20dh}-\frac{pr(g+hx)^5}{25h}-\frac{qr(g+hx)^5}{25h}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -1/5*((b*g - a*h)^4*p*r*x)/b^4 - ((d*g - c*h)^4*q*r*x)/(5*d^4) - ((b*g - a*
h)^3*p*r*(g + h*x)^2)/(10*b^3*h) - ((d*g - c*h)^3*q*r*(g + h*x)^2)/(10*d^3*
h) - ((b*g - a*h)^2*p*r*(g + h*x)^3)/(15*b^2*h) - ((d*g - c*h)^2*q*r*(g + h
*x)^3)/(15*d^2*h) - ((b*g - a*h)*p*r*(g + h*x)^4)/(20*b*h) - ((d*g - c*h)*q
*r*(g + h*x)^4)/(20*d*h) - (p*r*(g + h*x)^5)/(25*h) - (q*r*(g + h*x)^5)/(25
*h) - ((b*g - a*h)^5*p*r*Log[a + b*x])/(5*b^5*h) - ((d*g - c*h)^5*q*r*Log[c
 + d*x])/(5*d^5*h) + ((g + h*x)^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(5*
h)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
```

```
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\int (g+hx)^4 \log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \frac{(g+hx)^5\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5h} - \frac{(bpr)\int \frac{(g+hx)^5}{a+bx}\,dx}{5h}$$

$$= \frac{(g+hx)^5\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5h} - \frac{(bpr)\int \left(\frac{h(bg-ah)^4}{b^5}\right.}{}$$

$$= -\frac{(bg-ah)^4prx}{5b^4} - \frac{(dg-ch)^4qrx}{5d^4} - \frac{(bg-ah)^3pr(g+hx)^2}{10b^3h}$$

Mathematica [A]
time = 0.22, size = 275, normalized size = 0.82

$$\frac{-\frac{pr\left(60bh(bg-ah)^4x+30b^2(bg-ah)^3(g+hx)^2+20b^3(bg-ah)^2(g+hx)^3+15b^4(bg-ah)(g+hx)^4+12b^5(g+hx)^5+60(bg-ah)^5\log(a+bx)\right)}{60b^5}-\frac{qr\left(60dh(dg-ch)^4x+30d^2(dg-ch)^3(g+hx)^2+20d^3(dg-ch)^2(g+hx)^3+15d^4(dg-ch)(g+hx)^4+12d^5(g+hx)^5+60(dg-ch)^5\log(c+dx)\right)}{60d^5}+(g+hx)^5\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{5h}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] (-1/60*(p*r*(60*b*h*(b*g - a*h)^4*x + 30*b^2*(b*g - a*h)^3*(g + h*x)^2 + 20
*b^3*(b*g - a*h)^2*(g + h*x)^3 + 15*b^4*(b*g - a*h)*(g + h*x)^4 + 12*b^5*(g
 + h*x)^5 + 60*(b*g - a*h)^5*Log[a + b*x]))/b^5 - (q*r*(60*d*h*(d*g - c*h)^
4*x + 30*d^2*(d*g - c*h)^3*(g + h*x)^2 + 20*d^3*(d*g - c*h)^2*(g + h*x)^3 +
 15*d^4*(d*g - c*h)*(g + h*x)^4 + 12*d^5*(g + h*x)^5 + 60*(d*g - c*h)^5*Log
[c + d*x]))/(60*d^5) + (g + h*x)^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(5
*h)
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int (hx+g)^4 \ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

```
[Out] int((h*x+g)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 625 vs. 2(309) = 618.
time = 0.29, size = 625, normalized size = 1.87

$$\frac{1}{5}\left(h^4x^5+5gh^3x^4+10g^2h^2x^3+10g^3hx^2+5g^4x\right)\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)+\frac{r\left(\frac{60\left(5ab^4fg^4p-10a^2b^3fg^3hp+10a^3b^2fg^2h^2p-5a^4bfgh^3p+a^5fh^4p\right)\log(bx+a)}{b^5}+\frac{60\left(5cd^4fg^4q-10c^2d^3fg^3hq+10c^3d^2fg^2h^2q-5c^4dfgh^3q+c^5fh^4q\right)\log(dx+c)}{d^5}-\frac{12b^4d^4fh^4(p+q)x^5-15\left(ab^3d^4fh^4p-\left(5d^4fgh^3(p+q)-cd^3fh^4q\right)b^4\right)x^4-20\left(5ab^3d^4fgh^3p-a^2b^2d^4fh^4p-\left(10d^4fg^2h^2(p+q)-5cd^3fgh^3q+c^2d^2fh^4q\right)b^4\right)x^3-30\left(10ab^3d^4fg^2h^2p-5a^2b^2d^4fgh^3p+a^3bd^4fh^4p-\left(10d^4fg^3h(p+q)-10cd^3fg^2h^2q+5c^2d^2fgh^3q-c^3dfh^4q\right)b^4\right)x^2-60\left(10ab^3d^4fg^3hp-10a^2b^2d^4fg^2h^2p+5a^3bd^4fgh^3p-a^4d^4fh^4p-\left(5d^4fg^4(p+q)-10cd^3fg^3hq+10c^2d^2fg^2h^2q-5c^3dfgh^3q+c^4fh^4q\right)b^4\right)x}{b^4d^4}\right)}{300f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] 1/5*(h^4*x^5 + 5*g*h^3*x^4 + 10*g^2*h^2*x^3 + 10*g^3*h*x^2 + 5*g^4*x)*log((
(b*x + a)^p*(d*x + c)^q*f)^r*e) + 1/300*r*(60*(5*a*b^4*f*g^4*p - 10*a^2*b^3
*f*g^3*h*p + 10*a^3*b^2*f*g^2*h^2*p - 5*a^4*b*f*g*h^3*p + a^5*f*h^4*p)*log(
b*x + a)/b^5 + 60*(5*c*d^4*f*g^4*q - 10*c^2*d^3*f*g^3*h*q + 10*c^3*d^2*f*g^
2*h^2*q - 5*c^4*d*f*g*h^3*q + c^5*f*h^4*q)*log(d*x + c)/d^5 - (12*b^4*d^4*f
*h^4*(p + q)*x^5 - 15*(a*b^3*d^4*f*h^4*p - (5*d^4*f*g*h^3*(p + q) - c*d^3*f
*h^4*q)*b^4)*x^4 - 20*(5*a*b^3*d^4*f*g*h^3*p - a^2*b^2*d^4*f*h^4*p - (10*d^
4*f*g^2*h^2*(p + q) - 5*c*d^3*f*g*h^3*q + c^2*d^2*f*h^4*q)*b^4)*x^3 - 30*(1
0*a*b^3*d^4*f*g^2*h^2*p - 5*a^2*b^2*d^4*f*g*h^3*p + a^3*b*d^4*f*h^4*p - (10
*d^4*f*g^3*h*(p + q) - 10*c*d^3*f*g^2*h^2*q + 5*c^2*d^2*f*g*h^3*q - c^3*d*f
*h^4*q)*b^4)*x^2 - 60*(10*a*b^3*d^4*f*g^3*h*p - 10*a^2*b^2*d^4*f*g^2*h^2*p
+ 5*a^3*b*d^4*f*g*h^3*p - a^4*d^4*f*h^4*p - (5*d^4*f*g^4*(p + q) - 10*c*d^3
*f*g^3*h*q + 10*c^2*d^2*f*g^2*h^2*q - 5*c^3*d*f*g*h^3*q + c^4*f*h^4*q)*b^4)
*x)/(b^4*d^4))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 943 vs. $2(309) = 618$.
time = 0.41, size = 943, normalized size = 2.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] 1/300*(12*(5*b^5*d^5*h^4 - (b^5*d^5*h^4*p + b^5*d^5*h^4*q)*r)*x^5 + 15*(20*
b^5*d^5*g*h^3 - ((5*b^5*d^5*g*h^3 - a*b^4*d^5*h^4)*p + (5*b^5*d^5*g*h^3 - b
^5*c*d^4*h^4)*q)*r)*x^4 + 20*(30*b^5*d^5*g^2*h^2 - ((10*b^5*d^5*g^2*h^2 - 5
*a*b^4*d^5*g*h^3 + a^2*b^3*d^5*h^4)*p + (10*b^5*d^5*g^2*h^2 - 5*b^5*c*d^4*g
*h^3 + b^5*c^2*d^3*h^4)*q)*r)*x^3 + 30*(20*b^5*d^5*g^3*h - ((10*b^5*d^5*g^3
*h - 10*a*b^4*d^5*g^2*h^2 + 5*a^2*b^3*d^5*g*h^3 - a^3*b^2*d^5*h^4)*p + (10*
b^5*d^5*g^3*h - 10*b^5*c*d^4*g^2*h^2 + 5*b^5*c^2*d^3*g*h^3 - b^5*c^3*d^2*h^
4)*q)*r)*x^2 + 60*(5*b^5*d^5*g^4 - ((5*b^5*d^5*g^4 - 10*a*b^4*d^5*g^3*h + 1
0*a^2*b^3*d^5*g^2*h^2 - 5*a^3*b^2*d^5*g*h^3 + a^4*b*d^5*h^4)*p + (5*b^5*d^5
*g^4 - 10*b^5*c*d^4*g^3*h + 10*b^5*c^2*d^3*g^2*h^2 - 5*b^5*c^3*d^2*g*h^3 +
b^5*c^4*d*h^4)*q)*r)*x + 60*(b^5*d^5*h^4*p*r*x^5 + 5*b^5*d^5*g*h^3*p*r*x^4
+ 10*b^5*d^5*g^2*h^2*p*r*x^3 + 10*b^5*d^5*g^3*h*p*r*x^2 + 5*b^5*d^5*g^4*p*r
*x + (5*a*b^4*d^5*g^4 - 10*a^2*b^3*d^5*g^3*h + 10*a^3*b^2*d^5*g^2*h^2 - 5*a
^4*b*d^5*g*h^3 + a^5*d^5*h^4)*p*r)*log(b*x + a) + 60*(b^5*d^5*h^4*q*r*x^5 +
 5*b^5*d^5*g*h^3*q*r*x^4 + 10*b^5*d^5*g^2*h^2*q*r*x^3 + 10*b^5*d^5*g^3*h*q*
r*x^2 + 5*b^5*d^5*g^4*q*r*x + (5*b^5*c*d^4*g^4 - 10*b^5*c^2*d^3*g^3*h + 10*
b^5*c^3*d^2*g^2*h^2 - 5*b^5*c^4*d*g*h^3 + b^5*c^5*h^4)*q*r)*log(d*x + c) +
60*(b^5*d^5*h^4*r*x^5 + 5*b^5*d^5*g*h^3*r*x^4 + 10*b^5*d^5*g^2*h^2*r*x^3 +
10*b^5*d^5*g^3*h*r*x^2 + 5*b^5*d^5*g^4*r*x)*log(f))/(b^5*d^5)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**4*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 1.03, size = 1128, normalized size = 3.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(g + h*x)^4,x)
```

```
[Out] log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(g^4*x + (h^4*x^5)/5 + 2*g^3*h*x^2 + g
*h^3*x^4 + 2*g^2*h^2*x^3) - x^2*(((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((h^3*r
*(b*c*h*p + 5*b*d*g*p + a*d*h*q + 5*b*d*g*q))/(5*b*d) - (h^4*r*(p + q)*(5*a
*d + 5*b*c))/(25*b*d)))/(5*b*d) - (g*h^2*r*(b*c*h*p + 2*b*d*g*p + a*d*h*q +
 2*b*d*g*q))/(b*d) + (a*c*h^4*r*(p + q))/(5*b*d)))/(10*b*d) - (a*c*((h^3*r*
(b*c*h*p + 5*b*d*g*p + a*d*h*q + 5*b*d*g*q))/(5*b*d) - (h^4*r*(p + q)*(5*a*
d + 5*b*c))/(25*b*d)))/(2*b*d) + (g^2*h*r*(b*c*h*p + b*d*g*p + a*d*h*q + b*
d*g*q))/(b*d)) - x^4*((h^3*r*(b*c*h*p + 5*b*d*g*p + a*d*h*q + 5*b*d*g*q))/(
20*b*d) - (h^4*r*(p + q)*(5*a*d + 5*b*c))/(100*b*d)) - x*((a*c*(((5*a*d + 5
*b*c)*((h^3*r*(b*c*h*p + 5*b*d*g*p + a*d*h*q + 5*b*d*g*q))/(5*b*d) - (h^4*r
*(p + q)*(5*a*d + 5*b*c))/(25*b*d)))/(5*b*d) - (g*h^2*r*(b*c*h*p + 2*b*d*g*
p + a*d*h*q + 2*b*d*g*q))/(b*d) + (a*c*h^4*r*(p + q))/(5*b*d)))/(b*d) - ((5
*a*d + 5*b*c)*(((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((h^3*r*(b*c*h*p + 5*b*d*
g*p + a*d*h*q + 5*b*d*g*q))/(5*b*d) - (h^4*r*(p + q)*(5*a*d + 5*b*c))/(25*b
*d)))/(5*b*d) - (g*h^2*r*(b*c*h*p + 2*b*d*g*p + a*d*h*q + 2*b*d*g*q))/(b*d)
 + (a*c*h^4*r*(p + q))/(5*b*d)))/(5*b*d) - (a*c*((h^3*r*(b*c*h*p + 5*b*d*g*
p + a*d*h*q + 5*b*d*g*q))/(5*b*d) - (h^4*r*(p + q)*(5*a*d + 5*b*c))/(25*b*d
```

```
)))/(b*d) + (2*g^2*h*r*(b*c*h*p + b*d*g*p + a*d*h*q + b*d*g*q))/(b*d)))/(5*
b*d) + (g^3*r*(2*b*c*h*p + b*d*g*p + 2*a*d*h*q + b*d*g*q))/(b*d)) + x^3*(((
5*a*d + 5*b*c)*((h^3*r*(b*c*h*p + 5*b*d*g*p + a*d*h*q + 5*b*d*g*q))/(5*b*d)
 - (h^4*r*(p + q)*(5*a*d + 5*b*c))/(25*b*d)))/(15*b*d) - (g*h^2*r*(b*c*h*p
+ 2*b*d*g*p + a*d*h*q + 2*b*d*g*q))/(3*b*d) + (a*c*h^4*r*(p + q))/(15*b*d))
 + (log(a + b*x)*((a^5*h^4*p*r)/5 + a*b^4*g^4*p*r + 2*a^3*b^2*g^2*h^2*p*r -
 a^4*b*g*h^3*p*r - 2*a^2*b^3*g^3*h*p*r))/b^5 + (log(c + d*x)*((c^5*h^4*q*r)
/5 + c*d^4*g^4*q*r + 2*c^3*d^2*g^2*h^2*q*r - c^4*d*g*h^3*q*r - 2*c^2*d^3*g^
3*h*q*r))/d^5 - (h^4*r*x^5*(p + q))/25
```

3.26 $\int (g+hx)^3 \log\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx$

Optimal. Leaf size=276

$$-\frac{(bg-ah)^3prx}{4b^3}-\frac{(dg-ch)^3qrx}{4d^3}-\frac{(bg-ah)^2pr(g+hx)^2}{8b^2h}-\frac{(dg-ch)^2qr(g+hx)^2}{8d^2h}-\frac{(bg-ah)pr(g+hx)^3}{12bh}-$$

```
[Out] -1/4*(-a*h+b*g)^3*p*r*x/b^3-1/4*(-c*h+d*g)^3*q*r*x/d^3-1/8*(-a*h+b*g)^2*p*r
*(h*x+g)^2/b^2/h-1/8*(-c*h+d*g)^2*q*r*(h*x+g)^2/d^2/h-1/12*(-a*h+b*g)*p*r*(
h*x+g)^3/b/h-1/12*(-c*h+d*g)*q*r*(h*x+g)^3/d/h-1/16*p*r*(h*x+g)^4/h-1/16*q*
r*(h*x+g)^4/h-1/4*(-a*h+b*g)^4*p*r*ln(b*x+a)/b^4/h-1/4*(-c*h+d*g)^4*q*r*ln(
d*x+c)/d^4/h+1/4*(h*x+g)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/h
```

Rubi [A]
time = 0.09, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2581, 45}

$$-\frac{pr(bg-ah)^4\log(a+bx)}{4b^4h}-\frac{prx(bg-ah)^3}{4b^3}-\frac{pr(g+hx)^2(bg-ah)^2}{8b^2h}+\frac{(g+hx)^4\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4h}-\frac{pr(g+hx)^3(bg-ah)}{12bh}-\frac{qr(dg-ch)^4\log(c+dx)}{4d^4h}-\frac{qrx(dg-ch)^3}{4d^3}-\frac{qr(g+hx)^2(dg-ch)^2}{8d^2h}-\frac{qr(g+hx)^3(dg-ch)}{12dh}-\frac{pr(g+hx)^4}{16h}-\frac{qr(g+hx)^4}{16h}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -1/4*((b*g - a*h)^3*p*r*x)/b^3 - ((d*g - c*h)^3*q*r*x)/(4*d^3) - ((b*g - a*
h)^2*p*r*(g + h*x)^2)/(8*b^2*h) - ((d*g - c*h)^2*q*r*(g + h*x)^2)/(8*d^2*h)
 - ((b*g - a*h)*p*r*(g + h*x)^3)/(12*b*h) - ((d*g - c*h)*q*r*(g + h*x)^3)/(
12*d*h) - (p*r*(g + h*x)^4)/(16*h) - (q*r*(g + h*x)^4)/(16*h) - ((b*g - a*h
)^4*p*r*Log[a + b*x])/(4*b^4*h) - ((d*g - c*h)^4*q*r*Log[c + d*x])/(4*d^4*h
) + ((g + h*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*h)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\int (g+hx)^3 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx = \frac{(g+hx)^4\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h} - \frac{(bpr)\int \frac{(g+hx)^4}{a+bx}\,dx}{4h}$$

$$= \frac{(g+hx)^4\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h} - \frac{(bpr)\int \left(\frac{h(bg-ah)^3}{b^4}\right.}{}$$

$$= -\frac{(bg-ah)^3prx}{4b^3} - \frac{(dg-ch)^3qrx}{4d^3} - \frac{(bg-ah)^2pr(g+hx)^2}{8b^2h}$$

Mathematica [A]
time = 0.19, size = 231, normalized size = 0.84

$$\frac{\frac{1}{12}r\left(-\frac{p\left(12bh(bg-ah)^3x+6b^2(bg-ah)^2(g+hx)^2+4b^3(bg-ah)(g+hx)^3+3b^4(g+hx)^4+12(bg-ah)^4\log(a+bx)\right)}{b^4}-\frac{q\left(12dh(dg-ch)^3x+6d^2(dg-ch)^2(g+hx)^2+4d^3(dg-ch)(g+hx)^3+3d^4(g+hx)^4+12(dg-ch)^4\log(c+dx)\right)}{d^4}\right)+(g+hx)^4\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] ((r*(-((p*(12*b*h*(b*g - a*h)^3*x + 6*b^2*(b*g - a*h)^2*(g + h*x)^2 + 4*b^3
*(b*g - a*h)*(g + h*x)^3 + 3*b^4*(g + h*x)^4 + 12*(b*g - a*h)^4*Log[a + b*x
]))/b^4) - (q*(12*d*h*(d*g - c*h)^3*x + 6*d^2*(d*g - c*h)^2*(g + h*x)^2 + 4
*d^3*(d*g - c*h)*(g + h*x)^3 + 3*d^4*(g + h*x)^4 + 12*(d*g - c*h)^4*Log[c +
 d*x]))/d^4))/12 + (g + h*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*h)
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int (hx+g)^3 \ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

```
[Out] int((h*x+g)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

Maxima [A]
time = 0.30, size = 432, normalized size = 1.57

$$\frac{1}{4}\left(h^3x^4+4\,gh^2x^3+6\,g^2hx^2+4\,g^3x\right)\log\left(\left(((bx+a)^p(dx+c)^q f\right)^r e\right)+\frac{r\left(\frac{12\left(4\,ab^3fg^3p-6\,a^2b^2fg^2hp+4\,a^3bfgh^2p-a^4fh^3p\right)\log(bx+a)}{b^4}+\frac{12\left(4\,cd^3fg^3q-6\,c^2d^2fg^2hq+4\,c^3dfgh^2q-c^4fh^3q\right)\log(dx+c)}{d^4}-\frac{3\,b^3d^3fh^3(p+q)x^4-4\left(ab^2d^3fh^3p-\left(4\,d^3fgh^2(p+q)-cd^2fh^3q\right)b^3\right)x^3-6\left(4\,ab^2d^3fgh^2p-a^2bd^3fh^3p-\left(6\,d^3fg^2h(p+q)-4\,cd^2fgh^2q+c^2dfh^3q\right)b^3\right)x^2-12\left(6\,ab^2d^3fg^2hp-4\,a^2bd^3fgh^2p+a^3d^3fh^3p-\left(4\,d^3fg^3(p+q)-6\,cd^2fg^2hq+4\,c^2dfgh^2q-c^3fh^3q\right)b^3\right)x}{b^3d^3}\right)}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] 1/4*(h^3*x^4 + 4*g*h^2*x^3 + 6*g^2*h*x^2 + 4*g^3*x)*log(((b*x + a)^p*(d*x +
 c)^q*f)^r*e) + 1/48*r*(12*(4*a*b^3*f*g^3*p - 6*a^2*b^2*f*g^2*h*p + 4*a^3*b
*f*g*h^2*p - a^4*f*h^3*p)*log(b*x + a)/b^4 + 12*(4*c*d^3*f*g^3*q - 6*c^2*d^
2*f*g^2*h*q + 4*c^3*d*f*g*h^2*q - c^4*f*h^3*q)*log(d*x + c)/d^4 - (3*b^3*d^
3*f*h^3*(p + q)*x^4 - 4*(a*b^2*d^3*f*h^3*p - (4*d^3*f*g*h^2*(p + q) - c*d^2
*f*h^3*q)*b^3)*x^3 - 6*(4*a*b^2*d^3*f*g*h^2*p - a^2*b*d^3*f*h^3*p - (6*d^3*
f*g^2*h*(p + q) - 4*c*d^2*f*g*h^2*q + c^2*d*f*h^3*q)*b^3)*x^2 - 12*(6*a*b^2
*d^3*f*g^2*h*p - 4*a^2*b*d^3*f*g*h^2*p + a^3*d^3*f*h^3*p - (4*d^3*f*g^3*(p
+ q) - 6*c*d^2*f*g^2*h*q + 4*c^2*d*f*g*h^2*q - c^3*f*h^3*q)*b^3)*x)/(b^3*d^
3))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(255) = 510$.
time = 0.41, size = 677, normalized size = 2.45

$$\frac{3(4b^4d^4h^3 - (b^4d^4h^3p + b^4d^4h^3q)r)x^4 + 4(12b^4d^4gh^2 - ((4b^4d^4gh^2 - ab^3d^4h^3)p + (4b^4d^4gh^2 - b^4cd^3h^3)q)r)x^3 + 6(12b^4d^4g^2h - ((6b^4d^4g^2h - 4ab^3d^4gh^2 + a^2b^2d^4h^3)p + (6b^4d^4g^2h - 4b^4cd^3gh^2 + b^4c^2d^2h^3)q)r)x^2 + 12(4b^4d^4g^3 - ((4b^4d^4g^3 - 6ab^3d^4g^2h + 4a^2b^2d^4gh^2 - a^3bd^4h^3)p + (4b^4d^4g^3 - 6b^4cd^3g^2h + 4b^4c^2d^2gh^2 - b^4c^3dh^3)q)r)x + 12(b^4d^4h^3prx^4 + 4b^4d^4gh^2prx^3 + 6b^4d^4g^2hprx^2 + 4b^4d^4g^3prx + (4ab^3d^4g^3 - 6a^2b^2d^4g^2h + 4a^3bd^4gh^2 - a^4d^4h^3)pr)\log(bx+a) + 12(b^4d^4h^3qrx^4 + 4b^4d^4gh^2qrx^3 + 6b^4d^4g^2hqrx^2 + 4b^4d^4g^3qrx + (4b^4cd^3g^3 - 6b^4c^2d^2g^2h + 4b^4c^3dgh^2 - b^4c^4h^3)qr)\log(dx+c) + 12(b^4d^4h^3rx^4 + 4b^4d^4gh^2rx^3 + 6b^4d^4g^2hrx^2 + 4b^4d^4g^3rx)\log(f)}{48b^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] 1/48*(3*(4*b^4*d^4*h^3 - (b^4*d^4*h^3*p + b^4*d^4*h^3*q)*r)*x^4 + 4*(12*b^4
*d^4*g*h^2 - ((4*b^4*d^4*g*h^2 - a*b^3*d^4*h^3)*p + (4*b^4*d^4*g*h^2 - b^4*
c*d^3*h^3)*q)*r)*x^3 + 6*(12*b^4*d^4*g^2*h - ((6*b^4*d^4*g^2*h - 4*a*b^3*d^
4*g*h^2 + a^2*b^2*d^4*h^3)*p + (6*b^4*d^4*g^2*h - 4*b^4*c*d^3*g*h^2 + b^4*c
^2*d^2*h^3)*q)*r)*x^2 + 12*(4*b^4*d^4*g^3 - ((4*b^4*d^4*g^3 - 6*a*b^3*d^4*g
^2*h + 4*a^2*b^2*d^4*g*h^2 - a^3*b*d^4*h^3)*p + (4*b^4*d^4*g^3 - 6*b^4*c*d^
3*g^2*h + 4*b^4*c^2*d^2*g*h^2 - b^4*c^3*d*h^3)*q)*r)*x + 12*(b^4*d^4*h^3*p*
r*x^4 + 4*b^4*d^4*g*h^2*p*r*x^3 + 6*b^4*d^4*g^2*h*p*r*x^2 + 4*b^4*d^4*g^3*p
*r*x + (4*a*b^3*d^4*g^3 - 6*a^2*b^2*d^4*g^2*h + 4*a^3*b*d^4*g*h^2 - a^4*d^4
*h^3)*p*r)*log(b*x + a) + 12*(b^4*d^4*h^3*q*r*x^4 + 4*b^4*d^4*g*h^2*q*r*x^3
 + 6*b^4*d^4*g^2*h*q*r*x^2 + 4*b^4*d^4*g^3*q*r*x + (4*b^4*c*d^3*g^3 - 6*b^4
*c^2*d^2*g^2*h + 4*b^4*c^3*d*g*h^2 - b^4*c^4*h^3)*q*r)*log(d*x + c) + 12*(b
^4*d^4*h^3*r*x^4 + 4*b^4*d^4*g*h^2*r*x^3 + 6*b^4*d^4*g^2*h*r*x^2 + 4*b^4*d^
4*g^3*r*x)*log(f))/(b^4*d^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 0.83, size = 641, normalized size = 2.32

$$\ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)\left(g^3\,x+\frac{3\,g^2\,h\,x^2}{2}+g\,h^2\,x^3+\frac{h^3\,x^4}{4}\right)-x\left(\frac{(4\,a\,d+4\,b\,c)\left(\frac{(4\,a\,d+4\,b\,c)\left(\frac{h^2\,r\,(b\,c\,h\,p+4\,b\,d\,g\,p+a\,d\,h\,q+4\,b\,d\,g\,q)}{4\,b\,d}-\frac{h^3\,r\,(p+q)\,(4\,a\,d+4\,b\,c)}{16\,b\,d}\right)}{4\,b\,d}-\frac{g\,h\,r\,(2\,b\,c\,h\,p+3\,b\,d\,g\,p+2\,a\,d\,h\,q+3\,b\,d\,g\,q)}{2\,b\,d}+\frac{a\,c\,h^3\,r\,(p+q)}{4\,b\,d}\right)}{4\,b\,d}+\frac{g^2\,r\,(3\,b\,c\,h\,p+2\,b\,d\,g\,p+3\,a\,d\,h\,q+2\,b\,d\,g\,q)}{2\,b\,d}-\frac{a\,c\left(\frac{h^2\,r\,(b\,c\,h\,p+4\,b\,d\,g\,p+a\,d\,h\,q+4\,b\,d\,g\,q)}{4\,b\,d}-\frac{h^3\,r\,(p+q)\,(4\,a\,d+4\,b\,c)}{16\,b\,d}\right)}{b\,d}\right)-x^3\left(\frac{h^2\,r\,(b\,c\,h\,p+4\,b\,d\,g\,p+a\,d\,h\,q+4\,b\,d\,g\,q)}{12\,b\,d}-\frac{h^3\,r\,(p+q)\,(4\,a\,d+4\,b\,c)}{48\,b\,d}\right)+x^2\left(\frac{(4\,a\,d+4\,b\,c)\left(\frac{h^2\,r\,(b\,c\,h\,p+4\,b\,d\,g\,p+a\,d\,h\,q+4\,b\,d\,g\,q)}{4\,b\,d}-\frac{h^3\,r\,(p+q)\,(4\,a\,d+4\,b\,c)}{16\,b\,d}\right)}{8\,b\,d}-\frac{g\,h\,r\,(2\,b\,c\,h\,p+3\,b\,d\,g\,p+2\,a\,d\,h\,q+3\,b\,d\,g\,q)}{4\,b\,d}+\frac{a\,c\,h^3\,r\,(p+q)}{8\,b\,d}\right)-\frac{\ln(a+b\,x)\,(p\,r\,a^4\,h^3-4\,p\,r\,a^3\,b\,g\,h^2+6\,p\,r\,a^2\,b^2\,g^2\,h-4\,p\,r\,a\,b^3\,g^3)}{4\,b^4}-\frac{\ln(c+d\,x)\,(q\,r\,c^4\,h^3-4\,q\,r\,c^3\,d\,g\,h^2+6\,q\,r\,c^2\,d^2\,g^2\,h-4\,q\,r\,c\,d^3\,g^3)}{4\,d^4}-\frac{h^3\,r\,x^4\,(p+q)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(g + h*x)^3,x)
```

```
[Out] log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(g^3*x + (h^3*x^4)/4 + (3*g^2*h*x^2)/2
 + g*h^2*x^3) - x*(((4*a*d + 4*b*c)*(((4*a*d + 4*b*c)*((h^2*r*(b*c*h*p + 4*
b*d*g*p + a*d*h*q + 4*b*d*g*q))/(4*b*d) - (h^3*r*(p + q)*(4*a*d + 4*b*c))/(
16*b*d)))/(4*b*d) - (g*h*r*(2*b*c*h*p + 3*b*d*g*p + 2*a*d*h*q + 3*b*d*g*q))
/(2*b*d) + (a*c*h^3*r*(p + q))/(4*b*d)))/(4*b*d) + (g^2*r*(3*b*c*h*p + 2*b*
d*g*p + 3*a*d*h*q + 2*b*d*g*q))/(2*b*d) - (a*c*((h^2*r*(b*c*h*p + 4*b*d*g*p
 + a*d*h*q + 4*b*d*g*q))/(4*b*d) - (h^3*r*(p + q)*(4*a*d + 4*b*c))/(16*b*d)
))/(b*d)) - x^3*((h^2*r*(b*c*h*p + 4*b*d*g*p + a*d*h*q + 4*b*d*g*q))/(12*b*
d) - (h^3*r*(p + q)*(4*a*d + 4*b*c))/(48*b*d)) + x^2*(((4*a*d + 4*b*c)*((h^
2*r*(b*c*h*p + 4*b*d*g*p + a*d*h*q + 4*b*d*g*q))/(4*b*d) - (h^3*r*(p + q)*(
4*a*d + 4*b*c))/(16*b*d)))/(8*b*d) - (g*h*r*(2*b*c*h*p + 3*b*d*g*p + 2*a*d*
h*q + 3*b*d*g*q))/(4*b*d) + (a*c*h^3*r*(p + q))/(8*b*d)) - (log(a + b*x)*(a
^4*h^3*p*r - 4*a*b^3*g^3*p*r - 4*a^3*b*g*h^2*p*r + 6*a^2*b^2*g^2*h*p*r))/(4
*b^4) - (log(c + d*x)*(c^4*h^3*q*r - 4*c*d^3*g^3*q*r - 4*c^3*d*g*h^2*q*r +
6*c^2*d^2*g^2*h*q*r))/(4*d^4) - (h^3*r*x^4*(p + q))/16
```

3.27 $\int (g+hx)^2 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx$

Optimal. Leaf size=218

$$-\frac{(bg-ah)^2prx}{3b^2}-\frac{(dg-ch)^2qrx}{3d^2}-\frac{(bg-ah)pr(g+hx)^2}{6bh}-\frac{(dg-ch)qr(g+hx)^2}{6dh}-\frac{pr(g+hx)^3}{9h}-\frac{qr(g+hx)^3}{9h}$$

```
[Out] -1/3*(-a*h+b*g)^2*p*r*x/b^2-1/3*(-c*h+d*g)^2*q*r*x/d^2-1/6*(-a*h+b*g)*p*r*(
h*x+g)^2/b/h-1/6*(-c*h+d*g)*q*r*(h*x+g)^2/d/h-1/9*p*r*(h*x+g)^3/h-1/9*q*r*(
h*x+g)^3/h-1/3*(-a*h+b*g)^3*p*r*ln(b*x+a)/b^3/h-1/3*(-c*h+d*g)^3*q*r*ln(d*x
+c)/d^3/h+1/3*(h*x+g)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/h
```

Rubi [A]
time = 0.07, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2581, 45}

$$-\frac{pr(bg-ah)^3\log(a+bx)}{3b^3h}-\frac{prx(bg-ah)^2}{3b^2}+\frac{(g+hx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h}-\frac{pr(g+hx)^2(bg-ah)}{6bh}-\frac{qr(dg-ch)^3\log(c+dx)}{3d^3h}-\frac{qrx(dg-ch)^2}{3d^2}-\frac{qr(g+hx)^2(dg-ch)}{6dh}-\frac{pr(g+hx)^3}{9h}-\frac{qr(g+hx)^3}{9h}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -1/3*((b*g - a*h)^2*p*r*x)/b^2 - ((d*g - c*h)^2*q*r*x)/(3*d^2) - ((b*g - a*
h)*p*r*(g + h*x)^2)/(6*b*h) - ((d*g - c*h)*q*r*(g + h*x)^2)/(6*d*h) - (p*r*
(g + h*x)^3)/(9*h) - (q*r*(g + h*x)^3)/(9*h) - ((b*g - a*h)^3*p*r*Log[a + b
*x])/(3*b^3*h) - ((d*g - c*h)^3*q*r*Log[c + d*x])/(3*d^3*h) + ((g + h*x)^3*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*h)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\int (g+hx)^2 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right) \, dx = \frac{(g+hx)^3 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h} - \frac{(bpr) \int \frac{(g+hx)^3}{a+bx} \, dx}{3h}$$

$$= \frac{(g+hx)^3 \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h} - \frac{(bpr) \int \left(\frac{h(bg-ah)}{b^3}\right.}{}$$

$$= -\frac{(bg-ah)^2 prx}{3b^2} - \frac{(dg-ch)^2 qrx}{3d^2} - \frac{(bg-ah)pr(g+hx)^2}{6bh}$$

Mathematica [A]
time = 0.16, size = 209, normalized size = 0.96

$$\frac{-\frac{r\left(6d^3(bg-ah)^3p\log(a+bx)+b\left(6a^2d^3h^3px-3abd^3hp\left(g^2+6ghx+h^2x^2\right)+b^2d\left(6c^2h^3qx-3cdhq\left(g^2+6ghx+h^2x^2\right)+d^2(p+q)\left(5g^3+18g^2hx+9gh^2x^2+2h^3x^3\right)\right)+6b^2(dg-ch)^3q\log(c+dx)\right)\right)}{6b^3d^3}+(g+hx)^3\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] (-1/6*(r*(6*d^3*(b*g - a*h)^3*p*Log[a + b*x] + b*(6*a^2*d^3*h^3*p*x - 3*a*b
*d^3*h*p*(g^2 + 6*g*h*x + h^2*x^2) + b^2*d*(6*c^2*h^3*q*x - 3*c*d*h*q*(g^2
+ 6*g*h*x + h^2*x^2) + d^2*(p + q)*(5*g^3 + 18*g^2*h*x + 9*g*h^2*x^2 + 2*h^
3*x^3)) + 6*b^2*(d*g - c*h)^3*q*Log[c + d*x])))/(b^3*d^3) + (g + h*x)^3*Log
[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*h)
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \left(hx+g\right)^{2} \ln \left(e\left(f\left(bx+a\right)^{p}\left(dx+c\right)^{q}\right)^{r}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

```
[Out] int((h*x+g)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

Maxima [A]
time = 0.29, size = 270, normalized size = 1.24

$$\frac{1}{3}\left(h^{2}x^{3}+3\,ghx^{2}+3\,g^{2}x\right)\log\left(\left(\left(bx+a\right)^{p}\left(dx+c\right)^{q}f\right)^{r}e\right)+\frac{r\left(\frac{6\left(3\,ab^{2}fg^{2}p-3\,a^{2}bfghp+a^{3}fh^{2}p\right)\log\left(bx+a\right)}{b^{3}}+\frac{6\left(3\,cd^{2}fg^{2}q-3\,c^{2}dfghq+c^{3}fh^{2}q\right)\log\left(dx+c\right)}{d^{3}}-\frac{2\,b^{2}d^{2}fh^{2}(p+q)x^{3}-3\left(abd^{2}fh^{2}p-\left(3\,d^{2}fgh(p+q)-cdfh^{2}q\right)b^{2}\right)x^{2}-6\left(3\,abd^{2}fghp-a^{2}d^{2}fh^{2}p-\left(3\,d^{2}fg^{2}(p+q)-3\,cdfghq+c^{2}fh^{2}q\right)b^{2}\right)x}{b^{2}d^{2}}\right)}{18\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] 1/3*(h^2*x^3 + 3*g*h*x^2 + 3*g^2*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e) +
1/18*r*(6*(3*a*b^2*f*g^2*p - 3*a^2*b*f*g*h*p + a^3*f*h^2*p)*log(b*x + a)/b^
```

```
3 + 6*(3*c*d^2*f*g^2*q - 3*c^2*d*f*g*h*q + c^3*f*h^2*q)*log(d*x + c)/d^3 -
(2*b^2*d^2*f*h^2*(p + q)*x^3 - 3*(a*b*d^2*f*h^2*p - (3*d^2*f*g*h*(p + q) -
c*d*f*h^2*q)*b^2)*x^2 - 6*(3*a*b*d^2*f*g*h*p - a^2*d^2*f*h^2*p - (3*d^2*f*g
^2*(p + q) - 3*c*d*f*g*h*q + c^2*f*h^2*q)*b^2)*x)/(b^2*d^2))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(201) = 402$.
time = 0.37, size = 439, normalized size = 2.01

$$\frac{2\,(3\,b^3d^3h^2 - (b^3d^3h^2p + b^3d^3h^2q)r)x^3 + 3\,(6\,b^3d^3gh - ((3\,b^3d^3gh - ab^2d^3h^2)p + (3\,b^3d^3gh - b^3cd^2h^2)q)r)x^2 + 6\,(3\,b^3d^3g^2 - ((3\,b^3d^3g^2 - 3\,ab^2d^3gh + a^2bd^3h^2)p + (3\,b^3d^3g^2 - 3\,b^3cd^2gh + b^3c^2dh^2)q)r)x + 6\,(b^3d^3h^2prx^3 + 3\,b^3d^3ghprx^2 + 3\,b^3d^3g^2prx + (3\,ab^2d^3g^2 - 3\,a^2bd^3gh + a^3d^3h^2)pr)\log(bx+a) + 6\,(b^3d^3h^2qrx^3 + 3\,b^3d^3ghqrx^2 + 3\,b^3d^3g^2qrx + (3\,b^3cd^2g^2 - 3\,b^3c^2dgh + b^3c^3h^2)qr)\log(dx+c) + 6\,(b^3d^3h^2rx^3 + 3\,b^3d^3ghrx^2 + 3\,b^3d^3g^2rx)\log(f)}{18\,b^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] 1/18*(2*(3*b^3*d^3*h^2 - (b^3*d^3*h^2*p + b^3*d^3*h^2*q)*r)*x^3 + 3*(6*b^3*
d^3*g*h - ((3*b^3*d^3*g*h - a*b^2*d^3*h^2)*p + (3*b^3*d^3*g*h - b^3*c*d^2*h
^2)*q)*r)*x^2 + 6*(3*b^3*d^3*g^2 - ((3*b^3*d^3*g^2 - 3*a*b^2*d^3*g*h + a^2*
b*d^3*h^2)*p + (3*b^3*d^3*g^2 - 3*b^3*c*d^2*g*h + b^3*c^2*d*h^2)*q)*r)*x +
6*(b^3*d^3*h^2*p*r*x^3 + 3*b^3*d^3*g*h*p*r*x^2 + 3*b^3*d^3*g^2*p*r*x + (3*a
*b^2*d^3*g^2 - 3*a^2*b*d^3*g*h + a^3*d^3*h^2)*p*r)*log(b*x + a) + 6*(b^3*d^
3*h^2*q*r*x^3 + 3*b^3*d^3*g*h*q*r*x^2 + 3*b^3*d^3*g^2*q*r*x + (3*b^3*c*d^2*
g^2 - 3*b^3*c^2*d*g*h + b^3*c^3*h^2)*q*r)*log(d*x + c) + 6*(b^3*d^3*h^2*r*x
^3 + 3*b^3*d^3*g*h*r*x^2 + 3*b^3*d^3*g^2*r*x)*log(f))/(b^3*d^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 0.65, size = 328, normalized size = 1.50

$$x\left(\frac{\left(\frac{h\,r\,(b\,c\,h\,p+3\,b\,d\,g\,p+a\,d\,h\,q+3\,b\,d\,g\,q)}{3\,b\,d}-\frac{h^2\,r\,(p+q)\,(3\,a\,d+3\,b\,c)}{9\,b\,d}\right)\,(3\,a\,d+3\,b\,c)}{3\,b\,d}-\frac{g\,r\,(b\,c\,h\,p+b\,d\,g\,p+a\,d\,h\,q+b\,d\,g\,q)}{b\,d}+\frac{a\,c\,h^2\,r\,(p+q)}{3\,b\,d}\right)-x^2\left(\frac{h\,r\,(b\,c\,h\,p+3\,b\,d\,g\,p+a\,d\,h\,q+3\,b\,d\,g\,q)}{6\,b\,d}-\frac{h^2\,r\,(p+q)\,(3\,a\,d+3\,b\,c)}{18\,b\,d}\right)+\ln\left(e\,{\left(f\,{(a+b\,x)}^p\,{(c+d\,x)}^q\right)}^r\right)\,\left(g^2\,x+g\,h\,x^2+\frac{h^2\,x^3}{3}\right)+\frac{\ln(a+b\,x)\,(p\,r\,a^3\,h^2-3\,p\,r\,a^2\,b\,g\,h+3\,p\,r\,a\,b^2\,g^2)}{3\,b^3}+\frac{\ln(c+d\,x)\,(q\,r\,c^3\,h^2-3\,q\,r\,c^2\,d\,g\,h+3\,q\,r\,c\,d^2\,g^2)}{3\,d^3}-\frac{h^2\,r\,x^3\,(p+q)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(g + h*x)^2,x)
```

```
[Out] x*((((h*r*(b*c*h*p + 3*b*d*g*p + a*d*h*q + 3*b*d*g*q))/(3*b*d) - (h^2*r*(p
+ q)*(3*a*d + 3*b*c))/(9*b*d))*(3*a*d + 3*b*c))/(3*b*d) - (g*r*(b*c*h*p + b
*d*g*p + a*d*h*q + b*d*g*q))/(b*d) + (a*c*h^2*r*(p + q))/(3*b*d)) - x^2*((h
*r*(b*c*h*p + 3*b*d*g*p + a*d*h*q + 3*b*d*g*q))/(6*b*d) - (h^2*r*(p + q)*(3
*a*d + 3*b*c))/(18*b*d)) + log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(g^2*x + (h
^2*x^3)/3 + g*h*x^2) + (log(a + b*x)*(a^3*h^2*p*r + 3*a*b^2*g^2*p*r - 3*a^2
*b*g*h*p*r))/(3*b^3) + (log(c + d*x)*(c^3*h^2*q*r + 3*c*d^2*g^2*q*r - 3*c^2
*d*g*h*q*r))/(3*d^3) - (h^2*r*x^3*(p + q))/9
```

3.28 $\int (g+hx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$

Optimal. Leaf size=160

$$-\frac{(bg-ah)prx}{2b}-\frac{(dg-ch)qrx}{2d}-\frac{pr(g+hx)^2}{4h}-\frac{qr(g+hx)^2}{4h}-\frac{(bg-ah)^2pr\log(a+bx)}{2b^2h}-\frac{(dg-ch)^2qr\log(c+$$

```
[Out] -1/2*(-a*h+b*g)*p*r*x/b-1/2*(-c*h+d*g)*q*r*x/d-1/4*p*r*(h*x+g)^2/h-1/4*q*r*
(h*x+g)^2/h-1/2*(-a*h+b*g)^2*p*r*ln(b*x+a)/b^2/h-1/2*(-c*h+d*g)^2*q*r*ln(d*
x+c)/d^2/h+1/2*(h*x+g)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/h
```

Rubi [A]
time = 0.05, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2581, 45}

$$-\frac{pr(bg-ah)^2\log(a+bx)}{2b^2h}+\frac{(g+hx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h}-\frac{prx(bg-ah)}{2b}-\frac{qr(dg-ch)^2\log(c+dx)}{2d^2h}-\frac{qrx(dg-ch)}{2d}-\frac{pr(g+hx)^2}{4h}-\frac{qr(g+hx)^2}{4h}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -1/2*((b*g - a*h)*p*r*x)/b - ((d*g - c*h)*q*r*x)/(2*d) - (p*r*(g + h*x)^2)/
(4*h) - (q*r*(g + h*x)^2)/(4*h) - ((b*g - a*h)^2*p*r*Log[a + b*x])/(2*b^2*h
) - ((d*g - c*h)^2*q*r*Log[c + d*x])/(2*d^2*h) + ((g + h*x)^2*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/(2*h)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\int (g+hx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \frac{(g+hx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h} - \frac{(bpr)\int \frac{(g+hx)^2}{a+bx}\,dx}{2h}$$

$$= \frac{(g+hx)^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h} - \frac{(bpr)\int \left(\frac{h(bg-ah)}{b^2}\right.}{}$$

$$= -\frac{(bg-ah)prx}{2b} - \frac{(dg-ch)qrx}{2d} - \frac{pr(g+hx)^2}{4h} - \frac{qr(g+hx}{4h}$$

Mathematica [A]
time = 0.13, size = 120, normalized size = 0.75

$$-\frac{2ad^2(-2bg+ah)pr\log(a+bx)+b(2bc(-2dg+ch)qr\log(c+dx)+dx(r(-2adhp-2bchq+bd(p+q)(4g+hx))-2bd(2g+hx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)))}{4b^2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -1/4*(2*a*d^2*(-2*b*g + a*h)*p*r*Log[a + b*x] + b*(2*b*c*(-2*d*g + c*h)*q*r
*Log[c + d*x] + d*x*(r*(-2*a*d*h*p - 2*b*c*h*q + b*d*(p + q)*(4*g + h*x)) -
 2*b*d*(2*g + h*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])))/(b^2*d^2)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (hx+g)\ln\left(e\left(f(bx+a)^p\,(dx+c)^q\right)^r\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

```
[Out] int((h*x+g)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

Maxima [A]
time = 0.28, size = 144, normalized size = 0.90

$$\frac{1}{2}\left(hx^2+2\,gx\right)\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)+\frac{r\left(\frac{2\,(2\,abfgp-a^2fhp)\log(bx+a)}{b^2}+\frac{2\,(2\,cdfgq-c^2fhq)\log(dx+c)}{d^2}-\frac{bdfh(p+q)x^2-2\,(adfhp-(2\,dfg(p+q)-cfhq)b)x}{bd}\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] 1/2*(h*x^2 + 2*g*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e) + 1/4*r*(2*(2*a*b*
f*g*p - a^2*f*h*p)*log(b*x + a)/b^2 + 2*(2*c*d*f*g*q - c^2*f*h*q)*log(d*x +
 c)/d^2 - (b*d*f*h*(p + q)*x^2 - 2*(a*d*f*h*p - (2*d*f*g*(p + q) - c*f*h*q)
*b)*x)/(b*d))/f
```

Fricas [A]
time = 0.40, size = 240, normalized size = 1.50

$$\frac{(2\,b^2d^2h-(b^2d^2hp+b^2d^2hq)r)x^2+2\,(2\,b^2d^2g-((2\,b^2d^2g-abd^2h)p+(2\,b^2d^2g-b^2cdh)q)r)x+2\,(b^2d^2hprx^2+2\,b^2d^2gprx+(2\,abd^2g-a^2d^2h)pr)\log(bx+a)+2\,(b^2d^2hqrx^2+2\,b^2d^2gqrx+(2\,b^2cdg-b^2c^2h)qr)\log(dx+c)+2\,(b^2d^2hrx^2+2\,b^2d^2grx)\log(f)}{4\,b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] 1/4*((2*b^2*d^2*h - (b^2*d^2*h*p + b^2*d^2*h*q)*r)*x^2 + 2*(2*b^2*d^2*g - (
(2*b^2*d^2*g - a*b*d^2*h)*p + (2*b^2*d^2*g - b^2*c*d*h)*q)*r)*x + 2*(b^2*d^
2*h*p*r*x^2 + 2*b^2*d^2*g*p*r*x + (2*a*b*d^2*g - a^2*d^2*h)*p*r)*log(b*x +
a) + 2*(b^2*d^2*h*q*r*x^2 + 2*b^2*d^2*g*q*r*x + (2*b^2*c*d*g - b^2*c^2*h)*q
*r)*log(d*x + c) + 2*(b^2*d^2*h*r*x^2 + 2*b^2*d^2*g*r*x)*log(f))/(b^2*d^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(141) = 282$.
time = 34.19, size = 502, normalized size = 3.14

$$\begin{cases} \left(gx+\frac{hx^2}{2}\right)\log\left(e(a^pc^qf)^r\right) & \text{for}\ b=0\wedge d=0 \\ -\frac{c^2h\log\left(e(a^pf(c+dx)^q)^r\right)}{2d^2}+\frac{cg\log\left(e(a^pf(c+dx)^q)^r\right)}{d}+\frac{chqrx}{2d}-gqrx+gx\log\left(e(a^pf(c+dx)^q)^r\right)-\frac{hqrx^2}{4}+\frac{hx^2\log\left(e(a^pf(c+dx)^q)^r\right)}{2} & \text{for}\ b=0 \\ -\frac{a^2h\log\left(e(c^qf(a+bx)^p)^r\right)}{2b^2}+\frac{ag\log\left(e(c^qf(a+bx)^p)^r\right)}{b}+\frac{ahprx}{2b}-gprx+gx\log\left(e(c^qf(a+bx)^p)^r\right)-\frac{hprx^2}{4}+\frac{hx^2\log\left(e(c^qf(a+bx)^p)^r\right)}{2} & \text{for}\ d=0 \\ \frac{a^2hqr\log\left(\frac{c}{d}+x\right)}{2b^2}-\frac{a^2h\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2b^2}-\frac{agqr\log\left(\frac{c}{d}+x\right)}{b}+\frac{ag\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}+\frac{ahprx}{2b}-\frac{c^2hqr\log\left(\frac{c}{d}+x\right)}{2d^2}+\frac{cgqr\log\left(\frac{c}{d}+x\right)}{d}+\frac{chqrx}{2d}-gprx-gqrx+gx\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)-\frac{hprx^2}{4}-\frac{hqrx^2}{4}+\frac{hx^2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Piecewise(((g*x + h*x**2/2)*log(e*(a**p*c**q*f)**r), Eq(b, 0) & Eq(d, 0)),
(-c**2*h*log(e*(a**p*f*(c + d*x)**q)**r)/(2*d**2) + c*g*log(e*(a**p*f*(c +
d*x)**q)**r)/d + c*h*q*r*x/(2*d) - g*q*r*x + g*x*log(e*(a**p*f*(c + d*x)**q
)**r) - h*q*r*x**2/4 + h*x**2*log(e*(a**p*f*(c + d*x)**q)**r)/2, Eq(b, 0)),
 (-a**2*h*log(e*(c**q*f*(a + b*x)**p)**r)/(2*b**2) + a*g*log(e*(c**q*f*(a +
 b*x)**p)**r)/b + a*h*p*r*x/(2*b) - g*p*r*x + g*x*log(e*(c**q*f*(a + b*x)**
p)**r) - h*p*r*x**2/4 + h*x**2*log(e*(c**q*f*(a + b*x)**p)**r)/2, Eq(d, 0))
, (a**2*h*q*r*log(c/d + x)/(2*b**2) - a**2*h*log(e*(f*(a + b*x)**p*(c + d*x
)**q)**r)/(2*b**2) - a*g*q*r*log(c/d + x)/b + a*g*log(e*(f*(a + b*x)**p*(c
+ d*x)**q)**r)/b + a*h*p*r*x/(2*b) - c**2*h*q*r*log(c/d + x)/(2*d**2) + c*g
*q*r*log(c/d + x)/d + c*h*q*r*x/(2*d) - g*p*r*x - g*q*r*x + g*x*log(e*(f*(a
 + b*x)**p*(c + d*x)**q)**r) - h*p*r*x**2/4 - h*q*r*x**2/4 + h*x**2*log(e*(
f*(a + b*x)**p*(c + d*x)**q)**r)/2, True))
```

Giac [A]
time = 17.81, size = 176, normalized size = 1.10

$$-\frac{1}{4}\,(hpr+hqr-2\,hr\log(f)-2\,h)x^2+\frac{1}{2}\,(hprx^2+2\,gprx)\log(bx+a)+\frac{1}{2}\,(hqrx^2+2\,gqrx)\log(dx+c)-\frac{(2\,bdgpr-adhpr+2\,bdgqr-bchqr-2\,bdgr\log(f)-2\,bdg)x}{2\,bd}+\frac{(2\,abgpr-a^2hpr)\log(-bx-a)}{2\,b^2}+\frac{(2\,cdgqr-c^2hqr)\log(dx+c)}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] -1/4*(h*p*r + h*q*r - 2*h*r*log(f) - 2*h)*x^2 + 1/2*(h*p*r*x^2 + 2*g*p*r*x)
*log(b*x + a) + 1/2*(h*q*r*x^2 + 2*g*q*r*x)*log(d*x + c) - 1/2*(2*b*d*g*p*r
 - a*d*h*p*r + 2*b*d*g*q*r - b*c*h*q*r - 2*b*d*g*r*log(f) - 2*b*d*g)*x/(b*d
) + 1/2*(2*a*b*g*p*r - a^2*h*p*r)*log(-b*x - a)/b^2 + 1/2*(2*c*d*g*q*r - c^
2*h*q*r)*log(d*x + c)/d^2
```

Mupad [B]
time = 0.51, size = 153, normalized size = 0.96

$$\ln\left(e\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)^r\right)\,\left(\frac{h\,x^2}{2}+g\,x\right)-x\,\left(\frac{r\,\left(b\,c\,h\,p+2\,b\,d\,g\,p+a\,d\,h\,q+2\,b\,d\,g\,q\right)}{2\,b\,d}-\frac{h\,r\,\left(p+q\right)\,\left(2\,a\,d+2\,b\,c\right)}{4\,b\,d}\right)-\frac{\ln\left(a+b\,x\right)\,\left(a^2\,h\,p\,r-2\,a\,b\,g\,p\,r\right)}{2\,b^2}-\frac{\ln\left(c+d\,x\right)\,\left(c^2\,h\,q\,r-2\,c\,d\,g\,q\,r\right)}{2\,d^2}-\frac{h\,r\,x^2\,\left(p+q\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(g + h*x),x)
```

```
[Out] log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(g*x + (h*x^2)/2) - x*((r*(b*c*h*p + 2
*b*d*g*p + a*d*h*q + 2*b*d*g*q))/(2*b*d) - (h*r*(p + q)*(2*a*d + 2*b*c))/(4
*b*d)) - (log(a + b*x)*(a^2*h*p*r - 2*a*b*g*p*r))/(2*b^2) - (log(c + d*x)*(
c^2*h*q*r - 2*c*d*g*q*r))/(2*d^2) - (h*r*x^2*(p + q))/4
```

3.29 $\int \log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$

Optimal. Leaf size=61

$$-((p+q)rx)+\frac{(bc-ad)qr\log(c+dx)}{bd}+\frac{(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}$$

```
[Out] -(p+q)*r*x+(-a*d+b*c)*q*r*ln(d*x+c)/b/d+(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q
)^r)/b
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2579, 31, 8}

$$\frac{(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}+\frac{qr(bc-ad)\log(c+dx)}{bd}-(rx(p+q))$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -((p + q)*r*x) + ((b*c - a*d)*q*r*Log[c + d*x])/(b*d) + ((a + b*x)*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2579

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[(a + b*x)*(Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s/b), x] + (Dist[q*r*s*((b*c - a*d)/b), Int[Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]^(s - 1)/(c + d*x), x], x] - Dist[r*s*(p + q), Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^(s - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p, q,
r, s}, x] && NeQ[b*c - a*d, 0] && NeQ[p + q, 0] && IGtQ[s, 0] && LtQ[s, 4]
```

Rubi steps

$$\begin{aligned}\int \log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx &= \frac{(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}+\frac{((bc-ad)qr)\int\frac{1}{c+dx}\,dx}{b}-((p\\ &= -(p+q)rx+\frac{(bc-ad)qr\log(c+dx)}{bd}+\frac{(a+bx)\log\left(e(f(a+bx)^p(c+\end{aligned}$$

Mathematica [A]
time = 0.04, size = 57, normalized size = 0.93

$$\frac{apr\log(a+bx)}{b}+\frac{cqr\log(c+dx)}{d}+x(-((p+q)r)+\log\left(e(f(a+bx)^p(c+dx)^q)^r\right))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] (a*p*r*Log[a + b*x])/b + (c*q*r*Log[c + d*x])/d + x*(-((p + q)*r) + Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r])
```

Maple [A]
time = 0.08, size = 61, normalized size = 1.00

method	result	size
default	$\ln\left(e\left(f(bx+a)^p\,(dx+c)^q\right)^r\right)x-r\left(px+qx-\frac{ap\ln(bx+a)}{b}-\frac{cq\ln(dx+c)}{d}\right)$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x,method=_RETURNVERBOSE)
```

```
[Out] ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*x-r*(p*x+q*x-a*p/b*ln(b*x+a)-c*q/d*ln(d*x+c
))
```

Maxima [A]
time = 0.29, size = 76, normalized size = 1.25

$$x\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)-\frac{\left(bfp\left(\frac{x}{b}-\frac{a\log(bx+a)}{b^2}\right)+dfq\left(\frac{x}{d}-\frac{c\log(dx+c)}{d^2}\right)\right)r}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] x*log(((b*x + a)^p*(d*x + c)^q*f)^r*e) - (b*f*p*(x/b - a*log(b*x + a)/b^2)
+ d*f*q*(x/d - c*log(d*x + c)/d^2))*r/f
```

Fricas [A]
time = 0.47, size = 71, normalized size = 1.16

$$\frac{bdrx\log(f)+(bd-(bdp+bdq)r)x+(bdprx+adpr)\log(bx+a)+(bdqrx+bcqr)\log(dx+c)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] (b*d*r*x*log(f) + (b*d - (b*d*p + b*d*q)*r)*x + (b*d*p*r*x + a*d*p*r)*log(b
*x + a) + (b*d*q*r*x + b*c*q*r)*log(d*x + c))/(b*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(54) = 108$.
time = 6.67, size = 184, normalized size = 3.02

$$\begin{cases} x\log\left(e(a^pc^qf)^r\right) & \text{for } b=0 \wedge d=0 \\ \frac{a\log\left(e(c^qf(a+bx)^p)^r\right)}{b} - prx + x\log\left(e(c^qf(a+bx)^p)^r\right) & \text{for } d=0 \\ \frac{c\log\left(e(a^pf(c+dx)^q)^r\right)}{d} - qrx + x\log\left(e(a^pf(c+dx)^q)^r\right) & \text{for } b=0 \\ -\frac{aqr\log(c+dx)}{b} + \frac{a\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b} + \frac{cqr\log(c+dx)}{d} - prx - qrx + x\log\left(e(f(a+bx)^p(c+dx)^q)^r\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Piecewise((x*log(e*(a**p*c**q*f)**r), Eq(b, 0) & Eq(d, 0)), (a*log(e*(c**q*
f*(a + b*x)**p)**r)/b - p*r*x + x*log(e*(c**q*f*(a + b*x)**p)**r), Eq(d, 0)
), (c*log(e*(a**p*f*(c + d*x)**q)**r)/d - q*r*x + x*log(e*(a**p*f*(c + d*x)
**q)**r), Eq(b, 0)), (-a*q*r*log(c + d*x)/b + a*log(e*(f*(a + b*x)**p*(c +
d*x)**q)**r)/b + c*q*r*log(c + d*x)/d - p*r*x - q*r*x + x*log(e*(f*(a + b*x
)**p*(c + d*x)**q)**r), True))
```

Giac [A]
time = 4.23, size = 66, normalized size = 1.08

$$prx\log(bx+a) + qrx\log(dx+c) + \frac{apr\log(bx+a)}{b} + \frac{cqr\log(-dx-c)}{d} - (pr+qr-r\log(f)-1)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] p*r*x*log(b*x + a) + q*r*x*log(d*x + c) + a*p*r*log(b*x + a)/b + c*q*r*log(
-d*x - c)/d - (p*r + q*r - r*log(f) - 1)*x
```

Mupad [B]
time = 0.22, size = 60, normalized size = 0.98

$$x\ln\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) - prx - qrx + \frac{apr\ln(a+bx)}{b} + \frac{cqr\ln(c+dx)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r),x)
```

```
[Out] x*log(e*(f*(a + b*x)^p*(c + d*x)^q)^r) - p*r*x - q*r*x + (a*p*r*log(a + b*x
))/b + (c*q*r*log(c + d*x))/d
```

3.30 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{g+hx} dx$

Optimal. Leaf size=148

$$-\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{h}-\frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(g+hx)}{h}+\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\log(g+hx)}{h}$$

```
[Out] -p*r*ln(-h*(b*x+a)/(-a*h+b*g))*ln(h*x+g)/h-q*r*ln(-h*(d*x+c)/(-c*h+d*g))*ln
(h*x+g)/h+ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*ln(h*x+g)/h-p*r*polylog(2,b*(h*x+
g)/(-a*h+b*g))/h-q*r*polylog(2,d*(h*x+g)/(-c*h+d*g))/h
```

Rubi [A]
time = 0.10, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2580, 2441, 2440, 2438}

$$-\frac{pr\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)}{h}-\frac{qr\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)}{h}+\frac{\log(g+hx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h}-\frac{pr\log(g+hx)\log\left(-\frac{h(a+bx)}{bg-ah}\right)}{h}-\frac{qr\log(g+hx)\log\left(-\frac{h(c+dx)}{dg-ch}\right)}{h}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x),x]
```

```
[Out] -((p*r*Log[-((h*(a + b*x))/(b*g - a*h))]*Log[g + h*x])/h) - (q*r*Log[-((h*(
c + d*x))/(d*g - c*h))]*Log[g + h*x])/h + (Log[e*(f*(a + b*x)^p*(c + d*x)^q
)^r]*Log[g + h*x])/h - (p*r*PolyLog[2, (b*(g + h*x))/(b*g - a*h)])/h - (q*r
*PolyLog[2, (d*(g + h*x))/(d*g - c*h)])/h
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2580

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/h), x] + (-Dist[b*p*(r/h), Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[d*q*(r/h), Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{g+hx}\,dx = \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\log(g+hx)}{h} - \frac{(bpr)\int \frac{\log(g+hx)}{a+bx}\,dx}{h} - \frac{(dq$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{h} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(g+hx)}{h} + \frac{\log$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{h} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(g+hx)}{h} + \frac{\log$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{h} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(g+hx)}{h} + \frac{\log$$

Mathematica [A]
time = 0.06, size = 163, normalized size = 1.10

$$\frac{-pr\log(a+bx)\log(g+hx) - qr\log(c+dx)\log(g+hx) + \log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\log(g+hx) + pr\log(a+bx)\log\left(\frac{b(g+hx)}{bg-ah}\right) + qr\log(c+dx)\log\left(\frac{d(g+hx)}{dg-ch}\right) + pr\mathrm{Li}_2\left(\frac{h(a+bx)}{-bg+ah}\right) + qr\mathrm{Li}_2\left(\frac{h(c+dx)}{-dg+ch}\right)}{h}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x),x]
```

```
[Out] (-(p*r*Log[a + b*x]*Log[g + h*x]) - q*r*Log[c + d*x]*Log[g + h*x] + Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g + h*x] + p*r*Log[a + b*x]*Log[(b*(g + h
*x))/(b*g - a*h)] + q*r*Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)] + p*r*P
olyLog[2, (h*(a + b*x))/(-(b*g) + a*h)] + q*r*PolyLog[2, (h*(c + d*x))/(-(d
*g) + c*h)])/h
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\left(bx+a\right)^{p}\left(dx+c\right)^{q}\right)^{r}\right)}{hx+g}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g),x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g),x)
```

Maxima [A]
time = 0.28, size = 187, normalized size = 1.26

$$\frac{\left(\frac{\left(\log(bx+a)\log\left(\frac{bhx+ah}{bg-ah}+1\right)+\mathrm{Li}_2\left(-\frac{bhx+ah}{bg-ah}\right)\right)fp}{h}+\frac{\left(\log(dx+c)\log\left(\frac{dhx+ch}{dg-ch}+1\right)+\mathrm{Li}_2\left(-\frac{dhx+ch}{dg-ch}\right)\right)fq}{h}\right)r}{f}-\frac{(fp\log(bx+a)+fq\log(dx+c))r\log(hx+g)}{fh}+\frac{\log(hx+g)\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)}{h}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g),x, algorithm="maxima")
```

```
[Out] ((log(b*x + a)*log((b*h*x + a*h)/(b*g - a*h) + 1) + dilog(-(b*h*x + a*h)/(b
*g - a*h)))*f*p/h + (log(d*x + c)*log((d*h*x + c*h)/(d*g - c*h) + 1) + dilo
g(-(d*h*x + c*h)/(d*g - c*h)))*f*q/h)*r/f - (f*p*log(b*x + a) + f*q*log(d*x
 + c))*r*log(h*x + g)/(f*h) + log(h*x + g)*log(((b*x + a)^p*(d*x + c)^q*f)^
r*e)/h
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g),x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*x + g), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*x+g),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g),x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*x + g), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln \left(e \left(f \left(a+b x\right)^{p} \left(c+d x\right)^{q}\right)^{r}\right)}{g+h x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(g + h*x),x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(g + h*x), x)
```

3.31 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(g+hx)^2} dx$

Optimal. Leaf size=128

$$\frac{bpr\log(a+bx)}{h(bg-ah)}+\frac{dqr\log(c+dx)}{h(dg-ch)}-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h(g+hx)}-\frac{bpr\log(g+hx)}{h(bg-ah)}-\frac{dqr\log(g+hx)}{h(dg-ch)}$$

```
[Out] b*p*r*ln(b*x+a)/h/(-a*h+b*g)+d*q*r*ln(d*x+c)/h/(-c*h+d*g)-ln(e*(f*(b*x+a)^p
*(d*x+c)^q)^r)/h/(h*x+g)-b*p*r*ln(h*x+g)/h/(-a*h+b*g)-d*q*r*ln(h*x+g)/h/(-c
*h+d*g)
```

Rubi [A]
time = 0.04, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2581, 36, 31}

$$-\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h(g+hx)}+\frac{bpr\log(a+bx)}{h(bg-ah)}-\frac{bpr\log(g+hx)}{h(bg-ah)}+\frac{dqr\log(c+dx)}{h(dg-ch)}-\frac{dqr\log(g+hx)}{h(dg-ch)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^2,x]
```

```
[Out] (b*p*r*Log[a + b*x])/(h*(b*g - a*h)) + (d*q*r*Log[c + d*x])/(h*(d*g - c*h))
 - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(g + h*x)) - (b*p*r*Log[g + h*x]
)/(h*(b*g - a*h)) - (d*q*r*Log[g + h*x])/(h*(d*g - c*h))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^2}\,dx = -\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h(g+hx)} + \frac{(bpr)\int \frac{1}{(a+bx)(g+hx)}\,dx}{h} + \frac{(dqr)\int \frac{1}{(c+}}{h}$$

$$= -\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h(g+hx)} - \frac{(bpr)\int \frac{1}{g+hx}\,dx}{bg-ah} + \frac{(b^2pr)\int \frac{1}{a+bx}\,dx}{h(bg-ah)}$$

$$= \frac{bpr\log(a+bx)}{h(bg-ah)} + \frac{dqr\log(c+dx)}{h(dg-ch)} - \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h(g+hx)} -$$

Mathematica [A]
time = 0.11, size = 93, normalized size = 0.73

$$\frac{-\frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{g+hx} + \frac{bpr(\log(a+bx)-\log(g+hx))}{bg-ah} + \frac{dqr(\log(c+dx)-\log(g+hx))}{dg-ch}}{h}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^2,x]
[Out] (-(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)) + (b*p*r*(Log[a + b*x] -
 Log[g + h*x]))/(b*g - a*h) + (d*q*r*(Log[c + d*x] - Log[g + h*x]))/(d*g -
c*h))/h
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\left(bx+a\right)^{p}\left(dx+c\right)^{q}\right)^{r}\right)}{\left(hx+g\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^2,x)
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^2,x)
```

Maxima [A]
time = 0.30, size = 124, normalized size = 0.97

$$\frac{\left(bfp\left(\frac{\log(bx+a)}{bg-ah} - \frac{\log(hx+g)}{bg-ah}\right) + dfq\left(\frac{\log(dx+c)}{dg-ch} - \frac{\log(hx+g)}{dg-ch}\right)\right)r}{fh} - \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{(hx+g)h}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^2,x, algorithm="maxima")
```

```
[Out] (b*f*p*(log(b*x + a)/(b*g - a*h) - log(h*x + g)/(b*g - a*h)) + d*f*q*(log(d
*x + c)/(d*g - c*h) - log(h*x + g)/(d*g - c*h)))*r/(f*h) - log(((b*x + a)^p
*(d*x + c)^q*f)^r*e)/((h*x + g)*h)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(129) = 258$.
time = 66.48, size = 276, normalized size = 2.16

$$-\frac{bdg^2 + ach^2 - (bc + ad)gh + (bdg^2 + ach^2 - (bc + ad)gh)r\log(f) - ((bdgh - bch^2)prx + (adgh - ach^2)pr)\log(bx + a) - ((bdgh - adh^2)qrx + (bcgh - ach^2)qr)\log(dx + c) + (((bdgh - bch^2)p + (bdgh - adh^2)q)rx + ((bdg^2 - bcgh)p + (bdg^2 - adgh)q)r)\log(hx + g)}{bdg^3h + acgh^3 - (bc + ad)g^2h^2 + (bdg^2h^2 + ach^4 - (bc + ad)gh^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^2,x, algorithm="fricas")
```

```
[Out] -(b*d*g^2 + a*c*h^2 - (b*c + a*d)*g*h + (b*d*g^2 + a*c*h^2 - (b*c + a*d)*g*
h)*r*log(f) - ((b*d*g*h - b*c*h^2)*p*r*x + (a*d*g*h - a*c*h^2)*p*r)*log(b*x
 + a) - ((b*d*g*h - a*d*h^2)*q*r*x + (b*c*g*h - a*c*h^2)*q*r)*log(d*x + c)
+ (((b*d*g*h - b*c*h^2)*p + (b*d*g*h - a*d*h^2)*q)*r*x + ((b*d*g^2 - b*c*g*
h)*p + (b*d*g^2 - a*d*g*h)*q)*r)*log(h*x + g))/(b*d*g^3*h + a*c*g*h^3 - (b*
c + a*d)*g^2*h^2 + (b*d*g^2*h^2 + a*c*h^4 - (b*c + a*d)*g*h^3)*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*x+g)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.42, size = 190, normalized size = 1.48

$$\frac{b^2pr\log(|bx + a|)}{b^2gh - abh^2} + \frac{d^2qr\log(|-dx - c|)}{d^2gh - cdh^2} - \frac{pr\log(bx + a)}{h^2x + gh} - \frac{qr\log(dx + c)}{h^2x + gh} - \frac{(bdgpr - bchpr + bdgqr - adhqr)\log(hx + g)}{bdg^2h - bcgh^2 - adgh^2 + ach^3} - \frac{r\log(f) + 1}{h^2x + gh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^2,x, algorithm="giac")
```

```
[Out] b^2*p*r*log(abs(b*x + a))/(b^2*g*h - a*b*h^2) + d^2*q*r*log(abs(-d*x - c))/
(d^2*g*h - c*d*h^2) - p*r*log(b*x + a)/(h^2*x + g*h) - q*r*log(d*x + c)/(h^
2*x + g*h) - (b*d*g*p*r - b*c*h*p*r + b*d*g*q*r - a*d*h*q*r)*log(h*x + g)/(
b*d*g^2*h - b*c*g*h^2 - a*d*g*h^2 + a*c*h^3) - (r*log(f) + 1)/(h^2*x + g*h)
```

Mupad [**B**]
time = 1.20, size = 152, normalized size = 1.19

$$\frac{\ln(g + hx)\,(bchpr - g\,(bdpr + bdqr) + adhqr)}{ach^3 - adgh^2 - bcgh^2 + bdg^2h} - \frac{\ln\left(e\left(f\,(a + bx)^p\,(c + dx)^q\right)^r\right)\,\left(x + \frac{g}{h}\right)}{(g + hx)^2} - \frac{bpr\ln(a + bx)}{ah^2 - bgh} - \frac{dqr\ln(c + dx)}{ch^2 - dgh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(g + h*x)^2,x)
```

```
[Out] (log(g + h*x)*(b*c*h*p*r - g*(b*d*p*r + b*d*q*r) + a*d*h*q*r))/(a*c*h^3 - a
*d*g*h^2 - b*c*g*h^2 + b*d*g^2*h) - (log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(
x + g/h))/(g + h*x)^2 - (b*p*r*log(a + b*x))/(a*h^2 - b*g*h) - (d*q*r*log(c
 + d*x))/(c*h^2 - d*g*h)
```

3.32 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(g+hx)^3}\,dx$

Optimal. Leaf size=202

$$\frac{bpr}{2h(bg-ah)(g+hx)}+\frac{dqr}{2h(dg-ch)(g+hx)}+\frac{b^2pr\log(a+bx)}{2h(bg-ah)^2}+\frac{d^2qr\log(c+dx)}{2h(dg-ch)^2}-\frac{\log(e(f(a+bx)^p(c+dx}{2h(g+hx)^2}$$

```
[Out] 1/2*b*p*r/h/(-a*h+b*g)/(h*x+g)+1/2*d*q*r/h/(-c*h+d*g)/(h*x+g)+1/2*b^2*p*r*l
n(b*x+a)/h/(-a*h+b*g)^2+1/2*d^2*q*r*ln(d*x+c)/h/(-c*h+d*g)^2-1/2*ln(e*(f*(b
*x+a)^p*(d*x+c)^q)^r)/h/(h*x+g)^2-1/2*b^2*p*r*ln(h*x+g)/h/(-a*h+b*g)^2-1/2*
d^2*q*r*ln(h*x+g)/h/(-c*h+d*g)^2
```

Rubi [A]
time = 0.09, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2581, 46}

$$\frac{b^2pr\log(a+bx)}{2h(bg-ah)^2}-\frac{b^2pr\log(g+hx)}{2h(bg-ah)^2}-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h(g+hx)^2}+\frac{bpr}{2h(g+hx)(bg-ah)}+\frac{d^2qr\log(c+dx)}{2h(dg-ch)^2}-\frac{d^2qr\log(g+hx)}{2h(dg-ch)^2}+\frac{dqr}{2h(g+hx)(dg-ch)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^3,x]
```

```
[Out] (b*p*r)/(2*h*(b*g - a*h)*(g + h*x)) + (d*q*r)/(2*h*(d*g - c*h)*(g + h*x)) +
 (b^2*p*r*Log[a + b*x])/(2*h*(b*g - a*h)^2) + (d^2*q*r*Log[c + d*x])/(2*h*(
d*g - c*h)^2) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(2*h*(g + h*x)^2) - (b
^2*p*r*Log[g + h*x])/(2*h*(b*g - a*h)^2) - (d^2*q*r*Log[g + h*x])/(2*h*(d*g
 - c*h)^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^3}\,dx = -\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2} + \frac{(bpr)\int \frac{1}{(a+bx)(g+hx)^2}\,dx}{2h} + \frac{(dqr)\int \frac{}{(c}}{}$$

$$= -\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2} + \frac{(bpr)\int\left(\frac{b^2}{(bg-ah)^2(a+bx)} - \frac{h}{(bg-ah)(g+hx}\right.}{2h}$$

$$= \frac{bpr}{2h(bg-ah)(g+hx)} + \frac{dqr}{2h(dg-ch)(g+hx)} + \frac{b^2pr\log(a+bx)}{2h(bg-ah)^2} + \frac{d^2}{2}$$

Mathematica [A]
time = 0.28, size = 206, normalized size = 1.02

$$\frac{-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right) + \frac{r(g+hx)((bc-ad)(bg-ah)(dg-ch)(bdg(p+q)-h(bcp+adq))-(g+hx)(-b^2(bc-ad)(dg-ch)^2p(\log(a+bx)-\log(g+hx))+d^2(-bc+ad)(bg-ah)^2q(\log(c+dx)-\log(g+hx))))}{(bc-ad)(bg-ah)^2(dg-ch)^2}}{2h(g+hx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^3,x]
```

```
[Out] (-Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (r*(g + h*x)*((b*c - a*d)*(b*g - a
*h)*(d*g - c*h)*(b*d*g*(p + q) - h*(b*c*p + a*d*q)) - (g + h*x)*(-(b^2*(b*c
 - a*d)*(d*g - c*h)^2*p*(Log[a + b*x] - Log[g + h*x])) + d^2*(-(b*c) + a*d)
*(b*g - a*h)^2*q*(Log[c + d*x] - Log[g + h*x]))))/((b*c - a*d)*(b*g - a*h)^
2*(d*g - c*h)^2))/(2*h*(g + h*x)^2)
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\left(bx+a\right)^p\left(dx+c\right)^q\right)^r\right)}{\left(hx+g\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^3,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^3,x)
```

Maxima [A]
time = 0.29, size = 233, normalized size = 1.15

$$\frac{\left(bfp\left(\frac{b\log(bx+a)}{b^2g^2-2\,abgh+a^2h^2} - \frac{b\log(hx+g)}{b^2g^2-2\,abgh+a^2h^2} + \frac{1}{bg^2-agh+(bgh-ah^2)x}\right) + dfq\left(\frac{d\log(dx+c)}{d^2g^2-2\,cdgh+c^2h^2} - \frac{d\log(hx+g)}{d^2g^2-2\,cdgh+c^2h^2} + \frac{1}{dg^2-cgh+(dgh-ch^2)x}\right)\right)r}{2\,fh} - \frac{\log\left(\left(\left(bx+a\right)^p\left(dx+c\right)^qf\right)^r e\right)}{2\,\left(hx+g\right)^2h}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b*f*p*(b*log(b*x + a)/(b^2*g^2 - 2*a*b*g*h + a^2*h^2) - b*log(h*x + g)
/(b^2*g^2 - 2*a*b*g*h + a^2*h^2) + 1/(b*g^2 - a*g*h + (b*g*h - a*h^2)*x)) +
 d*f*q*(d*log(d*x + c)/(d^2*g^2 - 2*c*d*g*h + c^2*h^2) - d*log(h*x + g)/(d^
2*g^2 - 2*c*d*g*h + c^2*h^2) + 1/(d*g^2 - c*g*h + (d*g*h - c*h^2)*x)))*r/(f
*h) - 1/2*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((h*x + g)^2*h)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*x+g)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 595 vs. 2(189) = 378.
time = 4.81, size = 595, normalized size = 2.95

$$\frac{b^3pr\log\left(|bx+a|\right)}{2\left(b^3g^2h-2\,ab^2gh^2+a^2bh^3\right)}+\frac{d^3qr\log\left(|dx+c|\right)}{2\left(d^3g^2h-2\,cd^2gh^2+c^2dh^3\right)}-\frac{pr\log\left(bx+a\right)}{2\left(h^3x^2+2\,gh^2x+g^2h\right)}-\frac{qr\log\left(dx+c\right)}{2\left(h^3x^2+2\,gh^2x+g^2h\right)}-\frac{\left(b^2d^2g^2pr-2\,b^2cdghpr+b^2c^2h^2pr+b^2d^2g^2qr-2\,abd^2ghqr+a^2d^2h^2qr\right)\log\left(hx+g\right)}{2\left(b^2d^2g^4h-2\,b^2cdg^3h^2-2\,abd^2g^3h^2+b^2c^2g^2h^3+4\,abcdg^2h^3+a^2d^2g^2h^3-2\,abc^2gh^4-2\,a^2cdgh^4+a^2c^2h^5\right)}+\frac{bdghprx-bch^2prx+bdghqrx-adh^2qrx+bdg^2pr-bcghpr+bdg^2qr-adghqr-bdg^2r\log\left(f\right)+bcghr\log\left(f\right)+adghr\log\left(f\right)-ach^2r\log\left(f\right)-bdg^2+bcgh+adgh-ach^2}{2\left(bdg^2h^3x^2-bcgh^4x^2-adgh^4x^2+ach^5x^2+2\,bdg^3h^2x-2\,bcg^2h^3x-2\,adg^2h^3x+2\,acgh^4x+bdg^4h-bcg^3h^2-adg^3h^2+acg^2h^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^3*p*r*log(abs(b*x + a))/(b^3*g^2*h - 2*a*b^2*g*h^2 + a^2*b*h^3) + 1/2
*d^3*q*r*log(abs(d*x + c))/(d^3*g^2*h - 2*c*d^2*g*h^2 + c^2*d*h^3) - 1/2*p*
r*log(b*x + a)/(h^3*x^2 + 2*g*h^2*x + g^2*h) - 1/2*q*r*log(d*x + c)/(h^3*x^
2 + 2*g*h^2*x + g^2*h) - 1/2*(b^2*d^2*g^2*p*r - 2*b^2*c*d*g*h*p*r + b^2*c^2
*h^2*p*r + b^2*d^2*g^2*q*r - 2*a*b*d^2*g*h*q*r + a^2*d^2*h^2*q*r)*log(h*x +
 g)/(b^2*d^2*g^4*h - 2*b^2*c*d*g^3*h^2 - 2*a*b*d^2*g^3*h^2 + b^2*c^2*g^2*h^
3 + 4*a*b*c*d*g^2*h^3 + a^2*d^2*g^2*h^3 - 2*a*b*c^2*g*h^4 - 2*a^2*c*d*g*h^4
 + a^2*c^2*h^5) + 1/2*(b*d*g*h*p*r*x - b*c*h^2*p*r*x + b*d*g*h*q*r*x - a*d*
h^2*q*r*x + b*d*g^2*p*r - b*c*g*h*p*r + b*d*g^2*q*r - a*d*g*h*q*r - b*d*g^2
*r*log(f) + b*c*g*h*r*log(f) + a*d*g*h*r*log(f) - a*c*h^2*r*log(f) - b*d*g^
2 + b*c*g*h + a*d*g*h - a*c*h^2)/(b*d*g^2*h^3*x^2 - b*c*g*h^4*x^2 - a*d*g*h
```

```
^4*x^2 + a*c*h^5*x^2 + 2*b*d*g^3*h^2*x - 2*b*c*g^2*h^3*x - 2*a*d*g^2*h^3*x
+ 2*a*c*g*h^4*x + b*d*g^4*h - b*c*g^3*h^2 - a*d*g^3*h^2 + a*c*g^2*h^3)
```

Mupad [B]
time = 2.71, size = 384, normalized size = 1.90

$$\frac{b^2\,p\,r\,\ln\left(a+b\,x\right)}{2\,a^2\,h^3-4\,a\,b\,g\,h^2+2\,b^2\,g^2\,h}-\frac{\ln\left(g+h\,x\right)\,\left(h^2\,\left(q\,r\,a^2\,d^2+p\,r\,b^2\,c^2\right)-h\,\left(2\,c\,g\,p\,r\,b^2\,d+2\,a\,g\,q\,r\,b\,d^2\right)+b^2\,d^2\,g^2\,p\,r+b^2\,d^2\,g^2\,q\,r\right)}{2\,a^2\,c^2\,h^5-4\,a^2\,c\,d\,g\,h^4+2\,a^2\,d^2\,g^2\,h^3-4\,a\,b\,c^2\,g\,h^4+8\,a\,b\,c\,d\,g^2\,h^3-4\,a\,b\,d^2\,g^3\,h^2+2\,b^2\,c^2\,g^2\,h^3-4\,b^2\,c\,d\,g^3\,h^2+2\,b^2\,d^2\,g^4\,h}-\frac{\ln\left(e\,{\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)}^r\right)\,\left(\frac{x}{2}+\frac{g}{2\,h}\right)}{{\left(g+h\,x\right)}^3}-\frac{b\,c\,h\,p\,r-b\,d\,g\,p\,r+a\,d\,h\,q\,r-b\,d\,g\,q\,r}{\left(2\,x\,h^2+2\,g\,h\right)\,\left(a\,c\,h^2+b\,d\,g^2-a\,d\,g\,h-b\,c\,g\,h\right)}+\frac{d^2\,q\,r\,\ln\left(c+d\,x\right)}{2\,c^2\,h^3-4\,c\,d\,g\,h^2+2\,d^2\,g^2\,h}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(g + h*x)^3,x)
```

```
[Out] (b^2*p*r*log(a + b*x))/(2*a^2*h^3 + 2*b^2*g^2*h - 4*a*b*g*h^2) - (log(g + h
*x)*(h^2*(b^2*c^2*p*r + a^2*d^2*q*r) - h*(2*a*b*d^2*g*q*r + 2*b^2*c*d*g*p*r
) + b^2*d^2*g^2*p*r + b^2*d^2*g^2*q*r))/(2*a^2*c^2*h^5 + 2*b^2*d^2*g^4*h +
2*a^2*d^2*g^2*h^3 + 2*b^2*c^2*g^2*h^3 - 4*a*b*c^2*g*h^4 - 4*a^2*c*d*g*h^4 -
 4*a*b*d^2*g^3*h^2 - 4*b^2*c*d*g^3*h^2 + 8*a*b*c*d*g^2*h^3) - (log(e*(f*(a
+ b*x)^p*(c + d*x)^q)^r)*(x/2 + g/(2*h)))/(g + h*x)^3 - (b*c*h*p*r - b*d*g*
p*r + a*d*h*q*r - b*d*g*q*r)/((2*g*h + 2*h^2*x)*(a*c*h^2 + b*d*g^2 - a*d*g*
h - b*c*g*h)) + (d^2*q*r*log(c + d*x))/(2*c^2*h^3 + 2*d^2*g^2*h - 4*c*d*g*h
^2)
```

3.33 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(g+hx)^4}\,dx$

Optimal. Leaf size=260

$$\frac{bpr}{6h(bg-ah)(g+hx)^2}+\frac{dqr}{6h(dg-ch)(g+hx)^2}+\frac{b^2pr}{3h(bg-ah)^2(g+hx)}+\frac{d^2qr}{3h(dg-ch)^2(g+hx)}+\frac{b^3pr\log(a}{3h(bg-}$$

```
[Out] 1/6*b*p*r/h/(-a*h+b*g)/(h*x+g)^2+1/6*d*q*r/h/(-c*h+d*g)/(h*x+g)^2+1/3*b^2*p
*r/h/(-a*h+b*g)^2/(h*x+g)+1/3*d^2*q*r/h/(-c*h+d*g)^2/(h*x+g)+1/3*b^3*p*r*ln
(b*x+a)/h/(-a*h+b*g)^3+1/3*d^3*q*r*ln(d*x+c)/h/(-c*h+d*g)^3-1/3*ln(e*(f*(b*
x+a)^p*(d*x+c)^q)^r)/h/(h*x+g)^3-1/3*b^3*p*r*ln(h*x+g)/h/(-a*h+b*g)^3-1/3*d
^3*q*r*ln(h*x+g)/h/(-c*h+d*g)^3
```

Rubi [A]

time = 0.11, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2581, 46}

$$\frac{b^3pr\log(a+bx)}{3h(bg-ah)^3}-\frac{b^3pr\log(g+hx)}{3h(bg-ah)^3}+\frac{b^2pr}{3h(g+hx)(bg-ah)^2}-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h(g+hx)^3}+\frac{bpr}{6h(g+hx)^2(bg-ah)}+\frac{d^3qr\log(c+dx)}{3h(dg-ch)^3}-\frac{d^3qr\log(g+hx)}{3h(dg-ch)^3}+\frac{d^2qr}{3h(g+hx)(dg-ch)^2}+\frac{dqr}{6h(g+hx)^2(dg-ch)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^4,x]
```

```
[Out] (b*p*r)/(6*h*(b*g - a*h)*(g + h*x)^2) + (d*q*r)/(6*h*(d*g - c*h)*(g + h*x)^
2) + (b^2*p*r)/(3*h*(b*g - a*h)^2*(g + h*x)) + (d^2*q*r)/(3*h*(d*g - c*h)^2
*(g + h*x)) + (b^3*p*r*Log[a + b*x])/(3*h*(b*g - a*h)^3) + (d^3*q*r*Log[c +
 d*x])/(3*h*(d*g - c*h)^3) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(3*h*(g +
 h*x)^3) - (b^3*p*r*Log[g + h*x])/(3*h*(b*g - a*h)^3) - (d^3*q*r*Log[g + h*
x])/(3*h*(d*g - c*h)^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4}\,dx = -\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(g+hx)^3} + \frac{(bpr)\int \frac{1}{(a+bx)(g+hx)^3}\,dx}{3h} + \frac{(dqr)\int \frac{}{(c\cdots}$$

$$= -\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(g+hx)^3} + \frac{(bpr)\int\left(\frac{b^3}{(bg-ah)^3(a+bx)} - \frac{h}{(bg-ah)(g+hx\cdots}\right.}{}$$

$$= \frac{bpr}{6h(bg-ah)(g+hx)^2} + \frac{dqr}{6h(dg-ch)(g+hx)^2} + \frac{b^2pr}{3h(bg-ah)^2(g+h\cdots}$$

Mathematica [A]
time = 0.49, size = 254, normalized size = 0.98

$$\frac{-2\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)+\frac{r(g+hx)\left((bg-ah)^2(dg-ch)^2(bdg(p+q)-h(bcp+adq))-(g+hx)\left((bg-ah)(dg-ch)\left(4abd^2ghq-2a^2d^2h^2q-2b^2\left(-2cdghp+c^2h^2p+d^2g^2(p+q)\right)\right)-2(g+hx)\left(b^3(dg-ch)^3p(\log(a+bx)-\log(g+hx))+d^3(bg-ah)^3q(\log(c+dx)-\log(g+hx))\right)\right)\right)}{(bg-ah)^3(dg-ch)^3}}{6h(g+hx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^4,x]
```

```
[Out] (-2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (r*(g + h*x)*((b*g - a*h)^2*(d*g
 - c*h)^2*(b*d*g*(p + q) - h*(b*c*p + a*d*q)) - (g + h*x)*((b*g - a*h)*(d*g
 - c*h)*(4*a*b*d^2*g*h*q - 2*a^2*d^2*h^2*q - 2*b^2*(-2*c*d*g*h*p + c^2*h^2*
p + d^2*g^2*(p + q))) - 2*(g + h*x)*(b^3*(d*g - c*h)^3*p*(Log[a + b*x] - Lo
g[g + h*x]) + d^3*(b*g - a*h)^3*q*(Log[c + d*x] - Log[g + h*x])))))/((b*g -
 a*h)^3*(d*g - c*h)^3))/(6*h*(g + h*x)^3)
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\left(bx+a\right)^{p}\left(dx+c\right)^{q}\right)^{r}\right)}{\left(hx+g\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^4,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^4,x)
```

Maxima [A]
time = 0.35, size = 457, normalized size = 1.76

$$\frac{\left(\left(\frac{2\,b^2\log(bx+a)}{b^3g^3-3\,ab^2g^2h+3\,a^2bgh^2-a^3h^3}-\frac{2\,b^2\log(hx+g)}{b^3g^3-3\,ab^2g^2h+3\,a^2bgh^2-a^3h^3}+\frac{2\,bhx+3\,bg-ah}{b^2g^4-2\,abg^3h+a^2g^2h^2+(b^2g^2h^2-2\,abgh^3+a^2h^4)x^2+2\,(b^2g^3h-2\,abg^2h^2+a^2gh^3)x}\right)bfp+\left(\frac{2\,d^2\log(dx+c)}{d^3g^3-3\,cd^2g^2h+3\,c^2dgh^2-c^3h^3}-\frac{2\,d^2\log(hx+g)}{d^3g^3-3\,cd^2g^2h+3\,c^2dgh^2-c^3h^3}+\frac{2\,dhx+3\,dg-ch}{d^2g^4-2\,cdg^3h+c^2g^2h^2+(d^2g^2h^2-2\,cdgh^3+c^2h^4)x^2+2\,(d^2g^3h-2\,cdg^2h^2+c^2gh^3)x}\right)dfq\right)r}{6\,fh}-\frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{3\,(hx+g)^3h}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^4,x, algorithm="maxima")
```

```
[Out] 1/6*((2*b^2*log(b*x + a)/(b^3*g^3 - 3*a*b^2*g^2*h + 3*a^2*b*g*h^2 - a^3*h^3
) - 2*b^2*log(h*x + g)/(b^3*g^3 - 3*a*b^2*g^2*h + 3*a^2*b*g*h^2 - a^3*h^3)
+ (2*b*h*x + 3*b*g - a*h)/(b^2*g^4 - 2*a*b*g^3*h + a^2*g^2*h^2 + (b^2*g^2*h
^2 - 2*a*b*g*h^3 + a^2*h^4)*x^2 + 2*(b^2*g^3*h - 2*a*b*g^2*h^2 + a^2*g*h^3)
*x))*b*f*p + (2*d^2*log(d*x + c)/(d^3*g^3 - 3*c*d^2*g^2*h + 3*c^2*d*g*h^2 -
 c^3*h^3) - 2*d^2*log(h*x + g)/(d^3*g^3 - 3*c*d^2*g^2*h + 3*c^2*d*g*h^2 - c
^3*h^3) + (2*d*h*x + 3*d*g - c*h)/(d^2*g^4 - 2*c*d*g^3*h + c^2*g^2*h^2 + (d
^2*g^2*h^2 - 2*c*d*g*h^3 + c^2*h^4)*x^2 + 2*(d^2*g^3*h - 2*c*d*g^2*h^2 + c^
2*g*h^3)*x))*d*f*q)*r/(f*h) - 1/3*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((h*
x + g)^3*h)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*x+g)**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1765 vs. $2(243) = 486$.
time = 5.31, size = 1765, normalized size = 6.79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^4,x, algorithm="giac")
```

```
[Out] 1/3*b^4*p*r*log(abs(b*x + a))/(b^4*g^3*h - 3*a*b^3*g^2*h^2 + 3*a^2*b^2*g*h^
3 - a^3*b*h^4) + 1/3*d^4*q*r*log(abs(d*x + c))/(d^4*g^3*h - 3*c*d^3*g^2*h^2
 + 3*c^2*d^2*g*h^3 - c^3*d*h^4) - 1/3*p*r*log(b*x + a)/(h^4*x^3 + 3*g*h^3*x
^2 + 3*g^2*h^2*x + g^3*h) - 1/3*q*r*log(d*x + c)/(h^4*x^3 + 3*g*h^3*x^2 + 3
*g^2*h^2*x + g^3*h) - 1/3*(b^3*d^3*g^3*p*r - 3*b^3*c*d^2*g^2*h*p*r + 3*b^3*
c^2*d*g*h^2*p*r - b^3*c^3*h^3*p*r + b^3*d^3*g^3*q*r - 3*a*b^2*d^3*g^2*h*q*r
```

```
 + 3*a^2*b*d^3*g*h^2*q*r - a^3*d^3*h^3*q*r)*log(h*x + g)/(b^3*d^3*g^6*h - 3
*b^3*c*d^2*g^5*h^2 - 3*a*b^2*d^3*g^5*h^2 + 3*b^3*c^2*d*g^4*h^3 + 9*a*b^2*c*
d^2*g^4*h^3 + 3*a^2*b*d^3*g^4*h^3 - b^3*c^3*g^3*h^4 - 9*a*b^2*c^2*d*g^3*h^4
 - 9*a^2*b*c*d^2*g^3*h^4 - a^3*d^3*g^3*h^4 + 3*a*b^2*c^3*g^2*h^5 + 9*a^2*b*
c^2*d*g^2*h^5 + 3*a^3*c*d^2*g^2*h^5 - 3*a^2*b*c^3*g*h^6 - 3*a^3*c^2*d*g*h^6
 + a^3*c^3*h^7) + 1/6*(2*b^2*d^2*g^2*h^2*p*r*x^2 - 4*b^2*c*d*g*h^3*p*r*x^2
+ 2*b^2*c^2*h^4*p*r*x^2 + 2*b^2*d^2*g^2*h^2*q*r*x^2 - 4*a*b*d^2*g*h^3*q*r*x
^2 + 2*a^2*d^2*h^4*q*r*x^2 + 5*b^2*d^2*g^3*h*p*r*x - 10*b^2*c*d*g^2*h^2*p*r
*x - a*b*d^2*g^2*h^2*p*r*x + 5*b^2*c^2*g*h^3*p*r*x + 2*a*b*c*d*g*h^3*p*r*x
- a*b*c^2*h^4*p*r*x + 5*b^2*d^2*g^3*h*q*r*x - b^2*c*d*g^2*h^2*q*r*x - 10*a*
b*d^2*g^2*h^2*q*r*x + 2*a*b*c*d*g*h^3*q*r*x + 5*a^2*d^2*g*h^3*q*r*x - a^2*c
*d*h^4*q*r*x + 3*b^2*d^2*g^4*p*r - 6*b^2*c*d*g^3*h*p*r - a*b*d^2*g^3*h*p*r
+ 3*b^2*c^2*g^2*h^2*p*r + 2*a*b*c*d*g^2*h^2*p*r - a*b*c^2*g*h^3*p*r + 3*b^2
*d^2*g^4*q*r - b^2*c*d*g^3*h*q*r - 6*a*b*d^2*g^3*h*q*r + 2*a*b*c*d*g^2*h^2*
q*r + 3*a^2*d^2*g^2*h^2*q*r - a^2*c*d*g*h^3*q*r - 2*b^2*d^2*g^4*r*log(f) +
4*b^2*c*d*g^3*h*r*log(f) + 4*a*b*d^2*g^3*h*r*log(f) - 2*b^2*c^2*g^2*h^2*r*l
og(f) - 8*a*b*c*d*g^2*h^2*r*log(f) - 2*a^2*d^2*g^2*h^2*r*log(f) + 4*a*b*c^2
*g*h^3*r*log(f) + 4*a^2*c*d*g*h^3*r*log(f) - 2*a^2*c^2*h^4*r*log(f) - 2*b^2
*d^2*g^4 + 4*b^2*c*d*g^3*h + 4*a*b*d^2*g^3*h - 2*b^2*c^2*g^2*h^2 - 8*a*b*c*
d*g^2*h^2 - 2*a^2*d^2*g^2*h^2 + 4*a*b*c^2*g*h^3 + 4*a^2*c*d*g*h^3 - 2*a^2*c
^2*h^4)/(b^2*d^2*g^4*h^4*x^3 - 2*b^2*c*d*g^3*h^5*x^3 - 2*a*b*d^2*g^3*h^5*x^
3 + b^2*c^2*g^2*h^6*x^3 + 4*a*b*c*d*g^2*h^6*x^3 + a^2*d^2*g^2*h^6*x^3 - 2*a
*b*c^2*g*h^7*x^3 - 2*a^2*c*d*g*h^7*x^3 + a^2*c^2*h^8*x^3 + 3*b^2*d^2*g^5*h^
3*x^2 - 6*b^2*c*d*g^4*h^4*x^2 - 6*a*b*d^2*g^4*h^4*x^2 + 3*b^2*c^2*g^3*h^5*x
^2 + 12*a*b*c*d*g^3*h^5*x^2 + 3*a^2*d^2*g^3*h^5*x^2 - 6*a*b*c^2*g^2*h^6*x^2
 - 6*a^2*c*d*g^2*h^6*x^2 + 3*a^2*c^2*g*h^7*x^2 + 3*b^2*d^2*g^6*h^2*x - 6*b^
2*c*d*g^5*h^3*x - 6*a*b*d^2*g^5*h^3*x + 3*b^2*c^2*g^4*h^4*x + 12*a*b*c*d*g^
4*h^4*x + 3*a^2*d^2*g^4*h^4*x - 6*a*b*c^2*g^3*h^5*x - 6*a^2*c*d*g^3*h^5*x +
 3*a^2*c^2*g^2*h^6*x + b^2*d^2*g^7*h - 2*b^2*c*d*g^6*h^2 - 2*a*b*d^2*g^6*h^
2 + b^2*c^2*g^5*h^3 + 4*a*b*c*d*g^5*h^3 + a^2*d^2*g^5*h^3 - 2*a*b*c^2*g^4*h
^4 - 2*a^2*c*d*g^4*h^4 + a^2*c^2*g^3*h^5)
```

Mupad [B]
time = 5.48, size = 977, normalized size = 3.76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(g + h*x)^4,x)
```

```
[Out] ((3*b^2*d^2*g^3*p*r + 3*b^2*d^2*g^3*q*r - a*b*c^2*h^3*p*r - a^2*c*d*h^3*q*r
 + 3*b^2*c^2*g*h^2*p*r + 3*a^2*d^2*g*h^2*q*r - a*b*d^2*g^2*h*p*r - 6*a*b*d^
2*g^2*h*q*r - 6*b^2*c*d*g^2*h*p*r - b^2*c*d*g^2*h*q*r + 2*a*b*c*d*g*h^2*p*r
 + 2*a*b*c*d*g*h^2*q*r)/(2*(a^2*c^2*h^4 + b^2*d^2*g^4 + a^2*d^2*g^2*h^2 + b
^2*c^2*g^2*h^2 - 2*a*b*c^2*g*h^3 - 2*a*b*d^2*g^3*h - 2*a^2*c*d*g*h^3 - 2*b^
```

```
2*c*d*g^3*h + 4*a*b*c*d*g^2*h^2)) + (x*(b^2*c^2*h^3*p*r + a^2*d^2*h^3*q*r +
 b^2*d^2*g^2*h*p*r + b^2*d^2*g^2*h*q*r - 2*a*b*d^2*g*h^2*q*r - 2*b^2*c*d*g*
h^2*p*r))/(a^2*c^2*h^4 + b^2*d^2*g^4 + a^2*d^2*g^2*h^2 + b^2*c^2*g^2*h^2 -
2*a*b*c^2*g*h^3 - 2*a*b*d^2*g^3*h - 2*a^2*c*d*g*h^3 - 2*b^2*c*d*g^3*h + 4*a
*b*c*d*g^2*h^2))/(3*g^2*h + 3*h^3*x^2 + 6*g*h^2*x) + (log(g + h*x)*(g^2*(3*
a*b^2*d^3*h*q*r + 3*b^3*c*d^2*h*p*r) - g^3*(b^3*d^3*p*r + b^3*d^3*q*r) - g*
(3*a^2*b*d^3*h^2*q*r + 3*b^3*c^2*d*h^2*p*r) + b^3*c^3*h^3*p*r + a^3*d^3*h^3
*q*r))/(3*a^3*c^3*h^7 + 3*b^3*d^3*g^6*h - 3*a^3*d^3*g^3*h^4 - 3*b^3*c^3*g^3
*h^4 - 9*a^2*b*c^3*g*h^6 - 9*a^3*c^2*d*g*h^6 + 9*a*b^2*c^3*g^2*h^5 - 9*a*b^
2*d^3*g^5*h^2 + 9*a^2*b*d^3*g^4*h^3 + 9*a^3*c*d^2*g^2*h^5 - 9*b^3*c*d^2*g^5
*h^2 + 9*b^3*c^2*d*g^4*h^3 + 27*a*b^2*c*d^2*g^4*h^3 - 27*a*b^2*c^2*d*g^3*h^
4 - 27*a^2*b*c*d^2*g^3*h^4 + 27*a^2*b*c^2*d*g^2*h^5) - (log(e*(f*(a + b*x)^
p*(c + d*x)^q)^r)*(x/3 + g/(3*h)))/(g + h*x)^4 - (b^3*p*r*log(a + b*x))/(3*
a^3*h^4 - 3*b^3*g^3*h + 9*a*b^2*g^2*h^2 - 9*a^2*b*g*h^3) - (d^3*q*r*log(c +
 d*x))/(3*c^3*h^4 - 3*d^3*g^3*h + 9*c*d^2*g^2*h^2 - 9*c^2*d*g*h^3)
```

3.34 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(g+hx)^5}\,dx$

Optimal. Leaf size=318

$$\frac{bpr}{12h(bg-ah)(g+hx)^3}+\frac{dqr}{12h(dg-ch)(g+hx)^3}+\frac{b^2pr}{8h(bg-ah)^2(g+hx)^2}+\frac{d^2qr}{8h(dg-ch)^2(g+hx)^2}+\frac{}{4h(bg-}$$

```
[Out] 1/12*b*p*r/h/(-a*h+b*g)/(h*x+g)^3+1/12*d*q*r/h/(-c*h+d*g)/(h*x+g)^3+1/8*b^2
*p*r/h/(-a*h+b*g)^2/(h*x+g)^2+1/8*d^2*q*r/h/(-c*h+d*g)^2/(h*x+g)^2+1/4*b^3*
p*r/h/(-a*h+b*g)^3/(h*x+g)+1/4*d^3*q*r/h/(-c*h+d*g)^3/(h*x+g)+1/4*b^4*p*r*l
n(b*x+a)/h/(-a*h+b*g)^4+1/4*d^4*q*r*ln(d*x+c)/h/(-c*h+d*g)^4-1/4*ln(e*(f*(b
*x+a)^p*(d*x+c)^q)^r)/h/(h*x+g)^4-1/4*b^4*p*r*ln(h*x+g)/h/(-a*h+b*g)^4-1/4*
d^4*q*r*ln(h*x+g)/h/(-c*h+d*g)^4
```

Rubi [A]
time = 0.14, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2581, 46}

$$\frac{b^4pr\log(a+bx)}{4h(bg-ah)^4}-\frac{b^4pr\log(g+hx)}{4h(bg-ah)^4}+\frac{b^3pr}{4h(g+hx)(bg-ah)^3}+\frac{b^2pr}{8h(g+hx)^2(bg-ah)^2}-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h(g+hx)^4}+\frac{bpr}{12h(g+hx)^3(bg-ah)}+\frac{d^4qr\log(c+dx)}{4h(dg-ch)^4}-\frac{d^4qr\log(g+hx)}{4h(dg-ch)^4}+\frac{d^3qr}{4h(g+hx)(dg-ch)^3}+\frac{d^2qr}{8h(g+hx)^2(dg-ch)^2}+\frac{dqr}{12h(g+hx)^3(dg-ch)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^5,x]
```

```
[Out] (b*p*r)/(12*h*(b*g - a*h)*(g + h*x)^3) + (d*q*r)/(12*h*(d*g - c*h)*(g + h*x
)^3) + (b^2*p*r)/(8*h*(b*g - a*h)^2*(g + h*x)^2) + (d^2*q*r)/(8*h*(d*g - c*
h)^2*(g + h*x)^2) + (b^3*p*r)/(4*h*(b*g - a*h)^3*(g + h*x)) + (d^3*q*r)/(4*
h*(d*g - c*h)^3*(g + h*x)) + (b^4*p*r*Log[a + b*x])/(4*h*(b*g - a*h)^4) + (
d^4*q*r*Log[c + d*x])/(4*h*(d*g - c*h)^4) - Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]/(4*h*(g + h*x)^4) - (b^4*p*r*Log[g + h*x])/(4*h*(b*g - a*h)^4) - (d^4
*q*r*Log[g + h*x])/(4*h*(d*g - c*h)^4)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
```

```
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^5}\,dx = -\frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4h(g+hx)^4} + \frac{(bpr)\int \frac{1}{(a+bx)(g+hx)^4}\,dx}{4h} + \frac{(dqr)\int \cdots}{\cdots}$$

$$= -\frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4h(g+hx)^4} + \frac{(bpr)\int \left(\frac{b^4}{(bg-ah)^4(a+bx)} - \frac{h}{(bg-ah)(g+\cdots}\right.}{\cdots}$$

$$= \frac{bpr}{12h(bg-ah)(g+hx)^3} + \frac{dqr}{12h(dg-ch)(g+hx)^3} + \frac{b^2pr}{8h(bg-ah)^2(g+\cdots}$$

Mathematica [A]
time = 0.93, size = 480, normalized size = 1.51

$$\frac{-6\log\left(e(f(a+bx)^p(c+dx)^q)^r\right) + \frac{r(g+hx)\left(2(bg-ah)^3(dg-ch)^3(bdg(p+q)-h(bcp+adq)) - (g+hx)\left((bg-ah)^2(dg-ch)^2\left(6abd^2ghq-3a^2d^2h^2q-3b^2(-2cdghp+c^2h^2p+d^2g^2(p+q))\right) + 6(g+hx)\left(-\left((bg-ah)(-dg+ch)\left(3ab^2d^3g^2hq-3a^2bd^3gh^2q+a^3d^3h^3q-b^3(-3cd^2g^2hp+3c^2dgh^2p-c^3h^3p+d^3g^3(p+q))\right)\right) - (g+hx)\left(b^4(dg-ch)^4p\log(a+bx)+d^4(bg-ah)^4q\log(c+dx)-\left(-4ab^3d^4g^3hq+6a^2b^2d^4g^2h^2q-4a^3bd^4gh^3q+a^4d^4h^4q+b^4(-4cd^3g^3hp+6c^2d^2g^2h^2p-4c^3dgh^3p+c^4h^4p+d^4g^4(p+q))\right)\log(g+hx)\right)\right)\right)\right)}{(bg-ah)^4(dg-ch)^4}}{24h(g+hx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^5,x]
```

```
[Out] (-6*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (r*(g + h*x)*(2*(b*g - a*h)^3*(d
*g - c*h)^3*(b*d*g*(p + q) - h*(b*c*p + a*d*q)) - (g + h*x)*((b*g - a*h)^2*
(d*g - c*h)^2*(6*a*b*d^2*g*h*q - 3*a^2*d^2*h^2*q - 3*b^2*(-2*c*d*g*h*p + c^
2*h^2*p + d^2*g^2*(p + q))) + 6*(g + h*x)*(-((b*g - a*h)*(-(d*g) + c*h)*(3*
a*b^2*d^3*g^2*h*q - 3*a^2*b*d^3*g*h^2*q + a^3*d^3*h^3*q - b^3*(-3*c*d^2*g^2
*h*p + 3*c^2*d*g*h^2*p - c^3*h^3*p + d^3*g^3*(p + q)))) - (g + h*x)*(b^4*(d
*g - c*h)^4*p*Log[a + b*x] + d^4*(b*g - a*h)^4*q*Log[c + d*x] - (-4*a*b^3*d
^4*g^3*h*q + 6*a^2*b^2*d^4*g^2*h^2*q - 4*a^3*b*d^4*g*h^3*q + a^4*d^4*h^4*q
+ b^4*(-4*c*d^3*g^3*h*p + 6*c^2*d^2*g^2*h^2*p - 4*c^3*d*g*h^3*p + c^4*h^4*p
 + d^4*g^4*(p + q)))*Log[g + h*x])))))/((b*g - a*h)^4*(d*g - c*h)^4))/(24*h
*(g + h*x)^4)
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(e\left(f\left(bx+a\right)^{p}\left(dx+c\right)^{q}\right)^{r}\right)}{\left(hx+g\right)^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^5,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^5,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 777 vs. $2(297) = 594$.
time = 0.33, size = 777, normalized size = 2.44

$$\frac{\left(\left(\frac{6\,b^3\log(bx+a)}{b^4g^4-4\,ab^3g^3h+6\,a^2b^2g^2h^2-4\,a^3bgh^3+a^4h^4}-\frac{6\,b^3\log(hx+g)}{b^4g^4-4\,ab^3g^3h+6\,a^2b^2g^2h^2-4\,a^3bgh^3+a^4h^4}+\frac{6\,b^2h^2x^2+11\,b^2g^2-7\,abgh+2\,a^2h^2+3\,(5\,b^2gh-abh^2)x}{b^3g^6-3\,ab^2g^5h+3\,a^2bg^4h^2-a^3g^3h^3+(b^3g^3h^3-3\,ab^2g^2h^4+3\,a^2bgh^5-a^3h^6)x^3+3\,(b^3g^4h^2-3\,ab^2g^3h^3+3\,a^2bg^2h^4-a^3gh^5)x^2+3\,(b^3g^5h-3\,ab^2g^4h^2+3\,a^2bg^3h^3-a^3g^2h^4)x}\right)bfp+\left(\frac{6\,d^3\log(dx+c)}{d^4g^4-4\,cd^3g^3h+6\,c^2d^2g^2h^2-4\,c^3dgh^3+c^4h^4}-\frac{6\,d^3\log(hx+g)}{d^4g^4-4\,cd^3g^3h+6\,c^2d^2g^2h^2-4\,c^3dgh^3+c^4h^4}+\frac{6\,d^2h^2x^2+11\,d^2g^2-7\,cdgh+2\,c^2h^2+3\,(5\,d^2gh-cdh^2)x}{d^3g^6-3\,cd^2g^5h+3\,c^2dg^4h^2-c^3g^3h^3+(d^3g^3h^3-3\,cd^2g^2h^4+3\,c^2dgh^5-c^3h^6)x^3+3\,(d^3g^4h^2-3\,cd^2g^3h^3+3\,c^2dg^2h^4-c^3gh^5)x^2+3\,(d^3g^5h-3\,cd^2g^4h^2+3\,c^2dg^3h^3-c^3g^2h^4)x}\right)dfq\right)r}{24\,fh}-\frac{\log\left(\left((bx+a)^p(dx+c)^qf\right)^r e\right)}{4\,(hx+g)^4h}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^5,x, algorithm="maxima")
```

```
[Out] 1/24*((6*b^3*log(b*x + a)/(b^4*g^4 - 4*a*b^3*g^3*h + 6*a^2*b^2*g^2*h^2 - 4*
a^3*b*g*h^3 + a^4*h^4) - 6*b^3*log(h*x + g)/(b^4*g^4 - 4*a*b^3*g^3*h + 6*a^
2*b^2*g^2*h^2 - 4*a^3*b*g*h^3 + a^4*h^4) + (6*b^2*h^2*x^2 + 11*b^2*g^2 - 7*
a*b*g*h + 2*a^2*h^2 + 3*(5*b^2*g*h - a*b*h^2)*x)/(b^3*g^6 - 3*a*b^2*g^5*h +
 3*a^2*b*g^4*h^2 - a^3*g^3*h^3 + (b^3*g^3*h^3 - 3*a*b^2*g^2*h^4 + 3*a^2*b*g
*h^5 - a^3*h^6)*x^3 + 3*(b^3*g^4*h^2 - 3*a*b^2*g^3*h^3 + 3*a^2*b*g^2*h^4 -
a^3*g*h^5)*x^2 + 3*(b^3*g^5*h - 3*a*b^2*g^4*h^2 + 3*a^2*b*g^3*h^3 - a^3*g^2
*h^4)*x))*b*f*p + (6*d^3*log(d*x + c)/(d^4*g^4 - 4*c*d^3*g^3*h + 6*c^2*d^2*
g^2*h^2 - 4*c^3*d*g*h^3 + c^4*h^4) - 6*d^3*log(h*x + g)/(d^4*g^4 - 4*c*d^3*
g^3*h + 6*c^2*d^2*g^2*h^2 - 4*c^3*d*g*h^3 + c^4*h^4) + (6*d^2*h^2*x^2 + 11*
d^2*g^2 - 7*c*d*g*h + 2*c^2*h^2 + 3*(5*d^2*g*h - c*d*h^2)*x)/(d^3*g^6 - 3*c
*d^2*g^5*h + 3*c^2*d*g^4*h^2 - c^3*g^3*h^3 + (d^3*g^3*h^3 - 3*c*d^2*g^2*h^4
 + 3*c^2*d*g*h^5 - c^3*h^6)*x^3 + 3*(d^3*g^4*h^2 - 3*c*d^2*g^3*h^3 + 3*c^2*
d*g^2*h^4 - c^3*g*h^5)*x^2 + 3*(d^3*g^5*h - 3*c*d^2*g^4*h^2 + 3*c^2*d*g^3*h
^3 - c^3*g^2*h^4)*x))*d*f*q)*r/(f*h) - 1/4*log(((b*x + a)^p*(d*x + c)^q*f)^
r*e)/((h*x + g)^4*h)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*x+g)**5,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3911 vs. $2(297) = 594$.
time = 4.54, size = 3911, normalized size = 12.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^5,x, algorithm="giac")
```

```
[Out] 1/4*b^5*p*r*log(abs(b*x + a))/(b^5*g^4*h - 4*a*b^4*g^3*h^2 + 6*a^2*b^3*g^2*
h^3 - 4*a^3*b^2*g*h^4 + a^4*b*h^5) + 1/4*d^5*q*r*log(abs(d*x + c))/(d^5*g^4
*h - 4*c*d^4*g^3*h^2 + 6*c^2*d^3*g^2*h^3 - 4*c^3*d^2*g*h^4 + c^4*d*h^5) - 1
/4*p*r*log(b*x + a)/(h^5*x^4 + 4*g*h^4*x^3 + 6*g^2*h^3*x^2 + 4*g^3*h^2*x +
g^4*h) - 1/4*q*r*log(d*x + c)/(h^5*x^4 + 4*g*h^4*x^3 + 6*g^2*h^3*x^2 + 4*g^
3*h^2*x + g^4*h) - 1/4*(b^4*d^4*g^4*p*r - 4*b^4*c*d^3*g^3*h*p*r + 6*b^4*c^2
*d^2*g^2*h^2*p*r - 4*b^4*c^3*d*g*h^3*p*r + b^4*c^4*h^4*p*r + b^4*d^4*g^4*q*
r - 4*a*b^3*d^4*g^3*h*q*r + 6*a^2*b^2*d^4*g^2*h^2*q*r - 4*a^3*b*d^4*g*h^3*q
*r + a^4*d^4*h^4*q*r)*log(h*x + g)/(b^4*d^4*g^8*h - 4*b^4*c*d^3*g^7*h^2 - 4
*a*b^3*d^4*g^7*h^2 + 6*b^4*c^2*d^2*g^6*h^3 + 16*a*b^3*c*d^3*g^6*h^3 + 6*a^2
*b^2*d^4*g^6*h^3 - 4*b^4*c^3*d*g^5*h^4 - 24*a*b^3*c^2*d^2*g^5*h^4 - 24*a^2*
b^2*c*d^3*g^5*h^4 - 4*a^3*b*d^4*g^5*h^4 + b^4*c^4*g^4*h^5 + 16*a*b^3*c^3*d*
g^4*h^5 + 36*a^2*b^2*c^2*d^2*g^4*h^5 + 16*a^3*b*c*d^3*g^4*h^5 + a^4*d^4*g^4
*h^5 - 4*a*b^3*c^4*g^3*h^6 - 24*a^2*b^2*c^3*d*g^3*h^6 - 24*a^3*b*c^2*d^2*g^
3*h^6 - 4*a^4*c*d^3*g^3*h^6 + 6*a^2*b^2*c^4*g^2*h^7 + 16*a^3*b*c^3*d*g^2*h^
7 + 6*a^4*c^2*d^2*g^2*h^7 - 4*a^3*b*c^4*g*h^8 - 4*a^4*c^3*d*g*h^8 + a^4*c^4
*h^9) + 1/24*(6*b^3*d^3*g^3*h^3*p*r*x^3 - 18*b^3*c*d^2*g^2*h^4*p*r*x^3 + 18
*b^3*c^2*d*g*h^5*p*r*x^3 - 6*b^3*c^3*h^6*p*r*x^3 + 6*b^3*d^3*g^3*h^3*q*r*x^
3 - 18*a*b^2*d^3*g^2*h^4*q*r*x^3 + 18*a^2*b*d^3*g*h^5*q*r*x^3 - 6*a^3*d^3*h
^6*q*r*x^3 + 21*b^3*d^3*g^4*h^2*p*r*x^2 - 63*b^3*c*d^2*g^3*h^3*p*r*x^2 - 3*
a*b^2*d^3*g^3*h^3*p*r*x^2 + 63*b^3*c^2*d*g^2*h^4*p*r*x^2 + 9*a*b^2*c*d^2*g^
2*h^4*p*r*x^2 - 21*b^3*c^3*g*h^5*p*r*x^2 - 9*a*b^2*c^2*d*g*h^5*p*r*x^2 + 3*
a*b^2*c^3*h^6*p*r*x^2 + 21*b^3*d^3*g^4*h^2*q*r*x^2 - 3*b^3*c*d^2*g^3*h^3*q*
r*x^2 - 63*a*b^2*d^3*g^3*h^3*q*r*x^2 + 9*a*b^2*c*d^2*g^2*h^4*q*r*x^2 + 63*a
^2*b*d^3*g^2*h^4*q*r*x^2 - 9*a^2*b*c*d^2*g*h^5*q*r*x^2 - 21*a^3*d^3*g*h^5*q
*r*x^2 + 3*a^3*c*d^2*h^6*q*r*x^2 + 26*b^3*d^3*g^5*h*p*r*x - 78*b^3*c*d^2*g^
4*h^2*p*r*x - 10*a*b^2*d^3*g^4*h^2*p*r*x + 78*b^3*c^2*d*g^3*h^3*p*r*x + 30*
a*b^2*c*d^2*g^3*h^3*p*r*x + 2*a^2*b*d^3*g^3*h^3*p*r*x - 26*b^3*c^3*g^2*h^4*
p*r*x - 30*a*b^2*c^2*d*g^2*h^4*p*r*x - 6*a^2*b*c*d^2*g^2*h^4*p*r*x + 10*a*b
^2*c^3*g*h^5*p*r*x + 6*a^2*b*c^2*d*g*h^5*p*r*x - 2*a^2*b*c^3*h^6*p*r*x + 26
*b^3*d^3*g^5*h*q*r*x - 10*b^3*c*d^2*g^4*h^2*q*r*x - 78*a*b^2*d^3*g^4*h^2*q*
r*x + 2*b^3*c^2*d*g^3*h^3*q*r*x + 30*a*b^2*c*d^2*g^3*h^3*q*r*x + 78*a^2*b*d
^3*g^3*h^3*q*r*x - 6*a*b^2*c^2*d*g^2*h^4*q*r*x - 30*a^2*b*c*d^2*g^2*h^4*q*r
*x - 26*a^3*d^3*g^2*h^4*q*r*x + 6*a^2*b*c^2*d*g*h^5*q*r*x + 10*a^3*c*d^2*g*
h^5*q*r*x - 2*a^3*c^2*d*h^6*q*r*x + 11*b^3*d^3*g^6*p*r - 33*b^3*c*d^2*g^5*h
*p*r - 7*a*b^2*d^3*g^5*h*p*r + 33*b^3*c^2*d*g^4*h^2*p*r + 21*a*b^2*c*d^2*g^
```

```
4*h^2*p*r + 2*a^2*b*d^3*g^4*h^2*p*r - 11*b^3*c^3*g^3*h^3*p*r - 21*a*b^2*c^2
*d*g^3*h^3*p*r - 6*a^2*b*c*d^2*g^3*h^3*p*r + 7*a*b^2*c^3*g^2*h^4*p*r + 6*a^
2*b*c^2*d*g^2*h^4*p*r - 2*a^2*b*c^3*g*h^5*p*r + 11*b^3*d^3*g^6*q*r - 7*b^3*
c*d^2*g^5*h*q*r - 33*a*b^2*d^3*g^5*h*q*r + 2*b^3*c^2*d*g^4*h^2*q*r + 21*a*b
^2*c*d^2*g^4*h^2*q*r + 33*a^2*b*d^3*g^4*h^2*q*r - 6*a*b^2*c^2*d*g^3*h^3*q*r
 - 21*a^2*b*c*d^2*g^3*h^3*q*r - 11*a^3*d^3*g^3*h^3*q*r + 6*a^2*b*c^2*d*g^2*
h^4*q*r + 7*a^3*c*d^2*g^2*h^4*q*r - 2*a^3*c^2*d*g*h^5*q*r - 6*b^3*d^3*g^6*r
*log(f) + 18*b^3*c*d^2*g^5*h*r*log(f) + 18*a*b^2*d^3*g^5*h*r*log(f) - 18*b^
3*c^2*d*g^4*h^2*r*log(f) - 54*a*b^2*c*d^2*g^4*h^2*r*log(f) - 18*a^2*b*d^3*g
^4*h^2*r*log(f) + 6*b^3*c^3*g^3*h^3*r*log(f) + 54*a*b^2*c^2*d*g^3*h^3*r*log
(f) + 54*a^2*b*c*d^2*g^3*h^3*r*log(f) + 6*a^3*d^3*g^3*h^3*r*log(f) - 18*a*b
^2*c^3*g^2*h^4*r*log(f) - 54*a^2*b*c^2*d*g^2*h^4*r*log(f) - 18*a^3*c*d^2*g^
2*h^4*r*log(f) + 18*a^2*b*c^3*g*h^5*r*log(f) + 18*a^3*c^2*d*g*h^5*r*log(f)
- 6*a^3*c^3*h^6*r*log(f) - 6*b^3*d^3*g^6 + 18*b^3*c*d^2*g^5*h + 18*a*b^2*d^
3*g^5*h - 18*b^3*c^2*d*g^4*h^2 - 54*a*b^2*c*d^2*g^4*h^2 - 18*a^2*b*d^3*g^4*
h^2 + 6*b^3*c^3*g^3*h^3 + 54*a*b^2*c^2*d*g^3*h^3 + 54*a^2*b*c*d^2*g^3*h^3 +
 6*a^3*d^3*g^3*h^3 - 18*a*b^2*c^3*g^2*h^4 - 54*a^2*b*c^2*d*g^2*h^4 - 18*a^3
*c*d^2*g^2*h^4 + 18*a^2*b*c^3*g*h^5 + 18*a^3*c^2*d*g*h^5 - 6*a^3*c^3*h^6)/(
b^3*d^3*g^6*h^5*x^4 - 3*b^3*c*d^2*g^5*h^6*x^4 - 3*a*b^2*d^3*g^5*h^6*x^4 + 3
*b^3*c^2*d*g^4*h^7*x^4 + 9*a*b^2*c*d^2*g^4*h^7*x^4 + 3*a^2*b*d^3*g^4*h^7*x^
4 - b^3*c^3*g^3*h^8*x^4 - 9*a*b^2*c^2*d*g^3*h^8*x^4 - 9*a^2*b*c*d^2*g^3*h^8
*x^4 - a^3*d^3*g^3*h^8*x^4 + 3*a*b^2*c^3*g^2*h^9*x^4 + 9*a^2*b*c^2*d*g^2*h^
9*x^4 + 3*a^3*c*d^2*g^2*h^9*x^4 - 3*a^2*b*c^3*g*h^10*x^4 - 3*a^3*c^2*d*g*h^
10*x^4 + a^3*c^3*h^11*x^4 + 4*b^3*d^3*g^7*h^4*x^3 - 12*b^3*c*d^2*g^6*h^5*x^
3 - 12*a*b^2*d^3*g^6*h^5*x^3 + 12*b^3*c^2*d*g^5*h^6*x^3 + 36*a*b^2*c*d^2*g^
5*h^6*x^3 + 12*a^2*b*d^3*g^5*h^6*x^3 - 4*b^3*c^3*g^4*h^7*x^3 - 36*a*b^2*c^2
*d*g^4*h^7*x^3 - 36*a^2*b*c*d^2*g^4*h^7*x^3 - 4*a^3*d^3*g^4*h^7*x^3 + 12*a*
b^2*c^3*g^3*h^8*x^3 + 36*a^2*b*c^2*d*g^3*h^8*x^3 + 12*a^3*c*d^2*g^3*h^8*x^3
 - 12*a^2*b*c^3*g^2*h^9*x^3 - 12*a^3*c^2*d*g^2*h^9*x^3 + 4*a^3*c^3*g*h^10*x
^3 + 6*b^3*d^3*g^8*h^3*x^2 - 18*b^3*c*d^2*g^7*h^4*x^2 - 18*a*b^2*d^3*g^7*h^
4*x^2 + 18*b^3*c^2*d*g^6*h^5*x^2 + 54*a*b^2*c*d...
```

Mupad [B]
time = 10.15, size = 2215, normalized size = 6.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(g + h*x)^5,x)
```

```
[Out] ((11*b^3*d^3*g^5*p*r + 11*b^3*d^3*g^5*q*r - 11*b^3*c^3*g^2*h^3*p*r - 11*a^3
*d^3*g^2*h^3*q*r - 2*a^2*b*c^3*h^5*p*r - 2*a^3*c^2*d*h^5*q*r + 7*a*b^2*c^3*
g*h^4*p*r - 7*a*b^2*d^3*g^4*h*p*r - 33*a*b^2*d^3*g^4*h*q*r + 7*a^3*c*d^2*g*
h^4*q*r - 33*b^3*c*d^2*g^4*h*p*r - 7*b^3*c*d^2*g^4*h*q*r + 2*a^2*b*d^3*g^3*
h^2*p*r + 33*a^2*b*d^3*g^3*h^2*q*r + 33*b^3*c^2*d*g^3*h^2*p*r + 2*b^3*c^2*d
```

*g^3*h^2*q*r + 21*a*b^2*c*d^2*g^3*h^2*p*r - 21*a*b^2*c^2*d*g^2*h^3*p*r - 6*
a^2*b*c*d^2*g^2*h^3*p*r + 21*a*b^2*c*d^2*g^3*h^2*q*r - 6*a*b^2*c^2*d*g^2*h^
3*q*r - 21*a^2*b*c*d^2*g^2*h^3*q*r + 6*a^2*b*c^2*d*g*h^4*p*r + 6*a^2*b*c^2*
d*g*h^4*q*r)/(6*(a^3*c^3*h^6 + b^3*d^3*g^6 - a^3*d^3*g^3*h^3 - b^3*c^3*g^3*
h^3 - 3*a^2*b*c^3*g*h^5 - 3*a*b^2*d^3*g^5*h - 3*a^3*c^2*d*g*h^5 - 3*b^3*c*d
^2*g^5*h + 3*a*b^2*c^3*g^2*h^4 + 3*a^2*b*d^3*g^4*h^2 + 3*a^3*c*d^2*g^2*h^4
+ 3*b^3*c^2*d*g^4*h^2 + 9*a*b^2*c*d^2*g^4*h^2 - 9*a*b^2*c^2*d*g^3*h^3 - 9*a
^2*b*c*d^2*g^3*h^3 + 9*a^2*b*c^2*d*g^2*h^4)) - (x^2*(b^3*c^3*h^5*p*r + a^3*
d^3*h^5*q*r - b^3*d^3*g^3*h^2*p*r - b^3*d^3*g^3*h^2*q*r - 3*a^2*b*d^3*g*h^4
*q*r - 3*b^3*c^2*d*g*h^4*p*r + 3*a*b^2*d^3*g^2*h^3*q*r + 3*b^3*c*d^2*g^2*h^
3*p*r))/(a^3*c^3*h^6 + b^3*d^3*g^6 - a^3*d^3*g^3*h^3 - b^3*c^3*g^3*h^3 - 3*
a^2*b*c^3*g*h^5 - 3*a*b^2*d^3*g^5*h - 3*a^3*c^2*d*g*h^5 - 3*b^3*c*d^2*g^5*h
 + 3*a*b^2*c^3*g^2*h^4 + 3*a^2*b*d^3*g^4*h^2 + 3*a^3*c*d^2*g^2*h^4 + 3*b^3*
c^2*d*g^4*h^2 + 9*a*b^2*c*d^2*g^4*h^2 - 9*a*b^2*c^2*d*g^3*h^3 - 9*a^2*b*c*d
^2*g^3*h^3 + 9*a^2*b*c^2*d*g^2*h^4) + (x*(a*b^2*c^3*h^5*p*r + a^3*c*d^2*h^5
*q*r - 5*b^3*c^3*g*h^4*p*r - 5*a^3*d^3*g*h^4*q*r + 5*b^3*d^3*g^4*h*p*r + 5*
b^3*d^3*g^4*h*q*r - a*b^2*d^3*g^3*h^2*p*r - 15*a*b^2*d^3*g^3*h^2*q*r + 15*a
^2*b*d^3*g^2*h^3*q*r - 15*b^3*c*d^2*g^3*h^2*p*r + 15*b^3*c^2*d*g^2*h^3*p*r
- b^3*c*d^2*g^3*h^2*q*r + 3*a*b^2*c*d^2*g^2*h^3*p*r + 3*a*b^2*c*d^2*g^2*h^3
*q*r - 3*a*b^2*c^2*d*g*h^4*p*r - 3*a^2*b*c*d^2*g*h^4*q*r))/(2*(a^3*c^3*h^6
+ b^3*d^3*g^6 - a^3*d^3*g^3*h^3 - b^3*c^3*g^3*h^3 - 3*a^2*b*c^3*g*h^5 - 3*a
*b^2*d^3*g^5*h - 3*a^3*c^2*d*g*h^5 - 3*b^3*c*d^2*g^5*h + 3*a*b^2*c^3*g^2*h^
4 + 3*a^2*b*d^3*g^4*h^2 + 3*a^3*c*d^2*g^2*h^4 + 3*b^3*c^2*d*g^4*h^2 + 9*a*b
^2*c*d^2*g^4*h^2 - 9*a*b^2*c^2*d*g^3*h^3 - 9*a^2*b*c*d^2*g^3*h^3 + 9*a^2*b*
c^2*d*g^2*h^4)))/(4*g^3*h + 4*h^4*x^3 + 12*g^2*h^2*x + 12*g*h^3*x^2) - (log
(g + h*x)*(h^4*(b^4*c^4*p*r + a^4*d^4*q*r) - h*(4*a*b^3*d^4*g^3*q*r + 4*b^4
*c*d^3*g^3*p*r) + h^2*(6*a^2*b^2*d^4*g^2*q*r + 6*b^4*c^2*d^2*g^2*p*r) - h^3
*(4*a^3*b*d^4*g*q*r + 4*b^4*c^3*d*g*p*r) + b^4*d^4*g^4*p*r + b^4*d^4*g^4*q*
r))/(4*a^4*c^4*h^9 + 4*b^4*d^4*g^8*h + 4*a^4*d^4*g^4*h^5 + 4*b^4*c^4*g^4*h^
5 + 24*a^2*b^2*c^4*g^2*h^7 + 24*a^2*b^2*d^4*g^6*h^3 + 24*a^4*c^2*d^2*g^2*h^
7 + 24*b^4*c^2*d^2*g^6*h^3 - 16*a^3*b*c^4*g*h^8 - 16*a^4*c^3*d*g*h^8 - 16*a
*b^3*c^4*g^3*h^6 - 16*a*b^3*d^4*g^7*h^2 - 16*a^3*b*d^4*g^5*h^4 - 16*a^4*c*d
^3*g^3*h^6 - 16*b^4*c*d^3*g^7*h^2 - 16*b^4*c^3*d*g^5*h^4 + 64*a*b^3*c*d^3*g
^6*h^3 + 64*a*b^3*c^3*d*g^4*h^5 + 64*a^3*b*c*d^3*g^4*h^5 + 64*a^3*b*c^3*d*g
^2*h^7 - 96*a*b^3*c^2*d^2*g^5*h^4 - 96*a^2*b^2*c*d^3*g^5*h^4 - 96*a^2*b^2*c
^3*d*g^3*h^6 - 96*a^3*b*c^2*d^2*g^3*h^6 + 144*a^2*b^2*c^2*d^2*g^4*h^5) - (l
og(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(x/4 + g/(4*h)))/(g + h*x)^5 + (b^4*p*r
*log(a + b*x))/(4*a^4*h^5 + 4*b^4*g^4*h - 16*a*b^3*g^3*h^2 + 24*a^2*b^2*g^2
*h^3 - 16*a^3*b*g*h^4) + (d^4*q*r*log(c + d*x))/(4*c^4*h^5 + 4*d^4*g^4*h -
16*c*d^3*g^3*h^2 + 24*c^2*d^2*g^2*h^3 - 16*c^3*d*g*h^4)

3.35 $\int (g+hx)^3 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$

Optimal. Leaf size=2240

$$\frac{2(bg-ah)^3p^2r^2x}{b^3}+\frac{5(bg-ah)^3pqr^2x}{8b^3}+\frac{5(bg-ah)^2(dg-ch)pqr^2x}{12b^2d}+\frac{5(bg-ah)(dg-ch)^2pqr^2x}{12bd^2}+\frac{5(dg-ch)}{8d^3}$$

```
[Out] 5/8*(-a*h+b*g)^3*p*q*r^2*x/b^3+5/8*(-c*h+d*g)^3*p*q*r^2*x/d^3+3/4*h*(-a*h+b
*g)^2*p^2*r^2*(b*x+a)^2/b^4+2/9*h^2*(-a*h+b*g)*p^2*r^2*(b*x+a)^3/b^4+3/4*h*
(-c*h+d*g)^2*q^2*r^2*(d*x+c)^2/d^4+2/9*h^2*(-c*h+d*g)*q^2*r^2*(d*x+c)^3/d^4
+1/8*p*r*(h*x+g)^4*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q
)^r))/h+1/8*q*r*(h*x+g)^4*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d
*x+c)^q)^r))/h+3/16*(-a*h+b*g)^2*p*q*r^2*(h*x+g)^2/b^2/h+3/16*(-c*h+d*g)^2*
p*q*r^2*(h*x+g)^2/d^2/h+7/72*(-a*h+b*g)*p*q*r^2*(h*x+g)^3/b/h+7/72*(-c*h+d*
g)*p*q*r^2*(h*x+g)^3/d/h+1/4*(h*x+g)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/h-
1/2*(-c*h+d*g)^4*p*q*r^2*polylog(2,-d*(b*x+a)/(-a*d+b*c))/d^4/h-1/2*(-a*h+b
*g)^4*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c))/b^4/h+2*(-a*h+b*g)^3*p^2*r^2*
x/b^3+2*(-c*h+d*g)^3*q^2*r^2*x/d^3+1/32*h^3*p^2*r^2*(b*x+a)^4/b^4+1/32*h^3*
q^2*r^2*(d*x+c)^4/d^4+1/16*p*q*r^2*(h*x+g)^4/h+1/8*(-a*h+b*g)^4*p*q*r^2*ln(
b*x+a)/b^4/h-3/2*h*(-a*h+b*g)^2*p^2*r^2*(b*x+a)^2*ln(b*x+a)/b^4-2/3*h^2*(-a
*h+b*g)*p^2*r^2*(b*x+a)^3*ln(b*x+a)/b^4+1/8*(-c*h+d*g)^4*p*q*r^2*ln(d*x+c)/
d^4/h-3/2*h*(-c*h+d*g)^2*q^2*r^2*(d*x+c)^2*ln(d*x+c)/d^4-2/3*h^2*(-c*h+d*g)
*q^2*r^2*(d*x+c)^3*ln(d*x+c)/d^4+1/4*(-a*h+b*g)^2*p*r*(h*x+g)^2*(p*r*ln(b*x
+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/b^2/h+1/4*(-c*h+d*g)^2*q
*r*(h*x+g)^2*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/
d^2/h+1/6*(-a*h+b*g)*p*r*(h*x+g)^3*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*
x+a)^p*(d*x+c)^q)^r))/b/h+1/6*(-c*h+d*g)*q*r*(h*x+g)^3*(p*r*ln(b*x+a)+q*r*l
n(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/d/h+1/2*(-a*h+b*g)^4*p*r*ln(b*x+a
)*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/b^4/h+1/2*(
-c*h+d*g)^4*q*r*ln(d*x+c)*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d
*x+c)^q)^r))/d^4/h+1/6*(-a*h+b*g)^3*(-c*h+d*g)*p*q*r^2*ln(b*x+a)/b^3/d/h+1/
4*(-a*h+b*g)^2*(-c*h+d*g)^2*p*q*r^2*ln(b*x+a)/b^2/d^2/h+1/4*(-a*h+b*g)^2*(-
c*h+d*g)^2*p*q*r^2*ln(d*x+c)/b^2/d^2/h+1/6*(-a*h+b*g)*(-c*h+d*g)^3*p*q*r^2*
ln(d*x+c)/b/d^3/h-2*(-a*h+b*g)^3*p^2*r^2*(b*x+a)*ln(b*x+a)/b^4-1/8*h^3*p^2*
r^2*(b*x+a)^4*ln(b*x+a)/b^4-1/8*p*q*r^2*(h*x+g)^4*ln(b*x+a)/h-1/4*(-a*h+b*g
)^4*p^2*r^2*ln(b*x+a)^2/b^4/h-2*(-c*h+d*g)^3*q^2*r^2*(d*x+c)*ln(d*x+c)/d^4-
1/8*h^3*q^2*r^2*(d*x+c)^4*ln(d*x+c)/d^4-1/8*p*q*r^2*(h*x+g)^4*ln(d*x+c)/h-1
/4*(-c*h+d*g)^4*q^2*r^2*ln(d*x+c)^2/d^4/h+1/2*(-a*h+b*g)^3*p*r*x*(p*r*ln(b*
x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/b^3+1/2*(-c*h+d*g)^3*q*
r*x*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/d^3-1/2*(
-c*h+d*g)^3*p*q*r^2*(b*x+a)*ln(b*x+a)/b/d^3-1/4*(-c*h+d*g)^2*p*q*r^2*(h*x+g
)^2*ln(b*x+a)/d^2/h-1/6*(-c*h+d*g)*p*q*r^2*(h*x+g)^3*ln(b*x+a)/d/h-1/2*(-a*
h+b*g)^3*p*q*r^2*(d*x+c)*ln(d*x+c)/b^3/d-1/4*(-a*h+b*g)^2*p*q*r^2*(h*x+g)^2
*ln(d*x+c)/b^2/h-1/6*(-a*h+b*g)*p*q*r^2*(h*x+g)^3*ln(d*x+c)/b/h-1/2*(-a*h+b
```

```
*g)^4*p*q*r^2*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/b^4/h-1/2*(-c*h+d*g)^4*p*
q*r^2*ln(b*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/d^4/h+1/6*(-a*h+b*g)*(-c*h+d*g)*p*
q*r^2*(h*x+g)^2/b/d/h+5/12*(-a*h+b*g)^2*(-c*h+d*g)*p*q*r^2*x/b^2/d+5/12*(-a
*h+b*g)*(-c*h+d*g)^2*p*q*r^2*x/b/d^2
```

Rubi [A]

time = 1.70, antiderivative size = 2240, normalized size of antiderivative = 1.00, number of steps used = 49, number of rules used = 14, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {2584, 2593, 2458, 45, 2372, 12, 2338, 2465, 2436, 2332, 2441, 2440, 2438, 2442}

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]

[Out] (2*(b*g - a*h)^3*p^2*r^2*x)/b^3 + (5*(b*g - a*h)^3*p*q*r^2*x)/(8*b^3) + (5*
(b*g - a*h)^2*(d*g - c*h)*p*q*r^2*x)/(12*b^2*d) + (5*(b*g - a*h)*(d*g - c*h
)^2*p*q*r^2*x)/(12*b*d^2) + (5*(d*g - c*h)^3*p*q*r^2*x)/(8*d^3) + (2*(d*g -
 c*h)^3*q^2*r^2*x)/d^3 + (3*h*(b*g - a*h)^2*p^2*r^2*(a + b*x)^2)/(4*b^4) +
(2*h^2*(b*g - a*h)*p^2*r^2*(a + b*x)^3)/(9*b^4) + (h^3*p^2*r^2*(a + b*x)^4)
/(32*b^4) + (3*h*(d*g - c*h)^2*q^2*r^2*(c + d*x)^2)/(4*d^4) + (2*h^2*(d*g -
 c*h)*q^2*r^2*(c + d*x)^3)/(9*d^4) + (h^3*q^2*r^2*(c + d*x)^4)/(32*d^4) + (
3*(b*g - a*h)^2*p*q*r^2*(g + h*x)^2)/(16*b^2*h) + ((b*g - a*h)*(d*g - c*h)*
p*q*r^2*(g + h*x)^2)/(6*b*d*h) + (3*(d*g - c*h)^2*p*q*r^2*(g + h*x)^2)/(16*
d^2*h) + (7*(b*g - a*h)*p*q*r^2*(g + h*x)^3)/(72*b*h) + (7*(d*g - c*h)*p*q*
r^2*(g + h*x)^3)/(72*d*h) + (p*q*r^2*(g + h*x)^4)/(16*h) + ((b*g - a*h)^4*p
*q*r^2*Log[a + b*x])/(8*b^4*h) + ((b*g - a*h)^3*(d*g - c*h)*p*q*r^2*Log[a +
 b*x])/(6*b^3*d*h) + ((b*g - a*h)^2*(d*g - c*h)^2*p*q*r^2*Log[a + b*x])/(4*
b^2*d^2*h) - (2*(b*g - a*h)^3*p^2*r^2*(a + b*x)*Log[a + b*x])/b^4 - ((d*g -
 c*h)^3*p*q*r^2*(a + b*x)*Log[a + b*x])/(2*b*d^3) - (3*h*(b*g - a*h)^2*p^2*
r^2*(a + b*x)^2*Log[a + b*x])/(2*b^4) - (2*h^2*(b*g - a*h)*p^2*r^2*(a + b*x
)^3*Log[a + b*x])/(3*b^4) - (h^3*p^2*r^2*(a + b*x)^4*Log[a + b*x])/(8*b^4)
- ((d*g - c*h)^2*p*q*r^2*(g + h*x)^2*Log[a + b*x])/(4*d^2*h) - ((d*g - c*h)
*p*q*r^2*(g + h*x)^3*Log[a + b*x])/(6*d*h) - (p*q*r^2*(g + h*x)^4*Log[a + b
*x])/(8*h) - ((b*g - a*h)^4*p^2*r^2*Log[a + b*x]^2)/(4*b^4*h) + ((b*g - a*h
)^2*(d*g - c*h)^2*p*q*r^2*Log[c + d*x])/(4*b^2*d^2*h) + ((b*g - a*h)*(d*g -
 c*h)^3*p*q*r^2*Log[c + d*x])/(6*b*d^3*h) + ((d*g - c*h)^4*p*q*r^2*Log[c +
d*x])/(8*d^4*h) - ((b*g - a*h)^3*p*q*r^2*(c + d*x)*Log[c + d*x])/(2*b^3*d)
- (2*(d*g - c*h)^3*q^2*r^2*(c + d*x)*Log[c + d*x])/d^4 - (3*h*(d*g - c*h)^2
*q^2*r^2*(c + d*x)^2*Log[c + d*x])/(2*d^4) - (2*h^2*(d*g - c*h)*q^2*r^2*(c
+ d*x)^3*Log[c + d*x])/(3*d^4) - (h^3*q^2*r^2*(c + d*x)^4*Log[c + d*x])/(8*
d^4) - ((b*g - a*h)^2*p*q*r^2*(g + h*x)^2*Log[c + d*x])/(4*b^2*h) - ((b*g -
 a*h)*p*q*r^2*(g + h*x)^3*Log[c + d*x])/(6*b*h) - (p*q*r^2*(g + h*x)^4*Log[
c + d*x])/(8*h) - ((b*g - a*h)^4*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*
```

```
Log[c + d*x])/(2*b^4*h) - ((d*g - c*h)^4*q^2*r^2*Log[c + d*x]^2)/(4*d^4*h)
- ((d*g - c*h)^4*p*q*r^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(2*d^
4*h) + ((b*g - a*h)^3*p*r*x*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]))/(2*b^3) + ((d*g - c*h)^3*q*r*x*(p*r*Log[a +
b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(2*d^3) +
((b*g - a*h)^2*p*r*(g + h*x)^2*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(4*b^2*h) + ((d*g - c*h)^2*q*r*(g + h*x)^
2*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^
r]))/(4*d^2*h) + ((b*g - a*h)*p*r*(g + h*x)^3*(p*r*Log[a + b*x] + q*r*Log[c
 + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(6*b*h) + ((d*g - c*h)*q*r
*(g + h*x)^3*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c
 + d*x)^q)^r]))/(6*d*h) + (p*r*(g + h*x)^4*(p*r*Log[a + b*x] + q*r*Log[c +
d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(8*h) + (q*r*(g + h*x)^4*(p*r
*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(
8*h) + ((b*g - a*h)^4*p*r*Log[a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x]
 - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(2*b^4*h) + ((d*g - c*h)^4*q*r*Lo
g[c + d*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]))/(2*d^4*h) + ((g + h*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^2)/(4*h) - ((d*g - c*h)^4*p*q*r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))]
)/(2*d^4*h) - ((b*g - a*h)^4*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])
/(2*b^4*h)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
```

```
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2584

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Dist[b*p*r*(
s/(h*(m + 1))), Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1)/(a + b*x)), x], x] - Dist[d*q*r*(s/(h*(m + 1))), Int[(g + h*x)^(m
+ 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\int (g+hx)^3 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \frac{(g+hx)^4 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4h} - \frac{(bpr)\int \frac{(g+hx)^4 \mathrm{l}}{}}{}$$

$$= \frac{(g+hx)^4 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4h} - \frac{(bp^2r^2)\int \frac{(g+hx)}{}}{2h}$$

$$= \frac{(g+hx)^4 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4h} - \frac{(p^2r^2)\,\mathrm{Subst}\Big(}{}$$

$$= -\frac{p^2r^2\log(a+bx)\left(\frac{48h(bg-ah)^3(a+bx)}{b^4} + \frac{36h^2(bg-ah)^2(a+bx)^2}{b^4} + \right.}{24h}$$

$$= -\frac{(dg-ch)^2pqr^2(g+hx)^2\log(a+bx)}{4d^2h} - \frac{(dg-ch)pqr^2(g}{6}$$

$$= \frac{2(bg-ah)^3p^2r^2x}{b^3} + \frac{(bg-ah)^3pqr^2x}{2b^3} + \frac{(dg-ch)^3pqr^2x}{2d^3}$$

$$= \frac{2(bg-ah)^3p^2r^2x}{b^3} + \frac{5(bg-ah)^3pqr^2x}{8b^3} + \frac{5(bg-ah)^2(dg-}{12b^2d}$$

<u>Mathematica</u> [A]
time = 1.98, size = 1386, normalized size = 0.62

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (72*a*d^4*(-4*b^3*g^3 + 6*a*b^2*g^2*h - 4*a^2*b*g*h^2 + a^3*h^3)*p^2*r^2*Lo
g[a + b*x]^2 + 12*p*r*Log[a + b*x]*(12*b^4*c*(-4*d^3*g^3 + 6*c*d^2*g^2*h -
4*c^2*d*g*h^2 + c^3*h^3)*q*r*Log[c + d*x] - 12*(4*a*b^3*d^4*g^3 - 6*a^2*b^2
*d^4*g^2*h + 4*a^3*b*d^4*g*h^2 - a^4*d^4*h^3 + b^4*c*(-4*d^3*g^3 + 6*c*d^2*
g^2*h - 4*c^2*d*g*h^2 + c^3*h^3))*q*r*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*
((12*b^3*(-4*d^3*g^3 + 6*c*d^2*g^2*h - 4*c^2*d*g*h^2 + c^3*h^3)*q + a^3*d^3
*h^3*(25*p + 3*q) - 4*a^2*b*d^2*h^2*(22*d*g*p + 4*d*g*q - c*h*q) + 6*a*b^2*
d*h*(-4*c*d*g*h*q + c^2*h^2*q + 6*d^2*g^2*(3*p + q)))*r + 12*d^3*(4*b^3*g^3
 - 6*a*b^2*g^2*h + 4*a^2*b*g*h^2 - a^3*h^3)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r])) + b*(72*b^3*c*(-4*d^3*g^3 + 6*c*d^2*g^2*h - 4*c^2*d*g*h^2 + c^3*h^3
)*q^2*r^2*Log[c + d*x]^2 + 12*q*r*Log[c + d*x]*((12*a^3*c*d^3*h^3*p + 6*a^2
*b*c*d^2*h^2*(-8*d*g + c*h)*p + 4*a*b^2*d*(12*d^3*g^3 + 18*c*d^2*g^2*h - 6*
c^2*d*g*h^2 + c^3*h^3)*p + b^3*c*(-48*d^3*g^3*(p + q) + 36*c*d^2*g^2*h*(p +
```

```
3*q) - 8*c^2*d*g*h^2*(2*p + 11*q) + c^3*h^3*(3*p + 25*q)))*r - 12*b^3*c*(-
4*d^3*g^3 + 6*c*d^2*g^2*h - 4*c^2*d*g*h^2 + c^3*h^3)*Log[e*(f*(a + b*x)^p*(
c + d*x)^q)^r]) + d*(r^2*(-60*a^3*d^3*h^3*p*(5*p + 3*q)*x + 6*a^2*b*d^2*h^2
*p*x*(-20*c*h*q + 16*d*g*(11*p + 8*q) + d*h*(13*p + 9*q)*x) + b^3*x*(-60*c^
3*h^3*q*(3*p + 5*q) + 6*c^2*d*h^2*q*(16*g*(8*p + 11*q) + h*(9*p + 13*q)*x)
- 4*c*d^2*h*q*(p + q)*(324*g^2 + 60*g*h*x + 7*h^2*x^2) + d^3*(p + q)^2*(576
*g^3 + 216*g^2*h*x + 64*g*h^2*x^2 + 9*h^3*x^3)) - 4*a*b^2*p*(36*c^3*h^3*q +
 6*c^2*d*h^2*q*(-24*g + 5*h*x) - 12*c*d^2*h*q*(-18*g^2 + 12*g*h*x + h^2*x^2
) + d^3*(-144*g^3*q + 324*g^2*h*(p + q)*x + 60*g*h^2*(p + q)*x^2 + 7*h^3*(p
 + q)*x^3))) + 12*r*(12*a^3*d^3*h^3*p*x - 6*a^2*b*d^3*h^2*p*x*(8*g + h*x) +
 4*a*b^2*d^3*p*(-12*g^3 + 18*g^2*h*x + 6*g*h^2*x^2 + h^3*x^3) - b^3*x*(-12*
c^3*h^3*q + 6*c^2*d*h^2*q*(8*g + h*x) - 4*c*d^2*h*q*(18*g^2 + 6*g*h*x + h^2
*x^2) + d^3*(p + q)*(48*g^3 + 36*g^2*h*x + 16*g*h^2*x^2 + 3*h^3*x^3)))*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 72*b^3*d^3*x*(4*g^3 + 6*g^2*h*x + 4*g*h^
2*x^2 + h^3*x^3)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)) - 144*(4*a*b^3*d^
4*g^3 - 6*a^2*b^2*d^4*g^2*h + 4*a^3*b*d^4*g*h^2 - a^4*d^4*h^3 + b^4*c*(-4*d
^3*g^3 + 6*c*d^2*g^2*h - 4*c^2*d*g*h^2 + c^3*h^3))*p*q*r^2*PolyLog[2, (d*(a
 + b*x))/(-(b*c) + a*d)])/(288*b^4*d^4)
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \left(hx+g\right)^{3} \ln \left(e {\left(f \left(bx+a\right)^{p} \left(dx+c\right)^{q}\right)}^{r}\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

```
[Out] int((h*x+g)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

Maxima [A]
time = 0.36, size = 1801, normalized size = 0.80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima
")
```

```
[Out] 1/4*(h^3*x^4 + 4*g*h^2*x^3 + 6*g^2*h*x^2 + 4*g^3*x)*log(((b*x + a)^p*(d*x +
 c)^q*f)^r*e)^2 + 1/24*r*(12*(4*a*b^3*f*g^3*p - 6*a^2*b^2*f*g^2*h*p + 4*a^3
*b*f*g*h^2*p - a^4*f*h^3*p)*log(b*x + a)/b^4 + 12*(4*c*d^3*f*g^3*q - 6*c^2*
d^2*f*g^2*h*q + 4*c^3*d*f*g*h^2*q - c^4*f*h^3*q)*log(d*x + c)/d^4 - (3*b^3*
d^3*f*h^3*(p + q)*x^4 - 4*(a*b^2*d^3*f*h^3*p - (4*d^3*f*g*h^2*(p + q) - c*d
^2*f*h^3*q)*b^3)*x^3 - 6*(4*a*b^2*d^3*f*g*h^2*p - a^2*b*d^3*f*h^3*p - (6*d^
3*f*g^2*h*(p + q) - 4*c*d^2*f*g*h^2*q + c^2*d*f*h^3*q)*b^3)*x^2 - 12*(6*a*b
```

```
^2*d^3*f*g^2*h*p - 4*a^2*b*d^3*f*g*h^2*p + a^3*d^3*f*h^3*p - (4*d^3*f*g^3*(
p + q) - 6*c*d^2*f*g^2*h*q + 4*c^2*d*f*g*h^2*q - c^3*f*h^3*q)*b^3)*x)/(b^3*
d^3))*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/f + 1/288*r^2*(12*(12*a^3*c*d^3*
f^2*h^3*p*q - 6*(8*c*d^3*f^2*g*h^2*p*q - c^2*d^2*f^2*h^3*p*q)*a^2*b + 4*(18
*c*d^3*f^2*g^2*h*p*q - 6*c^2*d^2*f^2*g*h^2*p*q + c^3*d*f^2*h^3*p*q)*a*b^2 -
 (48*(p*q + q^2)*c*d^3*f^2*g^3 - 36*(p*q + 3*q^2)*c^2*d^2*f^2*g^2*h + 8*(2*
p*q + 11*q^2)*c^3*d*f^2*g*h^2 - (3*p*q + 25*q^2)*c^4*f^2*h^3)*b^3)*log(d*x
+ c)/(b^3*d^4) - 144*(4*a*b^3*d^4*f^2*g^3*p*q - 6*a^2*b^2*d^4*f^2*g^2*h*p*q
 + 4*a^3*b*d^4*f^2*g*h^2*p*q - a^4*d^4*f^2*h^3*p*q - (4*c*d^3*f^2*g^3*p*q -
 6*c^2*d^2*f^2*g^2*h*p*q + 4*c^3*d*f^2*g*h^2*p*q - c^4*f^2*h^3*p*q)*b^4)*(l
og(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c
- a*d)))/(b^4*d^4) + (9*(p^2 + 2*p*q + q^2)*b^4*d^4*f^2*h^3*x^4 - 144*(4*c*
d^3*f^2*g^3*p*q - 6*c^2*d^2*f^2*g^2*h*p*q + 4*c^3*d*f^2*g*h^2*p*q - c^4*f^2
*h^3*p*q)*b^4*log(b*x + a)*log(d*x + c) - 72*(4*c*d^3*f^2*g^3*q^2 - 6*c^2*d
^2*f^2*g^2*h*q^2 + 4*c^3*d*f^2*g*h^2*q^2 - c^4*f^2*h^3*q^2)*b^4*log(d*x + c
)^2 - 4*(7*(p^2 + p*q)*a*b^3*d^4*f^2*h^3 - (16*(p^2 + 2*p*q + q^2)*d^4*f^2*
g*h^2 - 7*(p*q + q^2)*c*d^3*f^2*h^3)*b^4)*x^3 + 6*((13*p^2 + 9*p*q)*a^2*b^2
*d^4*f^2*h^3 + 8*(c*d^3*f^2*h^3*p*q - 5*(p^2 + p*q)*d^4*f^2*g*h^2)*a*b^3 +
(36*(p^2 + 2*p*q + q^2)*d^4*f^2*g^2*h - 40*(p*q + q^2)*c*d^3*f^2*g*h^2 + (9
*p*q + 13*q^2)*c^2*d^2*f^2*h^3)*b^4)*x^2 - 72*(4*a*b^3*d^4*f^2*g^3*p^2 - 6*
a^2*b^2*d^4*f^2*g^2*h*p^2 + 4*a^3*b*d^4*f^2*g*h^2*p^2 - a^4*d^4*f^2*h^3*p^2
)*log(b*x + a)^2 - 12*(5*(5*p^2 + 3*p*q)*a^3*b*d^4*f^2*h^3 + 2*(5*c*d^3*f^2
*h^3*p*q - 4*(11*p^2 + 8*p*q)*d^4*f^2*g*h^2)*a^2*b^2 - 2*(24*c*d^3*f^2*g*h^
2*p*q - 5*c^2*d^2*f^2*h^3*p*q - 54*(p^2 + p*q)*d^4*f^2*g^2*h)*a*b^3 - (48*(
p^2 + 2*p*q + q^2)*d^4*f^2*g^3 - 108*(p*q + q^2)*c*d^3*f^2*g^2*h + 8*(8*p*q
 + 11*q^2)*c^2*d^2*f^2*g*h^2 - 5*(3*p*q + 5*q^2)*c^3*d*f^2*h^3)*b^4)*x + 12
*((25*p^2 + 3*p*q)*a^4*d^4*f^2*h^3 + 4*(c*d^3*f^2*h^3*p*q - 2*(11*p^2 + 2*p
*q)*d^4*f^2*g*h^2)*a^3*b - 6*(4*c*d^3*f^2*g*h^2*p*q - c^2*d^2*f^2*h^3*p*q -
 6*(3*p^2 + p*q)*d^4*f^2*g^2*h)*a^2*b^2 + 12*(6*c*d^3*f^2*g^2*h*p*q - 4*c^2
*d^2*f^2*g*h^2*p*q + c^3*d*f^2*h^3*p*q - 4*(p^2 + p*q)*d^4*f^2*g^3)*a*b^3)*
log(b*x + a))/(b^4*d^4))/f^2
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas
")
```

```
[Out] integral((h^3*x^3 + 3*g*h^2*x^2 + 3*g^2*h*x + g^3)*log(((b*x + a)^p*(d*x +
c)^q*f)^r*e)^2, x)
```

Sympy [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(e\left(f\left(a+b\,x\right)^p\left(c+d\,x\right)^q\right)^r\right)^2\left(g+h\,x\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(g + h*x)^3,x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(g + h*x)^3, x)
```

3.36 $\int (g+hx)^2 \log^2 \left(e(f(a+bx)^p(c+dx)^q)^r\right) dx$

Optimal. Leaf size=1645

$$\frac{2(bg-ah)^2p^2r^2x}{b^2}+\frac{8(bg-ah)^2pqr^2x}{9b^2}+\frac{2(bg-ah)(dg-ch)pqr^2x}{3bd}+\frac{8(dg-ch)^2pqr^2x}{9d^2}+\frac{2(dg-ch)^2q^2r^2x}{d^2}+$$

```
[Out] 8/9*(-a*h+b*g)^2*p*q*r^2*x/b^2+8/9*(-c*h+d*g)^2*p*q*r^2*x/d^2+1/2*h*(-a*h+b
*g)*p^2*r^2*(b*x+a)^2/b^3+1/2*h*(-c*h+d*g)*q^2*r^2*(d*x+c)^2/d^3+2/9*p*r*(h
*x+g)^3*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h+2/9
*q*r*(h*x+g)^3*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)
)/h+1/3*(h*x+g)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/h+5/18*(-a*h+b*g)*p*q*r
^2*(h*x+g)^2/b/h+5/18*(-c*h+d*g)*p*q*r^2*(h*x+g)^2/d/h-2/3*(-c*h+d*g)^3*p*q
*r^2*polylog(2,-d*(b*x+a)/(-a*d+b*c))/d^3/h-2/3*(-a*h+b*g)^3*p*q*r^2*polylo
g(2,b*(d*x+c)/(-a*d+b*c))/b^3/h+2*(-a*h+b*g)^2*p^2*r^2*x/b^2+2*(-c*h+d*g)^2
*q^2*r^2*x/d^2+2/27*h^2*p^2*r^2*(b*x+a)^3/b^3+2/27*h^2*q^2*r^2*(d*x+c)^3/d^
3+4/27*p*q*r^2*(h*x+g)^3/h+2/9*(-a*h+b*g)^3*p*q*r^2*ln(b*x+a)/b^3/h+2/9*(-c
*h+d*g)^3*p*q*r^2*ln(d*x+c)/d^3/h+1/3*(-a*h+b*g)*p*r*(h*x+g)^2*(p*r*ln(b*x+
a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/b/h+1/3*(-c*h+d*g)*q*r*(h
*x+g)^2*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/d/h+2
/3*(-a*h+b*g)^3*p*r*ln(b*x+a)*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^
p*(d*x+c)^q)^r))/b^3/h+2/3*(-c*h+d*g)^3*q*r*ln(d*x+c)*(p*r*ln(b*x+a)+q*r*ln
(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/d^3/h-h*(-a*h+b*g)*p^2*r^2*(b*x+a)
^2*ln(b*x+a)/b^3-h*(-c*h+d*g)*q^2*r^2*(d*x+c)^2*ln(d*x+c)/d^3+1/3*(-a*h+b*g
)^2*(-c*h+d*g)*p*q*r^2*ln(b*x+a)/b^2/d/h+1/3*(-a*h+b*g)*(-c*h+d*g)^2*p*q*r^
2*ln(d*x+c)/b/d^2/h-2*(-a*h+b*g)^2*p^2*r^2*(b*x+a)*ln(b*x+a)/b^3-2/9*h^2*p^
2*r^2*(b*x+a)^3*ln(b*x+a)/b^3-2/9*p*q*r^2*(h*x+g)^3*ln(b*x+a)/h-1/3*(-a*h+b
*g)^3*p^2*r^2*ln(b*x+a)^2/b^3/h-2*(-c*h+d*g)^2*q^2*r^2*(d*x+c)*ln(d*x+c)/d^
3-2/9*h^2*q^2*r^2*(d*x+c)^3*ln(d*x+c)/d^3-2/9*p*q*r^2*(h*x+g)^3*ln(d*x+c)/h
-1/3*(-c*h+d*g)^3*q^2*r^2*ln(d*x+c)^2/d^3/h+2/3*(-a*h+b*g)^2*p*r*x*(p*r*ln(
b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/b^2+2/3*(-c*h+d*g)^2*
q*r*x*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/d^2-2/3
*(-c*h+d*g)^2*p*q*r^2*(b*x+a)*ln(b*x+a)/b/d^2-1/3*(-c*h+d*g)*p*q*r^2*(h*x+g
)^2*ln(b*x+a)/d/h-2/3*(-a*h+b*g)^2*p*q*r^2*(d*x+c)*ln(d*x+c)/b^2/d-1/3*(-a*
h+b*g)*p*q*r^2*(h*x+g)^2*ln(d*x+c)/b/h-2/3*(-a*h+b*g)^3*p*q*r^2*ln(-d*(b*x+
a)/(-a*d+b*c))*ln(d*x+c)/b^3/h-2/3*(-c*h+d*g)^3*p*q*r^2*ln(b*x+a)*ln(b*(d*x
+c)/(-a*d+b*c))/d^3/h+2/3*(-a*h+b*g)*(-c*h+d*g)*p*q*r^2*x/b/d
```

Rubi [A]

time = 1.17, antiderivative size = 1645, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 15, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.484$, Rules used = {2584, 2593, 2458, 45, 2372, 12, 14, 2338, 2465, 2436, 2332, 2441, 2440, 2438, 2442}

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (2*(b*g - a*h)^2*p^2*r^2*x)/b^2 + (8*(b*g - a*h)^2*p*q*r^2*x)/(9*b^2) + (2*
(b*g - a*h)*(d*g - c*h)*p*q*r^2*x)/(3*b*d) + (8*(d*g - c*h)^2*p*q*r^2*x)/(9
*d^2) + (2*(d*g - c*h)^2*q^2*r^2*x)/d^2 + (h*(b*g - a*h)*p^2*r^2*(a + b*x)^
2)/(2*b^3) + (2*h^2*p^2*r^2*(a + b*x)^3)/(27*b^3) + (h*(d*g - c*h)*q^2*r^2*
(c + d*x)^2)/(2*d^3) + (2*h^2*q^2*r^2*(c + d*x)^3)/(27*d^3) + (5*(b*g - a*h
)*p*q*r^2*(g + h*x)^2)/(18*b*h) + (5*(d*g - c*h)*p*q*r^2*(g + h*x)^2)/(18*d
*h) + (4*p*q*r^2*(g + h*x)^3)/(27*h) + (2*(b*g - a*h)^3*p*q*r^2*Log[a + b*x
])/(9*b^3*h) + ((b*g - a*h)^2*(d*g - c*h)*p*q*r^2*Log[a + b*x])/(3*b^2*d*h)
 - (2*(b*g - a*h)^2*p^2*r^2*(a + b*x)*Log[a + b*x])/b^3 - (2*(d*g - c*h)^2*
p*q*r^2*(a + b*x)*Log[a + b*x])/(3*b*d^2) - (h*(b*g - a*h)*p^2*r^2*(a + b*x
)^2*Log[a + b*x])/b^3 - (2*h^2*p^2*r^2*(a + b*x)^3*Log[a + b*x])/(9*b^3) -
((d*g - c*h)*p*q*r^2*(g + h*x)^2*Log[a + b*x])/(3*d*h) - (2*p*q*r^2*(g + h*
x)^3*Log[a + b*x])/(9*h) - ((b*g - a*h)^3*p^2*r^2*Log[a + b*x]^2)/(3*b^3*h)
 + ((b*g - a*h)*(d*g - c*h)^2*p*q*r^2*Log[c + d*x])/(3*b*d^2*h) + (2*(d*g -
 c*h)^3*p*q*r^2*Log[c + d*x])/(9*d^3*h) - (2*(b*g - a*h)^2*p*q*r^2*(c + d*x
)*Log[c + d*x])/(3*b^2*d) - (2*(d*g - c*h)^2*q^2*r^2*(c + d*x)*Log[c + d*x]
)/d^3 - (h*(d*g - c*h)*q^2*r^2*(c + d*x)^2*Log[c + d*x])/d^3 - (2*h^2*q^2*r
^2*(c + d*x)^3*Log[c + d*x])/(9*d^3) - ((b*g - a*h)*p*q*r^2*(g + h*x)^2*Log
[c + d*x])/(3*b*h) - (2*p*q*r^2*(g + h*x)^3*Log[c + d*x])/(9*h) - (2*(b*g -
 a*h)^3*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(3*b^3*h) -
 ((d*g - c*h)^3*q^2*r^2*Log[c + d*x]^2)/(3*d^3*h) - (2*(d*g - c*h)^3*p*q*r^
2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(3*d^3*h) + (2*(b*g - a*h)^2
*p*r*x*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x
)^q)^r]))/(3*b^2) + (2*(d*g - c*h)^2*q*r*x*(p*r*Log[a + b*x] + q*r*Log[c +
d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(3*d^2) + ((b*g - a*h)*p*r*(g
 + h*x)^2*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]))/(3*b*h) + ((d*g - c*h)*q*r*(g + h*x)^2*(p*r*Log[a + b*x] + q*r
*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(3*d*h) + (2*p*r*(g
+ h*x)^3*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]))/(9*h) + (2*q*r*(g + h*x)^3*(p*r*Log[a + b*x] + q*r*Log[c + d*x]
 - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(9*h) + (2*(b*g - a*h)^3*p*r*Log[
a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]))/(3*b^3*h) + (2*(d*g - c*h)^3*q*r*Log[c + d*x]*(p*r*Log[a + b*x]
 + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(3*d^3*h) + ((
g + h*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/(3*h) - (2*(d*g - c*h)^3
*p*q*r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(3*d^3*h) - (2*(b*g - a*
h)^3*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(3*b^3*h)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2584

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Dist[b*p*r*(
s/(h*(m + 1))), Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1)/(a + b*x)), x], x] - Dist[d*q*r*(s/(h*(m + 1))), Int[(g + h*x)^(m
+ 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
```

```
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\int (g+hx)^2 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) dx = \frac{(g+hx)^3 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h} - \frac{(2bpr)\int \frac{(g+hx)^3 \cdots}{\cdots}}{\cdots}$$

$$= \frac{(g+hx)^3 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h} - \frac{(2bp^2r^2)\int \frac{(g+h \cdots}{\cdots}}{3h \cdots}$$

$$= \frac{(g+hx)^3 \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h} - \frac{(2p^2r^2)\,\text{Subst}\left(\cdots\right)}{\cdots}$$

$$= -\frac{p^2r^2\log(a+bx)\left(\frac{18h(bg-ah)^2(a+bx)}{b^3} + \frac{9h^2(bg-ah)(a+bx)^2}{b^3} + \frac{2h \cdots}{\cdots}\right)}{9h}$$

$$= -\frac{(dg-ch)pqr^2(g+hx)^2\log(a+bx)}{3dh} - \frac{2pqr^2(g+hx)^3 \log \cdots}{9h}$$

$$= \frac{2(bg-ah)^2pqr^2x}{3b^2} + \frac{2(dg-ch)^2pqr^2x}{3d^2} - \frac{2(dg-ch)^2pqr^2 \cdots}{3 \cdots}$$

$$= \frac{8(bg-ah)^2pqr^2x}{9b^2} + \frac{2(bg-ah)(dg-ch)pqr^2x}{3bd} + \frac{8(dg- \cdots}{9 \cdots}$$

$$= \frac{2(bg-ah)^2p^2r^2x}{b^2} + \frac{8(bg-ah)^2pqr^2x}{9b^2} + \frac{2(bg-ah)(dg- \cdots}{3bd}$$

Mathematica [A]
time = 1.21, size = 899, normalized size = 0.55

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (-18*a*d^3*(3*b^2*g^2 - 3*a*b*g*h + a^2*h^2)*p^2*r^2*Log[a + b*x]^2 - 6*p*r
*Log[a + b*x]*(6*b^3*c*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2)*q*r*Log[c + d*x] -
 6*(b*c - a*d)*(a^2*d^2*h^2 + a*b*d*h*(-3*d*g + c*h) + b^2*(3*d^2*g^2 - 3*c
```

```
*d*g*h + c^2*h^2))*q*r*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*((6*b^2*(3*d^2*
g^2 - 3*c*d*g*h + c^2*h^2)*q + a^2*d^2*h^2*(11*p + 2*q) - 3*a*b*d*h*(-(c*h*
q) + 3*d*g*(3*p + q)))*r - 6*d^2*(3*b^2*g^2 - 3*a*b*g*h + a^2*h^2)*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r])) + b*(-18*b^2*c*(3*d^2*g^2 - 3*c*d*g*h + c^2*
h^2)*q^2*r^2*Log[c + d*x]^2 - 6*q*r*Log[c + d*x]*((6*a^2*c*d^2*h^2*p - 3*a*
b*d*(6*d^2*g^2 + 6*c*d*g*h - c^2*h^2)*p + b^2*c*(18*d^2*g^2*(p + q) - 9*c*d
*g*h*(p + 3*q) + c^2*h^2*(2*p + 11*q)))*r - 6*b^2*c*(3*d^2*g^2 - 3*c*d*g*h
+ c^2*h^2)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) + d*(r^2*(6*a^2*d^2*h^2*p*
(11*p + 8*q)*x + b^2*x*(6*c^2*h^2*q*(8*p + 11*q) - 3*c*d*h*q*(p + q)*(54*g
+ 5*h*x) + d^2*(p + q)^2*(108*g^2 + 27*g*h*x + 4*h^2*x^2)) - 3*a*b*p*(-12*c
^2*h^2*q - 12*c*d*h*q*(-3*g + h*x) + d^2*(-36*g^2*q + 54*g*h*(p + q)*x + 5*
h^2*(p + q)*x^2))) - 6*r*(6*a^2*d^2*h^2*p*x + 3*a*b*d^2*p*(6*g^2 - 6*g*h*x
- h^2*x^2) + b^2*x*(6*c^2*h^2*q - 3*c*d*h*q*(6*g + h*x) + d^2*(p + q)*(18*g
^2 + 9*g*h*x + 2*h^2*x^2)))*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 18*b^2*d
^2*x*(3*g^2 + 3*g*h*x + h^2*x^2)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)) +
 36*(b*c - a*d)*(a^2*d^2*h^2 + a*b*d*h*(-3*d*g + c*h) + b^2*(3*d^2*g^2 - 3*
c*d*g*h + c^2*h^2))*p*q*r^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(54*b
^3*d^3)
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int (hx+g)^2 \ln\left(e(f(bx+a)^p\,(dx+c)^q)^r\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

```
[Out] int((h*x+g)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

Maxima [A]
time = 0.34, size = 1125, normalized size = 0.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima
")
```

```
[Out] 1/3*(h^2*x^3 + 3*g*h*x^2 + 3*g^2*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2
+ 1/9*r*(6*(3*a*b^2*f*g^2*p - 3*a^2*b*f*g*h*p + a^3*f*h^2*p)*log(b*x + a)/b
^3 + 6*(3*c*d^2*f*g^2*q - 3*c^2*d*f*g*h*q + c^3*f*h^2*q)*log(d*x + c)/d^3 -
 (2*b^2*d^2*f*h^2*(p + q)*x^3 - 3*(a*b*d^2*f*h^2*p - (3*d^2*f*g*h*(p + q) -
 c*d*f*h^2*q)*b^2)*x^2 - 6*(3*a*b*d^2*f*g*h*p - a^2*d^2*f*h^2*p - (3*d^2*f*
g^2*(p + q) - 3*c*d*f*g*h*q + c^2*f*h^2*q)*b^2)*x)/(b^2*d^2))*log(((b*x + a
)^p*(d*x + c)^q*f)^r*e)/f - 1/54*r^2*(6*(6*a^2*c*d^2*f^2*h^2*p*q - 3*(6*c*d
```

```
^2*f^2*g*h*p*q - c^2*d*f^2*h^2*p*q)*a*b + (18*(p*q + q^2)*c*d^2*f^2*g^2 - 9
*(p*q + 3*q^2)*c^2*d*f^2*g*h + (2*p*q + 11*q^2)*c^3*f^2*h^2)*b^2)*log(d*x +
 c)/(b^2*d^3) + 36*(3*a*b^2*d^3*f^2*g^2*p*q - 3*a^2*b*d^3*f^2*g*h*p*q + a^3
*d^3*f^2*h^2*p*q - (3*c*d^2*f^2*g^2*p*q - 3*c^2*d*f^2*g*h*p*q + c^3*f^2*h^2
*p*q)*b^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x
 + a*d)/(b*c - a*d)))/(b^3*d^3) - (4*(p^2 + 2*p*q + q^2)*b^3*d^3*f^2*h^2*x^
3 - 36*(3*c*d^2*f^2*g^2*p*q - 3*c^2*d*f^2*g*h*p*q + c^3*f^2*h^2*p*q)*b^3*lo
g(b*x + a)*log(d*x + c) - 18*(3*c*d^2*f^2*g^2*q^2 - 3*c^2*d*f^2*g*h*q^2 + c
^3*f^2*h^2*q^2)*b^3*log(d*x + c)^2 - 3*(5*(p^2 + p*q)*a*b^2*d^3*f^2*h^2 - (
9*(p^2 + 2*p*q + q^2)*d^3*f^2*g*h - 5*(p*q + q^2)*c*d^2*f^2*h^2)*b^3)*x^2 -
 18*(3*a*b^2*d^3*f^2*g^2*p^2 - 3*a^2*b*d^3*f^2*g*h*p^2 + a^3*d^3*f^2*h^2*p^
2)*log(b*x + a)^2 + 6*((11*p^2 + 8*p*q)*a^2*b*d^3*f^2*h^2 + 3*(2*c*d^2*f^2*
h^2*p*q - 9*(p^2 + p*q)*d^3*f^2*g*h)*a*b^2 + (18*(p^2 + 2*p*q + q^2)*d^3*f^
2*g^2 - 27*(p*q + q^2)*c*d^2*f^2*g*h + (8*p*q + 11*q^2)*c^2*d*f^2*h^2)*b^3)
*x - 6*((11*p^2 + 2*p*q)*a^3*d^3*f^2*h^2 + 3*(c*d^2*f^2*h^2*p*q - 3*(3*p^2
+ p*q)*d^3*f^2*g*h)*a^2*b - 6*(3*c*d^2*f^2*g*h*p*q - c^2*d*f^2*h^2*p*q - 3*
(p^2 + p*q)*d^3*f^2*g^2)*a*b^2)*log(b*x + a))/(b^3*d^3))/f^2
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas
")
```

```
[Out] integral((h^2*x^2 + 2*g*h*x + g^2)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2,
x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(e\left(f\left(a+b\,x\right)^p\left(c+d\,x\right)^q\right)^r\right)^2\left(g+h\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(g + h*x)^2,x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(g + h*x)^2, x)
```

3.37 $\int (g+hx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$

Optimal. Leaf size=1063

$$\frac{3(bg-ah)pqr^2x}{2b}+\frac{3(dg-ch)pqr^2x}{2d}+\frac{pqr^2(g+hx)^2}{2h}+\frac{p^2r^2(4bg-3ah+bhx)^2}{4b^2h}+\frac{q^2r^2(4dg-3ch+dhx)^2}{4d^2h}+($$

```
[Out] 1/2*p*r*(h*x+g)^2*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)
^r))/h+1/2*q*r*(h*x+g)^2*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*
x+c)^q)^r))/h+1/2*(h*x+g)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/h+3/2*(-a*h+b
*g)*p*q*r^2*x/b+3/2*(-c*h+d*g)*p*q*r^2*x/d+1/2*p*q*r^2*(h*x+g)^2/h+1/4*p^2*
r^2*(b*h*x-3*a*h+4*b*g)^2/b^2/h+1/4*q^2*r^2*(d*h*x-3*c*h+4*d*g)^2/d^2/h+1/2
*(-a*h+b*g)^2*p*q*r^2*ln(b*x+a)/b^2/h+1/2*(-c*h+d*g)^2*p*q*r^2*ln(d*x+c)/d^
2/h+(-a*h+b*g)^2*p*r*ln(b*x+a)*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)
^p*(d*x+c)^q)^r))/b^2/h+(-c*h+d*g)^2*q*r*ln(d*x+c)*(p*r*ln(b*x+a)+q*r*ln(d*
x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/d^2/h-(-c*h+d*g)^2*p*q*r^2*polylog(2,
-d*(b*x+a)/(-a*d+b*c))/d^2/h-(-a*h+b*g)^2*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d
+b*c))/b^2/h-1/2*p*q*r^2*(h*x+g)^2*ln(b*x+a)/h-1/2*(-a*h+b*g)^2*p^2*r^2*ln(
b*x+a)^2/b^2/h-2*(-c*h+d*g)*q^2*r^2*(d*x+c)*ln(d*x+c)/d^2-1/2*h*q^2*r^2*(d*
x+c)^2*ln(d*x+c)/d^2-1/2*p*q*r^2*(h*x+g)^2*ln(d*x+c)/h-1/2*(-c*h+d*g)^2*q^2
*r^2*ln(d*x+c)^2/d^2/h+(-a*h+b*g)*p*r*x*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(
f*(b*x+a)^p*(d*x+c)^q)^r))/b+(-c*h+d*g)*q*r*x*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-
ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/d-2*(-a*h+b*g)*p^2*r^2*(b*x+a)*ln(b*x+a)/b
^2-1/2*h*p^2*r^2*(b*x+a)^2*ln(b*x+a)/b^2-(-c*h+d*g)*p*q*r^2*(b*x+a)*ln(b*x+
a)/b/d-(-a*h+b*g)*p*q*r^2*(d*x+c)*ln(d*x+c)/b/d-(-a*h+b*g)^2*p*q*r^2*ln(-d*
(b*x+a)/(-a*d+b*c))*ln(d*x+c)/b^2/h-(-c*h+d*g)^2*p*q*r^2*ln(b*x+a)*ln(b*(d*
x+c)/(-a*d+b*c))/d^2/h
```

Rubi [A]
time = 0.78, antiderivative size = 1063, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 15, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.517$, Rules used = {2584, 2593, 2458, 45, 2372, 12, 14, 2338, 2465, 2436, 2332, 2441, 2440, 2438, 2442}

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (3*(b*g - a*h)*p*q*r^2*x)/(2*b) + (3*(d*g - c*h)*p*q*r^2*x)/(2*d) + (p*q*r^
2*(g + h*x)^2)/(2*h) + (p^2*r^2*(4*b*g - 3*a*h + b*h*x)^2)/(4*b^2*h) + (q^2
*r^2*(4*d*g - 3*c*h + d*h*x)^2)/(4*d^2*h) + ((b*g - a*h)^2*p*q*r^2*Log[a +
b*x])/(2*b^2*h) - (2*(b*g - a*h)*p^2*r^2*(a + b*x)*Log[a + b*x])/b^2 - ((d*
g - c*h)*p*q*r^2*(a + b*x)*Log[a + b*x])/(b*d) - (h*p^2*r^2*(a + b*x)^2*Log
[a + b*x])/(2*b^2) - (p*q*r^2*(g + h*x)^2*Log[a + b*x])/(2*h) - ((b*g - a*h
```

```
)^2*p^2*r^2*Log[a + b*x]^2)/(2*b^2*h) + ((d*g - c*h)^2*p*q*r^2*Log[c + d*x]
)/(2*d^2*h) - ((b*g - a*h)*p*q*r^2*(c + d*x)*Log[c + d*x])/(b*d) - (2*(d*g
- c*h)*q^2*r^2*(c + d*x)*Log[c + d*x])/d^2 - (h*q^2*r^2*(c + d*x)^2*Log[c +
 d*x])/(2*d^2) - (p*q*r^2*(g + h*x)^2*Log[c + d*x])/(2*h) - ((b*g - a*h)^2*
p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b^2*h) - ((d*g - c
*h)^2*q^2*r^2*Log[c + d*x]^2)/(2*d^2*h) - ((d*g - c*h)^2*p*q*r^2*Log[a + b*
x]*Log[(b*(c + d*x))/(b*c - a*d)])/(d^2*h) + ((b*g - a*h)*p*r*x*(p*r*Log[a
+ b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/b + ((d*
g - c*h)*q*r*x*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*
(c + d*x)^q)^r]))/d + (p*r*(g + h*x)^2*(p*r*Log[a + b*x] + q*r*Log[c + d*x]
 - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(2*h) + (q*r*(g + h*x)^2*(p*r*Log
[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(2*h)
 + ((b*g - a*h)^2*p*r*Log[a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - L
og[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(b^2*h) + ((d*g - c*h)^2*q*r*Log[c +
d*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]))/(d^2*h) + ((g + h*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/(2*h
) - ((d*g - c*h)^2*p*q*r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(d^2*h
) - ((b*g - a*h)^2*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b^2*h)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
```

```
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2584

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Dist[b*p*r*(
s/(h*(m + 1))), Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1)/(a + b*x)), x], x] - Dist[d*q*r*(s/(h*(m + 1))), Int[(g + h*x)^(m
+ 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\int (g+hx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \frac{(g+hx)^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h} - \frac{(bpr)\int \frac{(g+hx)^2 \log \cdots}{\cdots}}{\cdots}$$

$$= \frac{(g+hx)^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h} - \frac{(bp^2r^2)\int \frac{(g+hx)^2 \cdots}{a \cdots}}{h}$$

$$= \frac{(g+hx)^2\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h} - \frac{(p^2r^2)\,\mathrm{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{p^2r^2\log(a+bx)\left(\frac{4h(bg-ah)(a+bx)}{b^2}+\frac{h^2(a+bx)^2}{b^2}+\frac{2(bg-ah)^2\log(\cdots}{b^2}\cdots\right.}{2h}$$

$$= -\frac{pqr^2(g+hx)^2\log(a+bx)}{2h} - \frac{p^2r^2\log(a+bx)\left(\frac{4h(bg-ah)(a\cdots}{b^2}\cdots\right.}{\cdots}$$

$$= \frac{(bg-ah)pqr^2x}{b} + \frac{(dg-ch)pqr^2x}{d} - \frac{(dg-ch)pqr^2(a+bx\cdots}{bd}$$

$$= \frac{3(bg-ah)pqr^2x}{2b} + \frac{3(dg-ch)pqr^2x}{2d} + \frac{pqr^2(g+hx)^2}{2h} + \frac{p^2\cdots}{\cdots}$$

$$= \frac{3(bg-ah)pqr^2x}{2b} + \frac{3(dg-ch)pqr^2x}{2d} + \frac{pqr^2(g+hx)^2}{2h} + \frac{p^2\cdots}{\cdots}$$

Mathematica [A]
time = 0.56, size = 480, normalized size = 0.45

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (2*a*d^2*(-2*b*g + a*h)*p^2*r^2*Log[a + b*x]^2 + 2*p*r*Log[a + b*x]*(2*b^2*
c*(-2*d*g + c*h)*q*r*Log[c + d*x] - 2*(b*c - a*d)*(-2*b*d*g + b*c*h + a*d*h
)*q*r*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*(2*b*(-2*d*g + c*h)*q*r + a*d*h*
(3*p + q)*r + (4*b*d*g - 2*a*d*h)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])) +
b*(2*b*c*(-2*d*g + c*h)*q^2*r^2*Log[c + d*x]^2 + 2*q*r*Log[c + d*x]*(2*a*d*
(2*d*g + c*h)*p*r + b*c*(-4*d*g*(p + q) + c*h*(p + 3*q))*r - 2*b*c*(-2*d*g
+ c*h)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) + d*(r^2*(-2*a*p*(-4*d*g*q + 2
*c*h*q + 3*d*h*(p + q)*x) + b*(p + q)*x*(-6*c*h*q + d*(p + q)*(8*g + h*x)))
 - 2*r*(2*a*d*p*(2*g - h*x) + b*x*(-2*c*h*q + d*(p + q)*(4*g + h*x)))*Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r] + 2*b*d*x*(2*g + h*x)*Log[e*(f*(a + b*x)^p*
```

```
(c + d*x)^q)^r]^2)) - 4*(b*c - a*d)*(-2*b*d*g + b*c*h + a*d*h)*p*q*r^2*Poly
Log[2, (d*(a + b*x))/(-(b*c) + a*d)])/(4*b^2*d^2)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int (hx+g)\ln \left(e{\left(f{\left(bx+a\right)}^{p}{\left(dx+c\right)}^{q}\right)}^{r}\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

```
[Out] int((h*x+g)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

Maxima [A]
time = 0.31, size = 625, normalized size = 0.59

$$\frac{1}{2}\left(hx^{2}+2\,gx\right)\log\left(\left(\left(bx+a\right)^{p}\left(dx+c\right)^{q}f\right)^{r}e\right)^{2}+\frac{r\left(\frac{2\left(2\,abfgp-a^{2}fhp\right)\log\left(bx+a\right)}{b^{2}}+\frac{2\left(2\,cdfgq-c^{2}fhq\right)\log\left(dx+c\right)}{d^{2}}-\frac{bdfh\left(p+q\right)x^{2}-2\left(adfhp-\left(2\,dfg\left(p+q\right)-cfhq\right)b\right)x}{bd}\right)\log\left(\left(\left(bx+a\right)^{p}\left(dx+c\right)^{q}f\right)^{r}e\right)}{2\,f}+\frac{r^{2}\left(\frac{2\left(2\,acdf^{2}hpq-\left(4\left(pq+q^{2}\right)cdf^{2}g-\left(pq+3\,q^{2}\right)c^{2}f^{2}h\right)b\right)\log\left(dx+c\right)}{bd^{2}}-\frac{4\left(2\,abd^{2}f^{2}gpq-a^{2}d^{2}f^{2}hpq-\left(2\,cdf^{2}gpq-c^{2}f^{2}hpq\right)b^{2}\right)\left(\log\left(bx+a\right)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_{2}\left(-\frac{bdx+ad}{bc-ad}\right)\right)}{b^{2}d^{2}}+\frac{\left(p^{2}+2\,pq+q^{2}\right)b^{2}d^{2}f^{2}hx^{2}-4\left(2\,cdf^{2}gpq-c^{2}f^{2}hpq\right)b^{2}\log\left(bx+a\right)\log\left(dx+c\right)-2\left(2\,cdf^{2}gq^{2}-c^{2}f^{2}hq^{2}\right)b^{2}\log\left(dx+c\right)^{2}-2\left(2\,abd^{2}f^{2}gp^{2}-a^{2}d^{2}f^{2}hp^{2}\right)\log\left(bx+a\right)^{2}-2\left(3\left(p^{2}+pq\right)abd^{2}f^{2}h-\left(4\left(p^{2}+2\,pq+q^{2}\right)d^{2}f^{2}g-3\left(pq+q^{2}\right)cdf^{2}h\right)b^{2}\right)x+2\left(\left(3\,p^{2}+pq\right)a^{2}d^{2}f^{2}h+2\left(cdf^{2}hpq-2\left(p^{2}+pq\right)d^{2}f^{2}g\right)ab\right)\log\left(bx+a\right)}{b^{2}d^{2}}\right)}{4\,f^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(h*x^2 + 2*g*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2 + 1/2*r*(2*(2*a*
b*f*g*p - a^2*f*h*p)*log(b*x + a)/b^2 + 2*(2*c*d*f*g*q - c^2*f*h*q)*log(d*x
 + c)/d^2 - (b*d*f*h*(p + q)*x^2 - 2*(a*d*f*h*p - (2*d*f*g*(p + q) - c*f*h*
q)*b)*x)/(b*d))*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/f + 1/4*r^2*(2*(2*a*c*
d*f^2*h*p*q - (4*(p*q + q^2)*c*d*f^2*g - (p*q + 3*q^2)*c^2*f^2*h)*b)*log(d*
x + c)/(b*d^2) - 4*(2*a*b*d^2*f^2*g*p*q - a^2*d^2*f^2*h*p*q - (2*c*d*f^2*g*
p*q - c^2*f^2*h*p*q)*b^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1)
+ dilog(-(b*d*x + a*d)/(b*c - a*d)))/(b^2*d^2) + ((p^2 + 2*p*q + q^2)*b^2*d
^2*f^2*h*x^2 - 4*(2*c*d*f^2*g*p*q - c^2*f^2*h*p*q)*b^2*log(b*x + a)*log(d*x
 + c) - 2*(2*c*d*f^2*g*q^2 - c^2*f^2*h*q^2)*b^2*log(d*x + c)^2 - 2*(2*a*b*d
^2*f^2*g*p^2 - a^2*d^2*f^2*h*p^2)*log(b*x + a)^2 - 2*(3*(p^2 + p*q)*a*b*d^2
*f^2*h - (4*(p^2 + 2*p*q + q^2)*d^2*f^2*g - 3*(p*q + q^2)*c*d*f^2*h)*b^2)*x
 + 2*((3*p^2 + p*q)*a^2*d^2*f^2*h + 2*(c*d*f^2*h*p*q - 2*(p^2 + p*q)*d^2*f^
2*g)*a*b)*log(b*x + a))/(b^2*d^2))/f^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas")
```

```
[Out] integral((h*x + g)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (g+hx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
[Out] Integral((g + h*x)*log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(e\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right)^2\,(g+h\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(g + h*x),x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2*(g + h*x), x)
```

3.38 $\int \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx$

Optimal. Leaf size=269

$$2(p+q)^2r^2x-\frac{2(bc-ad)q(p+q)r^2\log(c+dx)}{bd}-\frac{2(bc-ad)pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{bd}-\frac{(bc-ad)q^2r^2\log}{bd}$$

```
[Out] 2*(p+q)^2*r^2*x-2*(-a*d+b*c)*q*(p+q)*r^2*ln(d*x+c)/b/d-2*(-a*d+b*c)*p*q*r^2
*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/b/d-(-a*d+b*c)*q^2*r^2*ln(d*x+c)^2/b/d
-2*(p+q)*r*(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b+2*(-a*d+b*c)*q*r*ln(d*
x+c)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/b/d+(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^
q)^r)^2/b-2*(-a*d+b*c)*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c))/b/d
```

Rubi [A]
time = 0.11, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2579, 2580, 2441, 2440, 2438, 2437, 2338, 31, 8}

$$-\frac{2pqr^2(bc-ad)\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bd}+\frac{(a+bx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}-\frac{2r(p+q)(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b}+\frac{2qr(bc-ad)\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{bd}-\frac{2qr^2(p+q)(bc-ad)\log(c+dx)}{bd}-\frac{2pqr^2(bc-ad)\log(c+dx)\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{bd}-\frac{q^2r^2(bc-ad)\log^2(c+dx)}{bd}+2r^2x(p+q)^2$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] 2*(p + q)^2*r^2*x - (2*(b*c - a*d)*q*(p + q)*r^2*Log[c + d*x])/(b*d) - (2*(
b*c - a*d)*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b*d) -
((b*c - a*d)*q^2*r^2*Log[c + d*x]^2)/(b*d) - (2*(p + q)*r*(a + b*x)*Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r])/b + (2*(b*c - a*d)*q*r*Log[c + d*x]*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r])/(b*d) + ((a + b*x)*Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^2)/b - (2*(b*c - a*d)*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*
d)])/(b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2579

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[(a + b*x)*(Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s/b), x] + (Dist[q*r*s*((b*c - a*d)/b), Int[Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]^(s - 1)/(c + d*x), x], x] - Dist[r*s*(p + q), Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^(s - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p, q,
r, s}, x] && NeQ[b*c - a*d, 0] && NeQ[p + q, 0] && IGtQ[s, 0] && LtQ[s, 4]
```

Rule 2580

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/h), x] + (-Dist[b*p*(r/h), Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[d*q*(r/h), Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\,dx = \frac{(a+bx)\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b} + \frac{(2(bc-ad)qr)\int \frac{\log(e(f(a+bx}{c+}}{b}$$

$$= -\frac{2(p+q)r(a+bx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{b} + \frac{2(bc-ad)qr\log(}{}$$

$$= 2(p+q)^2r^2x - \frac{2(bc-ad)q(p+q)r^2\log(c+dx)}{bd} - \frac{2(bc-ad)pqr^2\log}{}$$

$$= 2(p+q)^2r^2x - \frac{2(bc-ad)q(p+q)r^2\log(c+dx)}{bd} - \frac{2(bc-ad)pqr^2\log}{}$$

$$= 2(p+q)^2r^2x - \frac{2(bc-ad)q(p+q)r^2\log(c+dx)}{bd} - \frac{2(bc-ad)pqr^2\log}{}$$

Mathematica [A]
time = 0.16, size = 389, normalized size = 1.45

$$\frac{2adpqr^2 + 2bdp^2r^2x + 4bdpqr^2x + 2bdq^2r^2x - adp^2r^2\log^2(a+bx) - 2bcpqr^2\log(c+dx) + 2adpqr^2\log(c+dx) - 2bcq^2r^2\log(c+dx) - bcq^2r^2\log^2(c+dx) - 2pr\log(a+bx)\left(bcqr\log(c+dx) + (-bc+ad)qr\log\left(\frac{b(c+dx)}{bc-ad}\right) + ad(qr - \log\left(e(f(a+bx)^p(c+dx)^q)^r\right))\right) - 2adpr\log\left(e(f(a+bx)^p(c+dx)^q)^r\right) - 2bdprx\log\left(e(f(a+bx)^p(c+dx)^q)^r\right) - 2bdqrx\log\left(e(f(a+bx)^p(c+dx)^q)^r\right) + 2bcqr\log(c+dx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right) + bdx\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right) + 2(bc-ad)pqr^2\operatorname{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (2*a*d*p*q*r^2 + 2*b*d*p^2*r^2*x + 4*b*d*p*q*r^2*x + 2*b*d*q^2*r^2*x - a*d*
p^2*r^2*Log[a + b*x]^2 - 2*b*c*p*q*r^2*Log[c + d*x] + 2*a*d*p*q*r^2*Log[c +
 d*x] - 2*b*c*q^2*r^2*Log[c + d*x] - b*c*q^2*r^2*Log[c + d*x]^2 - 2*p*r*Log
[a + b*x]*(b*c*q*r*Log[c + d*x] + (-(b*c) + a*d)*q*r*Log[(b*(c + d*x))/(b*c
 - a*d)] + a*d*(q*r - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])) - 2*a*d*p*r*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*b*d*p*r*x*Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r] - 2*b*d*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 2*b*c*q*r*L
og[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + b*d*x*Log[e*(f*(a + b*x)
^p*(c + d*x)^q)^r]^2 + 2*(b*c - a*d)*p*q*r^2*PolyLog[2, (d*(a + b*x))/(-(b*
c) + a*d)])/(b*d)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \ln\left(e\left(f(bx+a)^p\,(dx+c)^q\right)^r\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

Maxima [A]
time = 0.30, size = 300, normalized size = 1.12

$$x \log \left(\left(\left((bx+a)^{p}(dx+c)^{q} f\right)^{r} e\right)^{2} - \frac{2 \left(f(p+q)x - \frac{afp\log(bx+a)}{b} - \frac{cfq\log(dx+c)}{d}\right) r \log \left(\left((bx+a)^{p}(dx+c)^{q} f\right)^{r} e\right)}{f} - \frac{\left(\frac{2\left(pq+q^{2}\right)cf^{2}\log(dx+c)}{d} - \frac{2\left(bcf^{2}pq-adf^{2}pq\right)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_{2}\left(-\frac{bdx+ad}{bc-ad}\right)\right)}{bd} + \frac{adf^{2}p^{2}\log(bx+a)^{2}+2\,bcf^{2}pq\log(bx+a)\log(dx+c)+bcf^{2}q^{2}\log(dx+c)^{2}-2\left(p^{2}+2\,pq+q^{2}\right)bdf^{2}x+2\left(p^{2}+pq\right)adf^{2}\log(bx+a)}{bd}\right)r^{2}}{f^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima")
```

```
[Out] x*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2 - 2*(f*(p + q)*x - a*f*p*log(b*x +
 a)/b - c*f*q*log(d*x + c)/d)*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/f - (2
*(p*q + q^2)*c*f^2*log(d*x + c)/d - 2*(b*c*f^2*p*q - a*d*f^2*p*q)*(log(b*x
+ a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d))
)/(b*d) + (a*d*f^2*p^2*log(b*x + a)^2 + 2*b*c*f^2*p*q*log(b*x + a)*log(d*x
+ c) + b*c*f^2*q^2*log(d*x + c)^2 - 2*(p^2 + 2*p*q + q^2)*b*d*f^2*x + 2*(p^
2 + p*q)*a*d*f^2*log(b*x + a))/(b*d))*r^2/f^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log \left(e\left(f\left(a+bx\right)^{p}\left(c+dx\right)^{q}\right)^{r}\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

```
[Out] Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(e\left(f\left(a+b\,x\right)^p\left(c+d\,x\right)^q\right)^r\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2,x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2, x)
```

3.39 $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{g+hx}\,dx$

Optimal. Leaf size=1471

$$\frac{pqr^2\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log^2\left(\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}\right)}{h}+\frac{p^2r^2\log^2(a+bx)\log(g+hx)}{h}+\frac{2pqr^2\log(a+bx)\log(c+dx)\log(g}{h}$$

```
[Out] -2*p*r*(q*r*ln((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))-ln(e*(f*(b*x+a)^p*(d*
x+c)^q)^r))*polylog(2,-h*(b*x+a)/(-a*h+b*g))/h+2*q*r*(p*r*ln((-a*h+b*g)*(d*
x+c)/(-c*h+d*g)/(b*x+a))+ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))*polylog(2,-h*(d*x
+c)/(-c*h+d*g))/h+p^2*r^2*ln(b*x+a)^2*ln(h*x+g)/h+q^2*r^2*ln(d*x+c)^2*ln(h*
x+g)/h-p^2*r^2*ln(b*x+a)^2*ln(b*(h*x+g)/(-a*h+b*g))/h-q^2*r^2*ln(d*x+c)^2*l
n(d*(h*x+g)/(-c*h+d*g))/h+ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2*ln(h*x+g)/h-2*p
^2*r^2*polylog(3,-h*(b*x+a)/(-a*h+b*g))/h-2*q^2*r^2*polylog(3,-h*(d*x+c)/(-
c*h+d*g))/h-2*p*q*r^2*polylog(3,-h*(b*x+a)/(-a*h+b*g))/h-2*p*q*r^2*polylog(
3,-h*(d*x+c)/(-c*h+d*g))/h-2*p*q*r^2*polylog(3,b*(d*x+c)/d/(b*x+a))/h+2*p*q
*r^2*polylog(3,(-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))/h+2*p*q*r^2*ln(b*x+a)
*ln(d*x+c)*ln(h*x+g)/h-2*p*q*r^2*ln(b*x+a)*ln(-h*(d*x+c)/(-c*h+d*g))*ln(b*(
h*x+g)/(-a*h+b*g))/h-2*p*q*r^2*ln(-h*(d*x+c)/(-c*h+d*g))*ln((-a*h+b*g)*(d*x
+c)/(-c*h+d*g)/(b*x+a))*ln(b*(h*x+g)/(-a*h+b*g))/h-2*p*q*r^2*ln(b*x+a)*ln(d
*x+c)*ln(d*(h*x+g)/(-c*h+d*g))/h+2*p*q*r^2*ln(b*x+a)*ln(-h*(d*x+c)/(-c*h+d*
g))*ln(d*(h*x+g)/(-c*h+d*g))/h+2*p*q*r^2*ln(-h*(d*x+c)/(-c*h+d*g))*ln((-a*h
+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))*ln(d*(h*x+g)/(-c*h+d*g))/h-2*p*r*ln(b*x+a
)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*ln(h*x+g)/h-2*q*r*ln(d*x+c)*ln(e*(f*(b*x+
a)^p*(d*x+c)^q)^r)*ln(h*x+g)/h+2*p*r*ln(b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)
^r)*ln(b*(h*x+g)/(-a*h+b*g))/h+2*q*r*ln(d*x+c)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)
^r)*ln(d*(h*x+g)/(-c*h+d*g))/h+2*p*q*r^2*ln((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(
b*x+a))*polylog(2,b*(d*x+c)/d/(b*x+a))/h-2*p*q*r^2*ln((-a*h+b*g)*(d*x+c)/(-
c*h+d*g)/(b*x+a))*polylog(2,(-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))/h+p*q*r^
2*ln((a*d-b*c)/d/(b*x+a))*ln((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^2/h+p*q
*r^2*ln(-h*(d*x+c)/(-c*h+d*g))^2*ln(b*(h*x+g)/(-a*h+b*g))/h+p*q*r^2*ln((-a*
h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^2*ln(b*(h*x+g)/(-a*h+b*g))/h-p*q*r^2*ln(
-h*(d*x+c)/(-c*h+d*g))^2*ln(d*(h*x+g)/(-c*h+d*g))/h-p*q*r^2*ln((-a*h+b*g)*(
d*x+c)/(-c*h+d*g)/(b*x+a))^2*ln(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))/h
```

Rubi [A]
time = 1.39, antiderivative size = 2096, normalized size of antiderivative = 1.42, number of steps used = 29, number of rules used = 14, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {2583, 2586, 2441, 2440, 2438, 2481, 2422, 2354, 2421, 6724, 2490, 2487, 2485, 2352}

Too large to display

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x),x]

[Out] -((Log[(a + b*x)^(p*r)]^2*Log[g + h*x])/h) - (2*p*q*r^2*Log[-((d*(a + b*x))
/(b*c - a*d))]*Log[c + d*x]*Log[g + h*x])/h - (2*p*q*r^2*Log[a + b*x]*Log[(
b*(c + d*x))/(b*c - a*d)]*Log[g + h*x])/h + (2*q*r*(p*r*Log[a + b*x] - Log[
(a + b*x)^(p*r)])*Log[-((h*(c + d*x))/(d*g - c*h))]*Log[g + h*x])/h + (2*p*
r*Log[-((h*(a + b*x))/(b*g - a*h))]*(q*r*Log[c + d*x] - Log[(c + d*x)^(q*r)
])*Log[g + h*x])/h - (Log[(c + d*x)^(q*r)]^2*Log[g + h*x])/h + (2*p*r*Log[-
((h*(a + b*x))/(b*g - a*h))]*(Log[(a + b*x)^(p*r)] + Log[(c + d*x)^(q*r)] -
 Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*Log[g + h*x])/h + (2*q*r*Log[-((h*(c
 + d*x))/(d*g - c*h))]*(Log[(a + b*x)^(p*r)] + Log[(c + d*x)^(q*r)] - Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r])*Log[g + h*x])/h + (Log[e*(f*(a + b*x)^p*(c
 + d*x)^q)^r]^2*Log[g + h*x])/h + (Log[(a + b*x)^(p*r)]^2*Log[(b*(g + h*x))
/(b*g - a*h)])/h + (Log[(c + d*x)^(q*r)]^2*Log[(d*(g + h*x))/(d*g - c*h)])/
h - (p*q*r^2*(Log[(b*(c + d*x))/(b*c - a*d)] + Log[(b*g - a*h)/(b*(g + h*x)
)] - Log[((b*g - a*h)*(c + d*x))/((b*c - a*d)*(g + h*x))])*Log[-(((b*c - a*
d)*(g + h*x))/((d*g - c*h)*(a + b*x)))]^2)/h + (p*q*r^2*(Log[(b*(c + d*x))/
(b*c - a*d)] - Log[-((h*(c + d*x))/(d*g - c*h))])*(Log[a + b*x] + Log[-(((b
*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b*x)))])^2)/h - (p*q*r^2*(Log[-((d*(
a + b*x))/(b*c - a*d))] + Log[(d*g - c*h)/(d*(g + h*x))] - Log[-(((d*g - c*
h)*(a + b*x))/((b*c - a*d)*(g + h*x)))])*Log[((b*c - a*d)*(g + h*x))/((b*g
- a*h)*(c + d*x))]^2)/h + (p*q*r^2*(Log[-((d*(a + b*x))/(b*c - a*d))] - Log
[-((h*(a + b*x))/(b*g - a*h))])*(Log[c + d*x] + Log[((b*c - a*d)*(g + h*x))
/((b*g - a*h)*(c + d*x))])^2)/h - (2*p*q*r^2*(Log[g + h*x] - Log[-(((b*c -
a*d)*(g + h*x))/((d*g - c*h)*(a + b*x)))])*PolyLog[2, -((d*(a + b*x))/(b*c
- a*d))])/h + (2*p*r*Log[(a + b*x)^(p*r)]*PolyLog[2, -((h*(a + b*x))/(b*g -
 a*h))])/h - (2*p*q*r^2*(Log[g + h*x] - Log[((b*c - a*d)*(g + h*x))/((b*g -
 a*h)*(c + d*x))])*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/h + (2*q*r*Log[(c
 + d*x)^(q*r)]*PolyLog[2, -((h*(c + d*x))/(d*g - c*h))])/h + (2*p*q*r^2*Log
[-(((b*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b*x)))]*PolyLog[2, (h*(a + b*x
))/(b*(g + h*x))])/h - (2*p*q*r^2*Log[-(((b*c - a*d)*(g + h*x))/((d*g - c*h
)*(a + b*x)))]*PolyLog[2, -(((d*g - c*h)*(a + b*x))/((b*c - a*d)*(g + h*x))
)])/h + (2*p*q*r^2*Log[((b*c - a*d)*(g + h*x))/((b*g - a*h)*(c + d*x))]*Pol
yLog[2, (h*(c + d*x))/(d*(g + h*x))])/h - (2*p*q*r^2*Log[((b*c - a*d)*(g +
h*x))/((b*g - a*h)*(c + d*x))]*PolyLog[2, ((b*g - a*h)*(c + d*x))/((b*c - a
*d)*(g + h*x))])/h + (2*p*r*(q*r*Log[c + d*x] - Log[(c + d*x)^(q*r)])*PolyL
og[2, (b*(g + h*x))/(b*g - a*h)])/h + (2*p*r*(Log[(a + b*x)^(p*r)] + Log[(c
 + d*x)^(q*r)] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*PolyLog[2, (b*(g + h
*x))/(b*g - a*h)])/h - (2*p*q*r^2*(Log[c + d*x] + Log[((b*c - a*d)*(g + h*x
))/((b*g - a*h)*(c + d*x))])*PolyLog[2, (b*(g + h*x))/(b*g - a*h)])/h + (2*
q*r*(p*r*Log[a + b*x] - Log[(a + b*x)^(p*r)])*PolyLog[2, (d*(g + h*x))/(d*g
 - c*h)])/h + (2*q*r*(Log[(a + b*x)^(p*r)] + Log[(c + d*x)^(q*r)] - Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r])*PolyLog[2, (d*(g + h*x))/(d*g - c*h)])/h - (
2*p*q*r^2*(Log[a + b*x] + Log[-(((b*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b
*x)))])*PolyLog[2, (d*(g + h*x))/(d*g - c*h)])/h + (2*p*q*r^2*PolyLog[3, -(
```

```
(d*(a + b*x))/(b*c - a*d))])/h - (2*p^2*r^2*PolyLog[3, -((h*(a + b*x))/(b*g
 - a*h))])/h + (2*p*q*r^2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)])/h - (2*q^2
*r^2*PolyLog[3, -((h*(c + d*x))/(d*g - c*h))])/h + (2*p*q*r^2*PolyLog[3, (h
*(a + b*x))/(b*(g + h*x))])/h - (2*p*q*r^2*PolyLog[3, -(((d*g - c*h)*(a + b
*x))/((b*c - a*d)*(g + h*x)))])/h + (2*p*q*r^2*PolyLog[3, (h*(c + d*x))/(d*
(g + h*x))])/h - (2*p*q*r^2*PolyLog[3, ((b*g - a*h)*(c + d*x))/((b*c - a*d)
*(g + h*x))])/h + (2*p*q*r^2*PolyLog[3, (b*(g + h*x))/(b*g - a*h)])/h + (2*
p*q*r^2*PolyLog[3, (d*(g + h*x))/(d*g - c*h)])/h
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2485

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[(-b)*(x/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1/2)*(Log[(-b)*(x/a
)] - Log[(-(b*c - a*d))*(x/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))]
)*Log[a*((c + d*x)/(c*(a + b*x)))]^2, x] - Simp[(1/2)*(Log[(-b)*(x/a)] - Lo
g[(-d)*(x/c)])*(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])^2, x] + Si
mp[(Log[c + d*x] - Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1 + b*(x/a)
], x] + Simp[(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1
 + d*(x/c)], x] + Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2, c*((a +
b*x)/(a*(c + d*x)))], x] - Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2,
 d*((a + b*x)/(b*(c + d*x)))], x] - Simp[PolyLog[3, 1 + b*(x/a)], x] - Simp
[PolyLog[3, 1 + d*(x/c)], x] + Simp[PolyLog[3, c*((a + b*x)/(a*(c + d*x)))]
, x] - Simp[PolyLog[3, d*((a + b*x)/(b*(c + d*x)))], x]) /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2487

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*Log[(h_.)*((i_.) + (j_.)*(x_))^(m
_.)])/(x_), x_Symbol] :> Dist[m, Int[Log[i + j*x]*(Log[c*(d + e*x)^n]/x), x
], x] - Dist[m*Log[i + j*x] - Log[h*(i + j*x)^m], Int[Log[c*(d + e*x)^n]/x,
 x], x] /; FreeQ[{c, d, e, h, i, j, m, n}, x] && NeQ[e*i - d*j, 0] && NeQ[i
 + j*x, h*(i + j*x)^m]
```

Rule 2490

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-(e*k - d*l)/l + e*(x/l))^n])*(f + g
```

```
*Log[h*(-(j*k - i*l)/l + j*(x/l))^m]), x], x, k + l*x], x] /; FreeQ[{a, b,
c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

Rule 2583

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^2/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a
 + b*x)^p*(c + d*x)^q)^r]^2/h), x] + (-Dist[2*b*p*(r/h), Int[Log[g + h*x]*(
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)), x], x] - Dist[2*d*q*(r/h),
 Int[Log[g + h*x]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(c + d*x)), x], x])
 /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2586

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.)))/((j_.) + (k
_.)*(x_)), x_Symbol] :> Dist[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - Log[(a
+ b*x)^(p*r)] - Log[(c + d*x)^(q*r)], Int[(s + t*Log[i*(g + h*x)^n])/(j + k
*x), x], x] + (Int[(Log[(a + b*x)^(p*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k
*x), x] + Int[(Log[(c + d*x)^(q*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k*x),
x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, n, p, q, r}, x] && NeQ
[b*c - a*d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{g+hx}\,dx = \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\log(g+hx)}{h} - \frac{(2bpr)\int \frac{\log(e(f(a+bx)^p(c+}{a+b}}{h}$$

$$= \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)\log(g+hx)}{h} - \frac{(2bpr)\int \frac{\log((a+bx)^{pr})\log(g}{a+bx}}{h}$$

$$= \frac{2pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right)-\log\left(e(f(a+\right.\right.}{h}$$

$$= -\frac{\log^2\left((a+bx)^{pr}\right)\log(g+hx)}{h} - \frac{\log^2\left((c+dx)^{qr}\right)\log(g+hx)}{h} + \frac{2pr}{}$$

$$= -\frac{\log^2\left((a+bx)^{pr}\right)\log(g+hx)}{h} - \frac{2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)\log}{h}$$

$$= -\frac{\log^2\left((a+bx)^{pr}\right)\log(g+hx)}{h} - \frac{2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)\log}{h}$$

$$= -\frac{\log^2\left((a+bx)^{pr}\right)\log(g+hx)}{h} - \frac{2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)\log}{h}$$

Mathematica [A]
time = 0.19, size = 1370, normalized size = 0.93

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x),x]
```

```
[Out] (p*q*r^2*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*g - a*h)*(c + d*x))/((d*
g - c*h)*(a + b*x))]^2 + p^2*r^2*Log[a + b*x]^2*Log[g + h*x] + 2*p*q*r^2*Lo
g[a + b*x]*Log[c + d*x]*Log[g + h*x] + q^2*r^2*Log[c + d*x]^2*Log[g + h*x]
- 2*p*r*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g + h*x] - 2*
q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g + h*x] + Log[e*
(f*(a + b*x)^p*(c + d*x)^q)^r]^2*Log[g + h*x] - p^2*r^2*Log[a + b*x]^2*Log[
(b*(g + h*x))/(b*g - a*h)] - 2*p*q*r^2*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*
g) + c*h)]*Log[(b*(g + h*x))/(b*g - a*h)] + p*q*r^2*Log[(h*(c + d*x))/(-(d*
g) + c*h)]^2*Log[(b*(g + h*x))/(b*g - a*h)] - 2*p*q*r^2*Log[(h*(c + d*x))/(
-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Log[(b*
(g + h*x))/(b*g - a*h)] + p*q*r^2*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*
(a + b*x))]^2*Log[(b*(g + h*x))/(b*g - a*h)] + 2*p*r*Log[a + b*x]*Log[e*(f*
```

```
(a + b*x)^p*(c + d*x)^q)^r]*Log[(b*(g + h*x))/(b*g - a*h)] - 2*p*q*r^2*Log[
a + b*x]*Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)] - q^2*r^2*Log[c + d*x]
^2*Log[(d*(g + h*x))/(d*g - c*h)] + 2*p*q*r^2*Log[a + b*x]*Log[(h*(c + d*x)
)/(-(d*g) + c*h)]*Log[(d*(g + h*x))/(d*g - c*h)] - p*q*r^2*Log[(h*(c + d*x)
)/(-(d*g) + c*h)]^2*Log[(d*(g + h*x))/(d*g - c*h)] + 2*p*q*r^2*Log[(h*(c +
d*x))/(-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*
Log[(d*(g + h*x))/(d*g - c*h)] + 2*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c
 + d*x)^q)^r]*Log[(d*(g + h*x))/(d*g - c*h)] - p*q*r^2*Log[((b*g - a*h)*(c
+ d*x))/((d*g - c*h)*(a + b*x))]^2*Log[((-(b*c) + a*d)*(g + h*x))/((d*g - c
*h)*(a + b*x))] + 2*p*r*(-(q*r*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a
+ b*x))]) + Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*PolyLog[2, (h*(a + b*x))/
(-(b*g) + a*h)] + 2*q*r*(p*r*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a +
b*x))] + Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*PolyLog[2, (h*(c + d*x))/(-(
d*g) + c*h)] + 2*p*q*r^2*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x)
)]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] - 2*p*q*r^2*Log[((b*g - a*h)*(c
+ d*x))/((d*g - c*h)*(a + b*x))]*PolyLog[2, ((b*g - a*h)*(c + d*x))/((d*g -
 c*h)*(a + b*x))] - 2*p^2*r^2*PolyLog[3, (h*(a + b*x))/(-(b*g) + a*h)] - 2*
p*q*r^2*PolyLog[3, (h*(a + b*x))/(-(b*g) + a*h)] - 2*p*q*r^2*PolyLog[3, (h*
(c + d*x))/(-(d*g) + c*h)] - 2*q^2*r^2*PolyLog[3, (h*(c + d*x))/(-(d*g) + c
*h)] - 2*p*q*r^2*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))] + 2*p*q*r^2*PolyLo
g[3, ((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))])/h
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(e\left(f\left(bx+a\right)^{p}\left(dx+c\right)^{q}\right)^{r}\right)^{2}}{hx+g}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g),x)
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g),x, algorithm="maxima")
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h*x + g), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g),x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h*x + g), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(h*x+g),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g),x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h*x + g), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)^r\right)^2}{g+h\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(g + h*x),x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(g + h*x), x)
```

3.40 $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{(g+hx)^2} \, dx$

Optimal. Leaf size=832

$$\frac{2bpqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{h(bg-ah)}+\frac{2dpqr^2\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{h(dg-ch)}-\frac{2bpr\log(a+bx)\left(pr\log(a+bx)+qr\right.}{h(}$$

```
[Out] 2*b*p*q*r^2*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/h/(-a*h+b*g)+2*d*p*q*r^2*ln
(b*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/h/(-c*h+d*g)-2*b*p*r*ln(b*x+a)*(p*r*ln(b*x
+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h/(-a*h+b*g)-2*d*q*r*ln(
d*x+c)*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h/(-c*
h+d*g)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/h/(h*x+g)+2*b*p*r*(p*r*ln(b*x+a)+q
*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))*ln(h*x+g)/h/(-a*h+b*g)+2*d*q*
r*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))*ln(h*x+g)/h
/(-c*h+d*g)-2*d*p*q*r^2*ln(b*x+a)*ln(b*(h*x+g)/(-a*h+b*g))/h/(-c*h+d*g)-2*b
*p*q*r^2*ln(d*x+c)*ln(d*(h*x+g)/(-c*h+d*g))/h/(-a*h+b*g)-2*b*p^2*r^2*ln(b*x
+a)*ln(1+(-a*h+b*g)/h/(b*x+a))/h/(-a*h+b*g)-2*d*q^2*r^2*ln(d*x+c)*ln(1+(-c*
h+d*g)/h/(d*x+c))/h/(-c*h+d*g)+2*b*p^2*r^2*polylog(2,(a*h-b*g)/h/(b*x+a))/h
/(-a*h+b*g)+2*d*p*q*r^2*polylog(2,-d*(b*x+a)/(-a*d+b*c))/h/(-c*h+d*g)-2*d*p
*q*r^2*polylog(2,-h*(b*x+a)/(-a*h+b*g))/h/(-c*h+d*g)+2*d*q^2*r^2*polylog(2,
(c*h-d*g)/h/(d*x+c))/h/(-c*h+d*g)+2*b*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c
))/h/(-a*h+b*g)-2*b*p*q*r^2*polylog(2,-h*(d*x+c)/(-c*h+d*g))/h/(-a*h+b*g)
```

Rubi [A]
time = 0.60, antiderivative size = 832, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {2584, 2593, 2458, 2379, 2438, 2465, 2441, 2440, 36, 31}

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x)^2,x]
```

```
[Out] (2*b*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(h*(b*g - a*h)
) + (2*d*p*q*r^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(h*(d*g - c*h
)) - (2*b*p*r*Log[a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*
(a + b*x)^p*(c + d*x)^q)^r]))/(h*(b*g - a*h)) - (2*d*q*r*Log[c + d*x]*(p*r*
Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(h
*(d*g - c*h)) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(h*(g + h*x)) + (2*b
*p*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r])*Log[g + h*x])/(h*(b*g - a*h)) + (2*d*q*r*(p*r*Log[a + b*x] + q*r*Log
[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*Log[g + h*x])/(h*(d*g - c
*h)) - (2*d*p*q*r^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)])/(h*(d*g -
c*h)) - (2*b*p*q*r^2*Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)])/(h*(b*g -
```

```
 a*h)) - (2*b*p^2*r^2*Log[a + b*x]*Log[1 + (b*g - a*h)/(h*(a + b*x))])/(h*(
b*g - a*h)) - (2*d*q^2*r^2*Log[c + d*x]*Log[1 + (d*g - c*h)/(h*(c + d*x))])
/(h*(d*g - c*h)) + (2*b*p^2*r^2*PolyLog[2, -((b*g - a*h)/(h*(a + b*x)))])/(
h*(b*g - a*h)) + (2*d*p*q*r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(h*
(d*g - c*h)) - (2*d*p*q*r^2*PolyLog[2, -((h*(a + b*x))/(b*g - a*h))])/(h*(d
*g - c*h)) + (2*d*q^2*r^2*PolyLog[2, -((d*g - c*h)/(h*(c + d*x)))])/(h*(d*g
 - c*h)) + (2*b*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(h*(b*g - a*
h)) - (2*b*p*q*r^2*PolyLog[2, -((h*(c + d*x))/(d*g - c*h))])/(h*(b*g - a*h)
)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2584

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Dist[b*p*r*(
s/(h*(m + 1))), Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1)/(a + b*x)), x], x] - Dist[d*q*r*(s/(h*(m + 1))), Int[(g + h*x)^(m
+ 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^2}\,dx = -\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h(g+hx)} + \frac{(2bpr)\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)(g+hx)}\,dx}{h} +$$

$$= -\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h(g+hx)} + \frac{(2bp^2r^2)\int \frac{\log(a+bx)}{(a+bx)(g+hx)}\,dx}{h} + \frac{(2bpq}{}$$

$$= -\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h(g+hx)} + \frac{(2p^2r^2)\,\mathrm{Subst}\left(\int \frac{\log(x)}{x\left(\frac{bg-ah}{b}+\frac{hx}{b}\right)}\,dx, x}{h}$$

$$= -\frac{2bpr\log(a+bx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)\right.\right.}{h(bg-ah)}$$

$$= \frac{bp^2r^2\log^2(a+bx)}{h(bg-ah)} + \frac{2bpqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{h(bg-ah)} + \frac{dq^2r^2\log^2(}{h(dg-}$$

$$= \frac{bp^2r^2\log^2(a+bx)}{h(bg-ah)} + \frac{2bpqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{h(bg-ah)} + \frac{dq^2r^2\log^2(}{h(dg-}$$

$$= \frac{bp^2r^2\log^2(a+bx)}{h(bg-ah)} + \frac{2bpqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{h(bg-ah)} + \frac{dq^2r^2\log^2(}{h(dg-}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2930 vs. $2(832) = 1664$.
time = 0.77, size = 2930, normalized size = 3.52

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x)^2,x]
```

```
[Out] (-(b*d*g^2*p^2*r^2*Log[a + b*x]^2) + b*c*g*h*p^2*r^2*Log[a + b*x]^2 - b*d*g
*h*p^2*r^2*x*Log[a + b*x]^2 + b*c*h^2*p^2*r^2*x*Log[a + b*x]^2 - 2*b*d*g^2*
p*q*r^2*Log[a + b*x]*Log[c + d*x] + 2*a*d*g*h*p*q*r^2*Log[a + b*x]*Log[c +
d*x] - 2*b*d*g*h*p*q*r^2*x*Log[a + b*x]*Log[c + d*x] + 2*a*d*h^2*p*q*r^2*x*
Log[a + b*x]*Log[c + d*x] - b*d*g^2*q^2*r^2*Log[c + d*x]^2 + a*d*g*h*q^2*r^
2*Log[c + d*x]^2 - b*d*g*h*q^2*r^2*x*Log[c + d*x]^2 + a*d*h^2*q^2*r^2*x*Log
[c + d*x]^2 + 2*b*c*g*h*p*q*r^2*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*g) + c*
h)] - 2*a*d*g*h*p*q*r^2*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*g) + c*h)] + 2*
b*c*h^2*p*q*r^2*x*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*g) + c*h)] - 2*a*d*h^
2*p*q*r^2*x*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*g) + c*h)] - b*c*g*h*p*q*r^
2*Log[(h*(c + d*x))/(-(d*g) + c*h)]^2 + a*d*g*h*p*q*r^2*Log[(h*(c + d*x))/(
```

-(d*g) + c*h)]^2 - b*c*h^2*p*q*r^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]^2 +
a*d*h^2*p*q*r^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]^2 + 2*b*c*g*h*p*q*r^2*L
og[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(
a + b*x))] - 2*a*d*g*h*p*q*r^2*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*g
- a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] + 2*b*c*h^2*p*q*r^2*x*Log[(-(b*c
) + a*d)/(d*(a + b*x))]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))
] - 2*a*d*h^2*p*q*r^2*x*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*g - a*h)*
(c + d*x))/((d*g - c*h)*(a + b*x))] + 2*b*c*g*h*p*q*r^2*Log[(h*(c + d*x))/(
-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] - 2*a*d
*g*h*p*q*r^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d*x))/
((d*g - c*h)*(a + b*x))] + 2*b*c*h^2*p*q*r^2*x*Log[(h*(c + d*x))/(-(d*g) +
c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] - 2*a*d*h^2*p*q*
r^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g -
 c*h)*(a + b*x))] - b*c*g*h*p*q*r^2*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h
)*(a + b*x))]^2 + a*d*g*h*p*q*r^2*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*
(a + b*x))]^2 - b*c*h^2*p*q*r^2*x*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*
(a + b*x))]^2 + a*d*h^2*p*q*r^2*x*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*
(a + b*x))]^2 + 2*b*d*g^2*p*r*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q
)^r] - 2*b*c*g*h*p*r*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 2*
b*d*g*h*p*r*x*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*b*c*h^2
*p*r*x*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 2*b*d*g^2*q*r*Lo
g[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*a*d*g*h*q*r*Log[c + d*x
]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 2*b*d*g*h*q*r*x*Log[c + d*x]*Log[e
(f(a + b*x)^p*(c + d*x)^q)^r] - 2*a*d*h^2*q*r*x*Log[c + d*x]*Log[e*(f*(a
+ b*x)^p*(c + d*x)^q)^r] - b*d*g^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 +
 b*c*g*h*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + a*d*g*h*Log[e*(f*(a + b*x
)^p*(c + d*x)^q)^r]^2 - a*c*h^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 2*
b*d*g^2*p*q*r^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)] + 2*a*d*g*h*p*q
*r^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)] - 2*b*d*g*h*p*q*r^2*x*Log[
a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)] + 2*a*d*h^2*p*q*r^2*x*Log[a + b*x]*
Log[(b*(g + h*x))/(b*g - a*h)] + 2*b*d*g^2*p*q*r^2*Log[(h*(c + d*x))/(-(d*g
) + c*h)]*Log[(b*(g + h*x))/(b*g - a*h)] - 2*b*c*g*h*p*q*r^2*Log[(h*(c + d*
x))/(-(d*g) + c*h)]*Log[(b*(g + h*x))/(b*g - a*h)] + 2*b*d*g*h*p*q*r^2*x*Lo
g[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(b*(g + h*x))/(b*g - a*h)] - 2*b*c*h^2*
p*q*r^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(b*(g + h*x))/(b*g - a*h)]
- 2*b*d*g^2*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[(b*(g + h*x))/(b*g
 - a*h)] + 2*b*c*g*h*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[(b*(g + h
*x))/(b*g - a*h)] - 2*b*d*g*h*p*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Lo
g[(b*(g + h*x))/(b*g - a*h)] + 2*b*c*h^2*p*r*x*Log[e*(f*(a + b*x)^p*(c + d*
x)^q)^r]*Log[(b*(g + h*x))/(b*g - a*h)] + 2*b*d*g^2*p*q*r^2*Log[a + b*x]*Lo
g[(d*(g + h*x))/(d*g - c*h)] - 2*a*d*g*h*p*q*r^2*Log[a + b*x]*Log[(d*(g + h
*x))/(d*g - c*h)] + 2*b*d*g*h*p*q*r^2*x*Log[a + b*x]*Log[(d*(g + h*x))/(d*g
 - c*h)] - 2*a*d*h^2*p*q*r^2*x*Log[a + b*x]*Log[(d*(g + h*x))/(d*g - c*h)]
- 2*b*d*g^2*p*q*r^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(d*(g + h*x))/(d*
g - c*h)] + 2*b*c*g*h*p*q*r^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(d*(g +

```
h*x))/(d*g - c*h)] - 2*b*d*g*h*p*q*r^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]
*Log[(d*(g + h*x))/(d*g - c*h)] + 2*b*c*h^2*p*q*r^2*x*Log[(h*(c + d*x))/(-(
d*g) + c*h)]*Log[(d*(g + h*x))/(d*g - c*h)] - 2*b*d*g^2*q*r*Log[e*(f*(a + b
*x)^p*(c + d*x)^q)^r]*Log[(d*(g + h*x))/(d*g - c*h)] + 2*a*d*g*h*q*r*Log[e*
(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[(d*(g + h*x))/(d*g - c*h)] - 2*b*d*g*h*q
*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[(d*(g + h*x))/(d*g - c*h)] +
2*a*d*h^2*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[(d*(g + h*x))/(d*g
 - c*h)] + 2*p*(b*c*h*p + a*d*h*q - b*d*g*(p + q))*r^2*(g + h*x)*PolyLog[2,
 (h*(a + b*x))/(-(b*g) + a*h)] + 2*q*(b*c*h*p + a*d*h*q - b*d*g*(p + q))*r^
2*(g + h*x)*PolyLog[2, (h*(c + d*x))/(-(d*g) + c*h)] + 2*b*c*g*h*p*q*r^2*Po
lyLog[2, (b*(c + d*x))/(d*(a + b*x))] - 2*a*d*g...
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(e\left(f\left(bx+a\right)^{p}\left(dx+c\right)^{q}\right)^{r}\right)^{2}}{\left(hx+g\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^2,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^2,x)
```

Maxima [A]
time = 0.34, size = 747, normalized size = 0.90

$$\frac{2\left(\frac{bfp\log(bx+a)}{bg-ah}+\frac{dfq\log(dx+c)}{dg-ch}-\frac{(adfhq-(dfg(p+q)-cfhp)b)\log(hx+g)}{(dgh-ch^2)a-(dg^2-cgh)b}\right)r\log\left(\left(\left(bx+a\right)^{p}\left(dx+c\right)^{q}f\right)^{r}e\right)}{fh}-\frac{\left(\frac{2\left(bcf^2hpq-adf^2hpq\right)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)}{(dgh-ch^2)a-(dg^2-cgh)b}+\frac{2\left(adf^2hpq+(cf^2hp^2-(p^2+pq)df^2g)b\right)\left(\log(bx+a)\log\left(\frac{bhx+ah}{bg-ah}+1\right)+\mathrm{Li}_2\left(-\frac{bhx+ah}{bg-ah}\right)\right)}{(dgh-ch^2)a-(dg^2-cgh)b}+\frac{2\left(adf^2hq^2+(cf^2hpq-(pq+q^2)df^2g)b\right)\left(\log(dx+c)\log\left(\frac{dhx+ch}{dg-ch}+1\right)+\mathrm{Li}_2\left(-\frac{dhx+ch}{dg-ch}\right)\right)}{(dgh-ch^2)a-(dg^2-cgh)b}-\frac{(df^2gp^2-cf^2hp^2)b\log(bx+a)^2+2\left(bdf^2gpq-adf^2hpq\right)\log(bx+a)\log(dx+c)+\left(bdf^2gq^2-adf^2hq^2\right)\log(dx+c)^2+2\left(\left(adf^2hpq+(cf^2hp^2-(p^2+pq)df^2g)b\right)\log(bx+a)+\left(adf^2hq^2+(cf^2hpq-(pq+q^2)df^2g)b\right)\log(dx+c)\right)\log(hx+g)}{(dgh-ch^2)a-(dg^2-cgh)b}\right)r^2}{f^2h}-\frac{\log\left(\left(\left(bx+a\right)^{p}\left(dx+c\right)^{q}f\right)^{r}e\right)^{2}}{(hx+g)h}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^2,x, algorithm="maxima
")
```

```
[Out] 2*(b*f*p*log(b*x + a)/(b*g - a*h) + d*f*q*log(d*x + c)/(d*g - c*h) - (a*d*f
*h*q - (d*f*g*(p + q) - c*f*h*p)*b)*log(h*x + g)/((d*g*h - c*h^2)*a - (d*g^
2 - c*g*h)*b))*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(f*h) - (2*(b*c*f^2*h
*p*q - a*d*f^2*h*p*q)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + di
log(-(b*d*x + a*d)/(b*c - a*d)))/((d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b) +
2*(a*d*f^2*h*p*q + (c*f^2*h*p^2 - (p^2 + p*q)*d*f^2*g)*b)*(log(b*x + a)*log
((b*h*x + a*h)/(b*g - a*h) + 1) + dilog(-(b*h*x + a*h)/(b*g - a*h)))/((d*g*
h - c*h^2)*a - (d*g^2 - c*g*h)*b) + 2*(a*d*f^2*h*q^2 + (c*f^2*h*p*q - (p*q
+ q^2)*d*f^2*g)*b)*(log(d*x + c)*log((d*h*x + c*h)/(d*g - c*h) + 1) + dilog
(-(d*h*x + c*h)/(d*g - c*h)))/((d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b) - ((d
*f^2*g*p^2 - c*f^2*h*p^2)*b*log(b*x + a)^2 + 2*(b*d*f^2*g*p*q - a*d*f^2*h*p
*q)*log(b*x + a)*log(d*x + c) + (b*d*f^2*g*q^2 - a*d*f^2*h*q^2)*log(d*x + c
)^2 + 2*((a*d*f^2*h*p*q + (c*f^2*h*p^2 - (p^2 + p*q)*d*f^2*g)*b)*log(b*x +
a) + (a*d*f^2*h*q^2 + (c*f^2*h*p*q - (p*q + q^2)*d*f^2*g)*b)*log(d*x + c))*
```

```
log(h*x + g))/((d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b))*r^2/(f^2*h) - log(((
b*x + a)^p*(d*x + c)^q*f)^r*e)^2/((h*x + g)*h)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^2,x, algorithm="fricas
")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h^2*x^2 + 2*g*h*x + g^2),
x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(h*x+g)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^2,x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h*x + g)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\,{\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)}^r\right)^2}{{\left(g+h\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(g + h*x)^2,x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(g + h*x)^2, x)
```

3.41 $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{(g+hx)^3} dx$

Optimal. Leaf size=1304

$$-\frac{bdpqr^2\log(a+bx)}{h(bg-ah)(dg-ch)}+\frac{dpqr^2\log(a+bx)}{h(dg-ch)(g+hx)}-\frac{bp^2r^2(a+bx)\log(a+bx)}{(bg-ah)^2(g+hx)}-\frac{bdpqr^2\log(c+dx)}{h(bg-ah)(dg-ch)}+\frac{bpqr^2\log(c}{h(bg-ah)(}$$

```
[Out] -1/2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/h/(h*x+g)^2-d^2*q^2*r^2*ln(d*x+c)*ln
(1+(-c*h+d*g)/h/(d*x+c))/h/(-c*h+d*g)^2+b^2*p^2*r^2*ln(h*x+g)/h/(-a*h+b*g)^
2+d^2*q^2*r^2*ln(h*x+g)/h/(-c*h+d*g)^2+b^2*p^2*r^2*polylog(2,(a*h-b*g)/h/(b
*x+a))/h/(-a*h+b*g)^2+d^2*q^2*r^2*polylog(2,(c*h-d*g)/h/(d*x+c))/h/(-c*h+d*
g)^2+d^2*p*q*r^2*polylog(2,-d*(b*x+a)/(-a*d+b*c))/h/(-c*h+d*g)^2-d^2*p*q*r^
2*polylog(2,-h*(b*x+a)/(-a*h+b*g))/h/(-c*h+d*g)^2+b^2*p*q*r^2*polylog(2,b*(
d*x+c)/(-a*d+b*c))/h/(-a*h+b*g)^2-b^2*p*q*r^2*polylog(2,-h*(d*x+c)/(-c*h+d*
g))/h/(-a*h+b*g)^2-b*p^2*r^2*(b*x+a)*ln(b*x+a)/(-a*h+b*g)^2/(h*x+g)-d*q^2*r
^2*(d*x+c)*ln(d*x+c)/(-c*h+d*g)^2/(h*x+g)-b*p*r*(p*r*ln(b*x+a)+q*r*ln(d*x+c
)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h/(-a*h+b*g)/(h*x+g)-d*q*r*(p*r*ln(b*x+a
)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h/(-c*h+d*g)/(h*x+g)-b^2*p
*r*ln(b*x+a)*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/
h/(-a*h+b*g)^2-d^2*q*r*ln(d*x+c)*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+
a)^p*(d*x+c)^q)^r))/h/(-c*h+d*g)^2+b^2*p*r*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(
e*(f*(b*x+a)^p*(d*x+c)^q)^r))*ln(h*x+g)/h/(-a*h+b*g)^2+d^2*q*r*(p*r*ln(b*x+
a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))*ln(h*x+g)/h/(-c*h+d*g)^2-
b^2*p^2*r^2*ln(b*x+a)*ln(1+(-a*h+b*g)/h/(b*x+a))/h/(-a*h+b*g)^2-b*d*p*q*r^2
*ln(b*x+a)/h/(-a*h+b*g)/(-c*h+d*g)-b*d*p*q*r^2*ln(d*x+c)/h/(-a*h+b*g)/(-c*h
+d*g)+2*b*d*p*q*r^2*ln(h*x+g)/h/(-a*h+b*g)/(-c*h+d*g)+d*p*q*r^2*ln(b*x+a)/h
/(-c*h+d*g)/(h*x+g)+b*p*q*r^2*ln(d*x+c)/h/(-a*h+b*g)/(h*x+g)+b^2*p*q*r^2*ln
(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/h/(-a*h+b*g)^2+d^2*p*q*r^2*ln(b*x+a)*ln(b
*(d*x+c)/(-a*d+b*c))/h/(-c*h+d*g)^2-d^2*p*q*r^2*ln(b*x+a)*ln(b*(h*x+g)/(-a*
h+b*g))/h/(-c*h+d*g)^2-b^2*p*q*r^2*ln(d*x+c)*ln(d*(h*x+g)/(-c*h+d*g))/h/(-a
*h+b*g)^2
```

Rubi [A]
time = 0.92, antiderivative size = 1304, normalized size of antiderivative = 1.00, number of steps used = 43, number of rules used = 14, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {2584, 2593, 2458, 2389, 2379, 2438, 2351, 31, 2465, 2441, 2440, 2442, 36, 46}

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x)^3,x]
```

```
[Out] -((b*d*p*q*r^2*Log[a + b*x])/(h*(b*g - a*h)*(d*g - c*h))) + (d*p*q*r^2*Log[
a + b*x])/(h*(d*g - c*h)*(g + h*x)) - (b*p^2*r^2*(a + b*x)*Log[a + b*x])/((
```

```
b*g - a*h)^2*(g + h*x)) - (b*d*p*q*r^2*Log[c + d*x])/(h*(b*g - a*h)*(d*g -
c*h)) + (b*p*q*r^2*Log[c + d*x])/(h*(b*g - a*h)*(g + h*x)) - (d*q^2*r^2*(c
+ d*x)*Log[c + d*x])/((d*g - c*h)^2*(g + h*x)) + (b^2*p*q*r^2*Log[-((d*(a +
 b*x))/(b*c - a*d))]*Log[c + d*x])/(h*(b*g - a*h)^2) + (d^2*p*q*r^2*Log[a +
 b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(h*(d*g - c*h)^2) - (b*p*r*(p*r*Log[a
 + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(h*(b*g
 - a*h)*(g + h*x)) - (d*q*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]))/(h*(d*g - c*h)*(g + h*x)) - (b^2*p*r*Log[a +
 b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)
^q)^r]))/(h*(b*g - a*h)^2) - (d^2*q*r*Log[c + d*x]*(p*r*Log[a + b*x] + q*r*
Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(h*(d*g - c*h)^2) - L
og[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(2*h*(g + h*x)^2) + (b^2*p^2*r^2*Log[
g + h*x])/(h*(b*g - a*h)^2) + (2*b*d*p*q*r^2*Log[g + h*x])/(h*(b*g - a*h)*(
d*g - c*h)) + (d^2*q^2*r^2*Log[g + h*x])/(h*(d*g - c*h)^2) + (b^2*p*r*(p*r*
Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*Log
[g + h*x])/(h*(b*g - a*h)^2) + (d^2*q*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x
] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*Log[g + h*x])/(h*(d*g - c*h)^2) -
 (d^2*p*q*r^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)])/(h*(d*g - c*h)^2
) - (b^2*p*q*r^2*Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)])/(h*(b*g - a*h
)^2) - (b^2*p^2*r^2*Log[a + b*x]*Log[1 + (b*g - a*h)/(h*(a + b*x))])/(h*(b*
g - a*h)^2) - (d^2*q^2*r^2*Log[c + d*x]*Log[1 + (d*g - c*h)/(h*(c + d*x))])
/(h*(d*g - c*h)^2) + (b^2*p^2*r^2*PolyLog[2, -((b*g - a*h)/(h*(a + b*x)))])
/(h*(b*g - a*h)^2) + (d^2*p*q*r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])
/(h*(d*g - c*h)^2) - (d^2*p*q*r^2*PolyLog[2, -((h*(a + b*x))/(b*g - a*h))])
/(h*(d*g - c*h)^2) + (d^2*q^2*r^2*PolyLog[2, -((d*g - c*h)/(h*(c + d*x)))])
/(h*(d*g - c*h)^2) + (b^2*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(h
*(b*g - a*h)^2) - (b^2*p*q*r^2*PolyLog[2, -((h*(c + d*x))/(d*g - c*h))])/(h
*(b*g - a*h)^2)
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

<u>Rule 36</u>

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

<u>Rule 46</u>

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
```

```
eQ[q, -1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2584

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Dist[b*p*r*(
s/(h*(m + 1))), Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1)/(a + b*x)), x], x] - Dist[d*q*r*(s/(h*(m + 1))), Int[(g + h*x)^(m
+ 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\int \frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^3}\,dx = -\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2} + \frac{(bpr)\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)(g+hx)^2}\,dx}{h} +$$

$$= -\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2} + \frac{(bp^2r^2)\int \frac{\log(a+bx)}{(a+bx)(g+hx)^2}\,dx}{h} + \frac{(bpqr^2}{}$$

$$= -\frac{\log^2\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2h(g+hx)^2} + \frac{(p^2r^2)\,\mathrm{Subst}\left(\int \frac{\log(x)}{x\left(\frac{bg-ah}{b}+\frac{hx}{b}\right)^2}\,dx, x\right.}{h}$$

$$= -\frac{bpr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{h(bg-ah)(g+hx)}$$

$$= \frac{dpqr^2\log(a+bx)}{h(dg-ch)(g+hx)} - \frac{bp^2r^2(a+bx)\log(a+bx)}{(bg-ah)^2(g+hx)} + \frac{bpqr^2\log(c+dx}{h(bg-ah)(g+h}$$

$$= \frac{dpqr^2\log(a+bx)}{h(dg-ch)(g+hx)} - \frac{bp^2r^2(a+bx)\log(a+bx)}{(bg-ah)^2(g+hx)} + \frac{b^2p^2r^2\log^2(a+bx}{2h(bg-ah)^2}$$

$$= -\frac{bdpqr^2\log(a+bx)}{h(bg-ah)(dg-ch)} + \frac{dpqr^2\log(a+bx)}{h(dg-ch)(g+hx)} - \frac{bp^2r^2(a+bx)\log(a-}{(bg-ah)^2(g+h}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 12086 vs. $2(1304) = 2608$.
time = 3.98, size = 12086, normalized size = 9.27

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x)^3,x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f(bx+a)^p\,(dx+c)^q\right)^r\right)^2}{(hx+g)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^3,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^3,x)
```

Maxima [A]
time = 0.58, size = 1859, normalized size = 1.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^3,x, algorithm="maxima
")

[Out] (b^2*f*p*log(b*x + a)/(b^2*g^2 - 2*a*b*g*h + a^2*h^2) + d^2*f*q*log(d*x + c
)/(d^2*g^2 - 2*c*d*g*h + c^2*h^2) + (2*a*b*d^2*f*g*h*q - a^2*d^2*f*h^2*q -
(d^2*f*g^2*(p + q) - 2*c*d*f*g*h*p + c^2*f*h^2*p)*b^2)*log(h*x + g)/((d^2*g
^2*h^2 - 2*c*d*g*h^3 + c^2*h^4)*a^2 - 2*(d^2*g^3*h - 2*c*d*g^2*h^2 + c^2*g*
h^3)*a*b + (d^2*g^4 - 2*c*d*g^3*h + c^2*g^2*h^2)*b^2) + (a*d*f*h*q - (d*f*g
*(p + q) - c*f*h*p)*b)/((d*g^2*h - c*g*h^2)*a - (d*g^3 - c*g^2*h)*b + ((d*g
*h^2 - c*h^3)*a - (d*g^2*h - c*g*h^2)*b)*x))*r*log(((b*x + a)^p*(d*x + c)^q
*f)^r*e)/(f*h) + 1/2*(2*(2*a*b*d^2*f^2*g*h*p*q - a^2*d^2*f^2*h^2*p*q - (2*c
*d*f^2*g*h*p*q - c^2*f^2*h^2*p*q)*b^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c
 - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/((d*g*h^2 - c*h^3)*a^2 -
2*(d*g^2*h - c*g*h^2)*a*b + (d*g^3 - c*g^2*h)*b^2) - 2*(2*a*b*d^2*f^2*g*h*p
*q - a^2*d^2*f^2*h^2*p*q + (2*c*d*f^2*g*h*p^2 - c^2*f^2*h^2*p^2 - (p^2 + p*
q)*d^2*f^2*g^2)*b^2)*(log(b*x + a)*log((b*h*x + a*h)/(b*g - a*h) + 1) + dil
og(-(b*h*x + a*h)/(b*g - a*h)))/((d*g*h^2 - c*h^3)*a^2 - 2*(d*g^2*h - c*g*h
^2)*a*b + (d*g^3 - c*g^2*h)*b^2) + 2*(2*a*b*d^2*f^2*g*h*q^2 - a^2*d^2*f^2*h
^2*q^2 + (2*c*d*f^2*g*h*p*q - c^2*f^2*h^2*p*q - (p*q + q^2)*d^2*f^2*g^2)*b^
2)*(log(d*x + c)*log((d*h*x + c*h)/(d*g - c*h) + 1) + dilog(-(d*h*x + c*h)/
(d*g - c*h)))/((d^2*g^2*h^2 - 2*c*d*g*h^3 + c^2*h^4)*a^2 - 2*(d^2*g^3*h - 2
*c*d*g^2*h^2 + c^2*g*h^3)*a*b + (d^2*g^4 - 2*c*d*g^3*h + c^2*g^2*h^2)*b^2)
 - 2*(a*d^2*f^2*h*q^2 + (c*d*f^2*h*p*q - (p*q + q^2)*d^2*f^2*g)*b)*log(d*x +
 c)/((d^2*g^2*h - 2*c*d*g*h^2 + c^2*h^3)*a - (d^2*g^3 - 2*c*d*g^2*h + c^2*g
*h^2)*b) + 2*(a^2*d^2*f^2*h^2*q^2 + 2*(c*d*f^2*h^2*p*q - (p*q + q^2)*d^2*f^
2*g*h)*a*b + (c^2*f^2*h^2*p^2 + (p^2 + 2*p*q + q^2)*d^2*f^2*g^2 - 2*(p^2 +
p*q)*c*d*f^2*g*h)*b^2)*log(h*x + g)/((d^2*g^2*h^2 - 2*c*d*g*h^3 + c^2*h^4)*
a^2 - 2*(d^2*g^3*h - 2*c*d*g^2*h^2 + c^2*g*h^3)*a*b + (d^2*g^4 - 2*c*d*g^3*
h + c^2*g^2*h^2)*b^2) + ((d^3*f^2*g^3*p^2 - 3*c*d^2*f^2*g^2*h*p^2 + 3*c^2*d
*f^2*g*h^2*p^2 - c^3*f^2*h^3*p^2)*b^2*log(b*x + a)^2 - 2*(b^2*d^2*f^2*g^2*p
*q - 2*a*b*d^2*f^2*g*h*p*q + a^2*d^2*f^2*h^2*p*q)*log(b*x + a)*log(d*x + c)
 - (b^2*d^2*f^2*g^2*q^2 - 2*a*b*d^2*f^2*g*h*q^2 + a^2*d^2*f^2*h^2*q^2)*log(
d*x + c)^2 + 2*((d^2*f^2*g*h*p*q - c*d*f^2*h^2*p*q)*a*b - (c^2*f^2*h^2*p^2
+ (p^2 + p*q)*d^2*f^2*g^2 - (2*p^2 + p*q)*c*d*f^2*g*h)*b^2)*log(b*x + a) -
2*((2*a*b*d^2*f^2*g*h*p*q - a^2*d^2*f^2*h^2*p*q + (2*c*d*f^2*g*h*p^2 - c^2*
f^2*h^2*p^2 - (p^2 + p*q)*d^2*f^2*g^2)*b^2)*log(b*x + a) + (2*a*b*d^2*f^2*g
*h*q^2 - a^2*d^2*f^2*h^2*q^2 + (2*c*d*f^2*g*h*p*q - c^2*f^2*h^2*p*q - (p*q
```

```
+ q^2)*d^2*f^2*g^2)*b^2)*log(d*x + c))*log(h*x + g))/((d^2*g^2*h^2 - 2*c*d*
g*h^3 + c^2*h^4)*a^2 - 2*(d^2*g^3*h - 2*c*d*g^2*h^2 + c^2*g*h^3)*a*b + (d^2
*g^4 - 2*c*d*g^3*h + c^2*g^2*h^2)*b^2))*r^2/(f^2*h) - 1/2*log(((b*x + a)^p*
(d*x + c)^q*f)^r*e)^2/((h*x + g)^2*h)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^3,x, algorithm="fricas
")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h^3*x^3 + 3*g*h^2*x^2 + 3*
g^2*h*x + g^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(h*x+g)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^3,x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h*x + g)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\,{\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)}^r\right)^2}{{\left(g+h\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(g + h*x)^3,x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(g + h*x)^3, x)
```

3.42 $\int \frac{\log^2(e(f(a+bx)^p(c+dx)^q)^r)}{(g+hx)^4}\,dx$

Optimal. Leaf size=1957

$$-\frac{b^2p^2r^2}{3h(bg-ah)^2(g+hx)}-\frac{2bdpqr^2}{3h(bg-ah)(dg-ch)(g+hx)}-\frac{d^2q^2r^2}{3h(dg-ch)^2(g+hx)}-\frac{b^3p^2r^2\log(a+bx)}{3h(bg-ah)^3}-\frac{2bc}{3h(b}$$

```
[Out] -1/3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/h/(h*x+g)^3-1/3*b^2*p^2*r^2/h/(-a*h+
b*g)^2/(h*x+g)-1/3*d^2*q^2*r^2/h/(-c*h+d*g)^2/(h*x+g)+2/3*b^3*p^2*r^2*polyl
og(2,(a*h-b*g)/h/(b*x+a))/h/(-a*h+b*g)^3+2/3*d^3*q^2*r^2*polylog(2,(c*h-d*g
)/h/(d*x+c))/h/(-c*h+d*g)^3-1/3*b^3*p^2*r^2*ln(b*x+a)/h/(-a*h+b*g)^3-1/3*d^
3*q^2*r^2*ln(d*x+c)/h/(-c*h+d*g)^3+b^3*p^2*r^2*ln(h*x+g)/h/(-a*h+b*g)^3+d^3
*q^2*r^2*ln(h*x+g)/h/(-c*h+d*g)^3+2/3*d^3*p*q*r^2*polylog(2,-d*(b*x+a)/(-a*
d+b*c))/h/(-c*h+d*g)^3-2/3*d^3*p*q*r^2*polylog(2,-h*(b*x+a)/(-a*h+b*g))/h/(
-c*h+d*g)^3+2/3*b^3*p*q*r^2*polylog(2,b*(d*x+c)/(-a*d+b*c))/h/(-a*h+b*g)^3-
2/3*b^3*p*q*r^2*polylog(2,-h*(d*x+c)/(-c*h+d*g))/h/(-a*h+b*g)^3+1/3*b*p^2*r
^2*ln(b*x+a)/h/(-a*h+b*g)/(h*x+g)^2-2/3*b^2*p^2*r^2*(b*x+a)*ln(b*x+a)/(-a*h
+b*g)^3/(h*x+g)+1/3*d*q^2*r^2*ln(d*x+c)/h/(-c*h+d*g)/(h*x+g)^2-2/3*d^2*q^2*
r^2*(d*x+c)*ln(d*x+c)/(-c*h+d*g)^3/(h*x+g)-1/3*b*p*r*(p*r*ln(b*x+a)+q*r*ln(
d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h/(-a*h+b*g)/(h*x+g)^2-1/3*d*q*r*(p
*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h/(-c*h+d*g)/(h
*x+g)^2-2/3*b^2*p*r*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^
q)^r))/h/(-a*h+b*g)^2/(h*x+g)-2/3*d^2*q*r*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e
*(f*(b*x+a)^p*(d*x+c)^q)^r))/h/(-c*h+d*g)^2/(h*x+g)-2/3*b^3*p*r*ln(b*x+a)*(
p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))/h/(-a*h+b*g)^3
-2/3*d^3*q*r*ln(d*x+c)*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+
c)^q)^r))/h/(-c*h+d*g)^3+2/3*b^3*p*r*(p*r*ln(b*x+a)+q*r*ln(d*x+c)-ln(e*(f*(
b*x+a)^p*(d*x+c)^q)^r))*ln(h*x+g)/h/(-a*h+b*g)^3+2/3*d^3*q*r*(p*r*ln(b*x+a)
+q*r*ln(d*x+c)-ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r))*ln(h*x+g)/h/(-c*h+d*g)^3-2/
3*b^3*p^2*r^2*ln(b*x+a)*ln(1+(-a*h+b*g)/h/(b*x+a))/h/(-a*h+b*g)^3-2/3*d^3*q
^2*r^2*ln(d*x+c)*ln(1+(-c*h+d*g)/h/(d*x+c))/h/(-c*h+d*g)^3-2/3*b^2*d*p*q*r^
2*ln(d*x+c)/h/(-a*h+b*g)^2/(-c*h+d*g)+b*d^2*p*q*r^2*ln(h*x+g)/h/(-a*h+b*g)/
(-c*h+d*g)^2+b^2*d*p*q*r^2*ln(h*x+g)/h/(-a*h+b*g)^2/(-c*h+d*g)-2/3*b*d^2*p*
q*r^2*ln(b*x+a)/h/(-a*h+b*g)/(-c*h+d*g)^2-1/3*b^2*d*p*q*r^2*ln(b*x+a)/h/(-a
*h+b*g)^2/(-c*h+d*g)-1/3*b*d^2*p*q*r^2*ln(d*x+c)/h/(-a*h+b*g)/(-c*h+d*g)^2+
2/3*b^3*p*q*r^2*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/h/(-a*h+b*g)^3+2/3*d^3*
p*q*r^2*ln(b*x+a)*ln(b*(d*x+c)/(-a*d+b*c))/h/(-c*h+d*g)^3-2/3*d^3*p*q*r^2*l
n(b*x+a)*ln(b*(h*x+g)/(-a*h+b*g))/h/(-c*h+d*g)^3-2/3*b^3*p*q*r^2*ln(d*x+c)*
ln(d*(h*x+g)/(-c*h+d*g))/h/(-a*h+b*g)^3+1/3*d*p*q*r^2*ln(b*x+a)/h/(-c*h+d*g
)/(h*x+g)^2+2/3*d^2*p*q*r^2*ln(b*x+a)/h/(-c*h+d*g)^2/(h*x+g)+1/3*b*p*q*r^2*
ln(d*x+c)/h/(-a*h+b*g)/(h*x+g)^2+2/3*b^2*p*q*r^2*ln(d*x+c)/h/(-a*h+b*g)^2/(
h*x+g)-2/3*b*d*p*q*r^2/h/(-a*h+b*g)/(-c*h+d*g)/(h*x+g)
```

Rubi [A]

time = 1.43, antiderivative size = 1957, normalized size of antiderivative = 1.00, number of steps used = 57, number of rules used = 15, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.484$, Rules used = {2584, 2593, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46, 2465, 2441, 2440, 2442, 36}

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x)^4,x]
```

```
[Out] -1/3*(b^2*p^2*r^2)/(h*(b*g - a*h)^2*(g + h*x)) - (2*b*d*p*q*r^2)/(3*h*(b*g
- a*h)*(d*g - c*h)*(g + h*x)) - (d^2*q^2*r^2)/(3*h*(d*g - c*h)^2*(g + h*x))
 - (b^3*p^2*r^2*Log[a + b*x])/(3*h*(b*g - a*h)^3) - (2*b*d^2*p*q*r^2*Log[a
+ b*x])/(3*h*(b*g - a*h)*(d*g - c*h)^2) - (b^2*d*p*q*r^2*Log[a + b*x])/(3*h
*(b*g - a*h)^2*(d*g - c*h)) + (b*p^2*r^2*Log[a + b*x])/(3*h*(b*g - a*h)*(g
+ h*x)^2) + (d*p*q*r^2*Log[a + b*x])/(3*h*(d*g - c*h)*(g + h*x)^2) + (2*d^2
*p*q*r^2*Log[a + b*x])/(3*h*(d*g - c*h)^2*(g + h*x)) - (2*b^2*p^2*r^2*(a +
b*x)*Log[a + b*x])/(3*(b*g - a*h)^3*(g + h*x)) - (b*d^2*p*q*r^2*Log[c + d*x
])/(3*h*(b*g - a*h)*(d*g - c*h)^2) - (2*b^2*d*p*q*r^2*Log[c + d*x])/(3*h*(b
*g - a*h)^2*(d*g - c*h)) - (d^3*q^2*r^2*Log[c + d*x])/(3*h*(d*g - c*h)^3) +
 (b*p*q*r^2*Log[c + d*x])/(3*h*(b*g - a*h)*(g + h*x)^2) + (d*q^2*r^2*Log[c
+ d*x])/(3*h*(d*g - c*h)*(g + h*x)^2) + (2*b^2*p*q*r^2*Log[c + d*x])/(3*h*(
b*g - a*h)^2*(g + h*x)) - (2*d^2*q^2*r^2*(c + d*x)*Log[c + d*x])/(3*(d*g -
c*h)^3*(g + h*x)) + (2*b^3*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c
+ d*x])/(3*h*(b*g - a*h)^3) + (2*d^3*p*q*r^2*Log[a + b*x]*Log[(b*(c + d*x))
/(b*c - a*d)])/(3*h*(d*g - c*h)^3) - (b*p*r*(p*r*Log[a + b*x] + q*r*Log[c +
 d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(3*h*(b*g - a*h)*(g + h*x)^2
) - (d*q*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]))/(3*h*(d*g - c*h)*(g + h*x)^2) - (2*b^2*p*r*(p*r*Log[a + b*x]
+ q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(3*h*(b*g - a*h
)^2*(g + h*x)) - (2*d^2*q*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]))/(3*h*(d*g - c*h)^2*(g + h*x)) - (2*b^3*p*r*L
og[a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c
+ d*x)^q)^r]))/(3*h*(b*g - a*h)^3) - (2*d^3*q*r*Log[c + d*x]*(p*r*Log[a + b
*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(3*h*(d*g -
 c*h)^3) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(3*h*(g + h*x)^3) + (b^3*
p^2*r^2*Log[g + h*x])/(h*(b*g - a*h)^3) + (b*d^2*p*q*r^2*Log[g + h*x])/(h*(
b*g - a*h)*(d*g - c*h)^2) + (b^2*d*p*q*r^2*Log[g + h*x])/(h*(b*g - a*h)^2*(
d*g - c*h)) + (d^3*q^2*r^2*Log[g + h*x])/(h*(d*g - c*h)^3) + (2*b^3*p*r*(p*
r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*L
og[g + h*x])/(3*h*(b*g - a*h)^3) + (2*d^3*q*r*(p*r*Log[a + b*x] + q*r*Log[c
 + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*Log[g + h*x])/(3*h*(d*g - c
*h)^3) - (2*d^3*p*q*r^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)])/(3*h*(
d*g - c*h)^3) - (2*b^3*p*q*r^2*Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)])
/(3*h*(b*g - a*h)^3) - (2*b^3*p^2*r^2*Log[a + b*x]*Log[1 + (b*g - a*h)/(h*(
a + b*x))])/(3*h*(b*g - a*h)^3) - (2*d^3*q^2*r^2*Log[c + d*x]*Log[1 + (d*g
```

```
- c*h)/(h*(c + d*x))])/(3*h*(d*g - c*h)^3) + (2*b^3*p^2*r^2*PolyLog[2, -((b
*g - a*h)/(h*(a + b*x)))])/(3*h*(b*g - a*h)^3) + (2*d^3*p*q*r^2*PolyLog[2,
-((d*(a + b*x))/(b*c - a*d))])/(3*h*(d*g - c*h)^3) - (2*d^3*p*q*r^2*PolyLog
[2, -((h*(a + b*x))/(b*g - a*h))])/(3*h*(d*g - c*h)^3) + (2*d^3*q^2*r^2*Pol
yLog[2, -((d*g - c*h)/(h*(c + d*x)))])/(3*h*(d*g - c*h)^3) + (2*b^3*p*q*r^2
*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(3*h*(b*g - a*h)^3) - (2*b^3*p*q*r^
2*PolyLog[2, -((h*(c + d*x))/(d*g - c*h))])/(3*h*(b*g - a*h)^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
```

```
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2584

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1
)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(h*(m + 1))), x] + (-Dist[b*p*r*(
s/(h*(m + 1))), Int[(g + h*x)^(m + 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1)/(a + b*x)), x], x] - Dist[d*q*r*(s/(h*(m + 1))), Int[(g + h*x)^(m
+ 1)*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\int \frac{\log^2 \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(g+hx)^4}\,dx = -\frac{\log^2 \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(g+hx)^3} + \frac{(2bpr)\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(a+bx)(g+hx)^3}\,dx}{3h} +$$

$$= -\frac{\log^2 \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(g+hx)^3} + \frac{(2bp^2r^2)\int \frac{\log(a+bx)}{(a+bx)(g+hx)^3}\,dx}{3h} + \frac{(2bp}{}$$

$$= -\frac{\log^2 \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(g+hx)^3} + \frac{(2p^2r^2)\,\mathrm{Subst}\left(\int \frac{\log(x)}{x\left(\frac{bg-ah}{b}+\frac{hx}{b}\right)^3}\,dx,\right.}{3h}$$

$$= -\frac{bpr(pr\log(a+bx)+qr\log(c+dx)-\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3h(bg-ah)(g+hx)^2}$$

$$= \frac{bp^2r^2\log(a+bx)}{3h(bg-ah)(g+hx)^2} + \frac{dpqr^2\log(a+bx)}{3h(dg-ch)(g+hx)^2} + \frac{2d^2pqr^2\log(a+b}{3h(dg-ch)^2(g+}$$

$$= \frac{bp^2r^2\log(a+bx)}{3h(bg-ah)(g+hx)^2} + \frac{dpqr^2\log(a+bx)}{3h(dg-ch)(g+hx)^2} + \frac{2d^2pqr^2\log(a+b}{3h(dg-ch)^2(g+}$$

$$= -\frac{b^2p^2r^2}{3h(bg-ah)^2(g+hx)} - \frac{2bdpqr^2}{3h(bg-ah)(dg-ch)(g+hx)} - \frac{}{3h(dg-}$$

$$= -\frac{b^2p^2r^2}{3h(bg-ah)^2(g+hx)} - \frac{2bdpqr^2}{3h(bg-ah)(dg-ch)(g+hx)} - \frac{}{3h(dg-}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 47127 vs. $2(1957) = 3914$.
time = 6.33, size = 47127, normalized size = 24.08

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x)^4,x]
```

```
[Out] Result too large to show
```

Maple [**F**]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f(bx+a)^p\,(dx+c)^q\right)^r\right)^2}{(hx+g)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^4,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^4,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 4734 vs. $2(1884) = 3768$.
time = 1.20, size = 4734, normalized size = 2.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^4,x, algorithm="maxima
")
```

```
[Out] 1/3*(2*b^3*f*p*log(b*x + a)/(b^3*g^3 - 3*a*b^2*g^2*h + 3*a^2*b*g*h^2 - a^3*
h^3) + 2*d^3*f*q*log(d*x + c)/(d^3*g^3 - 3*c*d^2*g^2*h + 3*c^2*d*g*h^2 - c^
3*h^3) - 2*(3*a*b^2*d^3*f*g^2*h*q - 3*a^2*b*d^3*f*g*h^2*q + a^3*d^3*f*h^3*q
 - (d^3*f*g^3*(p + q) - 3*c*d^2*f*g^2*h*p + 3*c^2*d*f*g*h^2*p - c^3*f*h^3*p
)*b^3)*log(h*x + g)/((d^3*g^3*h^3 - 3*c*d^2*g^2*h^4 + 3*c^2*d*g*h^5 - c^3*h
^6)*a^3 - 3*(d^3*g^4*h^2 - 3*c*d^2*g^3*h^3 + 3*c^2*d*g^2*h^4 - c^3*g*h^5)*a
^2*b + 3*(d^3*g^5*h - 3*c*d^2*g^4*h^2 + 3*c^2*d*g^3*h^3 - c^3*g^2*h^4)*a*b^
2 - (d^3*g^6 - 3*c*d^2*g^5*h + 3*c^2*d*g^4*h^2 - c^3*g^3*h^3)*b^3) + ((3*d^
2*f*g*h^2*q - c*d*f*h^3*q)*a^2 - (d^2*f*g^2*h*(p + 6*q) - 2*c*d*f*g*h^2*(p
+ q) + c^2*f*h^3*p)*a*b - (c*d*f*g^2*h*(6*p + q) - 3*d^2*f*g^3*(p + q) - 3*
c^2*f*g*h^2*p)*b^2 - 2*(2*a*b*d^2*f*g*h^2*q - a^2*d^2*f*h^3*q - (d^2*f*g^2*
h*(p + q) - 2*c*d*f*g*h^2*p + c^2*f*h^3*p)*b^2)*x)/((d^2*g^4*h^2 - 2*c*d*g^
3*h^3 + c^2*g^2*h^4)*a^2 - 2*(d^2*g^5*h - 2*c*d*g^4*h^2 + c^2*g^3*h^3)*a*b
+ (d^2*g^6 - 2*c*d*g^5*h + c^2*g^4*h^2)*b^2 + ((d^2*g^2*h^4 - 2*c*d*g*h^5 +
 c^2*h^6)*a^2 - 2*(d^2*g^3*h^3 - 2*c*d*g^2*h^4 + c^2*g*h^5)*a*b + (d^2*g^4*
h^2 - 2*c*d*g^3*h^3 + c^2*g^2*h^4)*b^2)*x^2 + 2*((d^2*g^3*h^3 - 2*c*d*g^2*h
^4 + c^2*g*h^5)*a^2 - 2*(d^2*g^4*h^2 - 2*c*d*g^3*h^3 + c^2*g^2*h^4)*a*b + (
d^2*g^5*h - 2*c*d*g^4*h^2 + c^2*g^3*h^3)*b^2)*x))*r*log(((b*x + a)^p*(d*x +
 c)^q*f)^r*e)/(f*h) + 1/3*(2*(3*a*b^2*d^3*f^2*g^2*h*p*q - 3*a^2*b*d^3*f^2*g
*h^2*p*q + a^3*d^3*f^2*h^3*p*q - (3*c*d^2*f^2*g^2*h*p*q - 3*c^2*d*f^2*g*h^2
*p*q + c^3*f^2*h^3*p*q)*b^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) +
1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/((d^3*g^3*h^3 - 3*c*d^2*g^2*h^4 + 3
*c^2*d*g*h^5 - c^3*h^6)*a^3 - 3*(d^3*g^4*h^2 - 3*c*d^2*g^3*h^3 + 3*c^2*d*g^
2*h^4 - c^3*g*h^5)*a^2*b + 3*(d^3*g^5*h - 3*c*d^2*g^4*h^2 + 3*c^2*d*g^3*h^3
 - c^3*g^2*h^4)*a*b^2 - (d^3*g^6 - 3*c*d^2*g^5*h + 3*c^2*d*g^4*h^2 - c^3*g^
3*h^3)*b^3) - 2*(3*a*b^2*d^3*f^2*g^2*h*p*q - 3*a^2*b*d^3*f^2*g*h^2*p*q + a^
3*d^3*f^2*h^3*p*q + (3*c*d^2*f^2*g^2*h*p^2 - 3*c^2*d*f^2*g*h^2*p^2 + c^3*f^
2*h^3*p^2 - (p^2 + p*q)*d^3*f^2*g^3)*b^3)*(log(b*x + a)*log((b*h*x + a*h)/(
b*g - a*h) + 1) + dilog(-(b*h*x + a*h)/(b*g - a*h)))/((d^3*g^3*h^3 - 3*c*d^
2*g^2*h^4 + 3*c^2*d*g*h^5 - c^3*h^6)*a^3 - 3*(d^3*g^4*h^2 - 3*c*d^2*g^3*h^3
```

```
 + 3*c^2*d*g^2*h^4 - c^3*g*h^5)*a^2*b + 3*(d^3*g^5*h - 3*c*d^2*g^4*h^2 + 3*
c^2*d*g^3*h^3 - c^3*g^2*h^4)*a*b^2 - (d^3*g^6 - 3*c*d^2*g^5*h + 3*c^2*d*g^4
*h^2 - c^3*g^3*h^3)*b^3) - 2*(3*a*b^2*d^3*f^2*g^2*h*q^2 - 3*a^2*b*d^3*f^2*g
*h^2*q^2 + a^3*d^3*f^2*h^3*q^2 + (3*c*d^2*f^2*g^2*h*p*q - 3*c^2*d*f^2*g*h^2
*p*q + c^3*f^2*h^3*p*q - (p*q + q^2)*d^3*f^2*g^3)*b^3)*(log(d*x + c)*log((d
*h*x + c*h)/(d*g - c*h) + 1) + dilog(-(d*h*x + c*h)/(d*g - c*h)))/((d^3*g^3
*h^3 - 3*c*d^2*g^2*h^4 + 3*c^2*d*g*h^5 - c^3*h^6)*a^3 - 3*(d^3*g^4*h^2 - 3*
c*d^2*g^3*h^3 + 3*c^2*d*g^2*h^4 - c^3*g*h^5)*a^2*b + 3*(d^3*g^5*h - 3*c*d^2
*g^4*h^2 + 3*c^2*d*g^3*h^3 - c^3*g^2*h^4)*a*b^2 - (d^3*g^6 - 3*c*d^2*g^5*h
+ 3*c^2*d*g^4*h^2 - c^3*g^3*h^3)*b^3) - (3*a^2*d^3*f^2*h^2*q^2 + (c*d^2*f^2
*h^2*p*q - (p*q + 6*q^2)*d^3*f^2*g*h)*a*b - (5*c*d^2*f^2*g*h*p*q - 2*c^2*d*
f^2*h^2*p*q - 3*(p*q + q^2)*d^3*f^2*g^2)*b^2)*log(d*x + c)/((d^3*g^3*h^2 -
3*c*d^2*g^2*h^3 + 3*c^2*d*g*h^4 - c^3*h^5)*a^2 - 2*(d^3*g^4*h - 3*c*d^2*g^3
*h^2 + 3*c^2*d*g^2*h^3 - c^3*g*h^4)*a*b + (d^3*g^5 - 3*c*d^2*g^4*h + 3*c^2*
d*g^3*h^2 - c^3*g^2*h^3)*b^2) + 3*(a^3*d^3*f^2*h^3*q^2 + (c*d^2*f^2*h^3*p*q
 - (p*q + 3*q^2)*d^3*f^2*g*h^2)*a^2*b - (4*c*d^2*f^2*g*h^2*p*q - c^2*d*f^2*
h^3*p*q - 3*(p*q + q^2)*d^3*f^2*g^2*h)*a*b^2 + (c^3*f^2*h^3*p^2 - (p^2 + 2*
p*q + q^2)*d^3*f^2*g^3 + 3*(p^2 + p*q)*c*d^2*f^2*g^2*h - (3*p^2 + p*q)*c^2*
d*f^2*g*h^2)*b^3)*log(h*x + g)/((d^3*g^3*h^3 - 3*c*d^2*g^2*h^4 + 3*c^2*d*g*
h^5 - c^3*h^6)*a^3 - 3*(d^3*g^4*h^2 - 3*c*d^2*g^3*h^3 + 3*c^2*d*g^2*h^4 - c
^3*g*h^5)*a^2*b + 3*(d^3*g^5*h - 3*c*d^2*g^4*h^2 + 3*c^2*d*g^3*h^3 - c^3*g^
2*h^4)*a*b^2 - (d^3*g^6 - 3*c*d^2*g^5*h + 3*c^2*d*g^4*h^2 - c^3*g^3*h^3)*b^
3) - ((d^3*f^2*g*h^3*q^2 - c*d^2*f^2*h^4*q^2)*a^3 - (2*c^2*d*f^2*h^4*p*q +
(2*p*q + 3*q^2)*d^3*f^2*g^2*h^2 - (4*p*q + 3*q^2)*c*d^2*f^2*g*h^3)*a^2*b -
(c^3*f^2*h^4*p^2 - (p^2 + 4*p*q + 3*q^2)*d^3*f^2*g^3*h + (3*p^2 + 8*p*q + 3
*q^2)*c*d^2*f^2*g^2*h^2 - (3*p^2 + 4*p*q)*c^2*d*f^2*g*h^3)*a*b^2 + (c^3*f^2
*g*h^3*p^2 - (p^2 + 2*p*q + q^2)*d^3*f^2*g^4 + (3*p^2 + 4*p*q + q^2)*c*d^2*
f^2*g^3*h - (3*p^2 + 2*p*q)*c^2*d*f^2*g^2*h^2)*b^3 - ((d^3*f^2*g^3*h*p^2 -
3*c*d^2*f^2*g^2*h^2*p^2 + 3*c^2*d*f^2*g*h^3*p^2 - c^3*f^2*h^4*p^2)*b^3*x +
(d^3*f^2*g^4*p^2 - 3*c*d^2*f^2*g^3*h*p^2 + 3*c^2*d*f^2*g^2*h^2*p^2 - c^3*f^
2*g*h^3*p^2)*b^3)*log(b*x + a)^2 - 2*(b^3*d^3*f^2*g^4*p*q - 3*a*b^2*d^3*f^2
*g^3*h*p*q + 3*a^2*b*d^3*f^2*g^2*h^2*p*q - a^3*d^3*f^2*g*h^3*p*q + (b^3*d^3
*f^2*g^3*h*p*q - 3*a*b^2*d^3*f^2*g^2*h^2*p*q + 3*a^2*b*d^3*f^2*g*h^3*p*q -
a^3*d^3*f^2*h^4*p*q)*x)*log(b*x + a)*log(d*x + c) - (b^3*d^3*f^2*g^4*q^2 -
3*a*b^2*d^3*f^2*g^3*h*q^2 + 3*a^2*b*d^3*f^2*g^2*h^2*q^2 - a^3*d^3*f^2*g*h^3
*q^2 + (b^3*d^3*f^2*g^3*h*q^2 - 3*a*b^2*d^3*f^2*g^2*h^2*q^2 + 3*a^2*b*d^3*f
^2*g*h^3*q^2 - a^3*d^3*f^2*h^4*q^2)*x)*log(d*x ...
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^4,x, algorithm="fricas
")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h^4*x^4 + 4*g*h^3*x^3 + 6*
g^2*h^2*x^2 + 4*g^3*h*x + g^4), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(h*x+g)**4,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^4,x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h*x + g)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\,{\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)}^r\right)^2}{{\left(g+h\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(g + h*x)^4,x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)^2/(g + h*x)^4, x)
```

3.43 $\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

Optimal. Leaf size=42

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^{1+n}}{bc(1+n)}$$

```
[Out] -(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^(1+n)/b/c/(1+n)
```

Rubi [A]
time = 0.09, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2573, 6818}

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^{n+1}}{bc(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

```
[Out] -((a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^(1 + n)/(b*c*(1 + n)))
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] && !Intege
rQ[n]
```

Rule 6818

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /; !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(a+b\log(x))^n}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$
$$= -\frac{\text{Subst}\left(\int x^n\,dx, x, a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc}$$
$$= -\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^{1+n}}{bc(1+n)}$$

Mathematica [A]
time = 0.02, size = 42, normalized size = 1.00

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^{1+n}}{bc(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
[Out] -((a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^(1 + n)/(b*c*(1 + n)))
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^n}{-c^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
[Out] int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algorit
hm="maxima")
```

```
[Out] -integrate((b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

Fricas [A]
time = 0.36, size = 56, normalized size = 1.33

$$-\frac{\left(b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)\left(b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^{n}}{bcn+bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algorit
hm="fricas")
```

```
[Out] -(b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)*(b*log(sqrt(-c*x + 1)/sqrt(c*x +
 1)) + a)^n/(b*c*n + b*c)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(32) = 64$.
time = 78.25, size = 94, normalized size = 2.24

$$\begin{cases} -\frac{a^{n}\operatorname{atan}\left(\frac{x}{\sqrt{-\frac{1}{c^{2}}}}\right)}{c^{2}\sqrt{-\frac{1}{c^{2}}}} & \text{for } b=0 \\ a^{n}x & \text{for } c=0 \\ -\frac{\begin{cases} \frac{\left(a+b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^{n+1}}{n+1} & \text{for } n\neq -1 \\ \log\left(a+b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right) & \text{otherwise} \end{cases}}{bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
```

```
[Out] Piecewise((-a**n*atan(x/sqrt(-1/c**2))/(c**2*sqrt(-1/c**2)), Eq(b, 0)), (a*
*n*x, Eq(c, 0)), (-Piecewise(((a + b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)))**(n
 + 1)/(n + 1), Ne(n, -1)), (log(a + b*log(sqrt(-c*x + 1)/sqrt(c*x + 1))), T
rue))/(b*c), True))
```

Giac [A]
time = 3.37, size = 38, normalized size = 0.90

$$-\frac{\left(-\frac{1}{2}\,b\log\left(cx+1\right)+\frac{1}{2}\,b\log\left(-cx+1\right)+a\right)^{n+1}}{bc(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algorit
hm="giac")
```

```
[Out] -(-1/2*b*log(c*x + 1) + 1/2*b*log(-c*x + 1) + a)^(n + 1)/(b*c*(n + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\left(a+b\,\ln\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^n}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1),x)
```

```
[Out] -int((a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1), x)
```

3.44 $\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

Optimal. Leaf size=37

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc}$$

```
[Out] -1/4*(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^4/b/c
```

Rubi [A]
time = 0.07, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2573, 2576, 12, 2339, 30}

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^4}{4bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] -1/4*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^4/(b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
```

```
rQ[n]
```

Rule 2576

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g = Coef
f[P2x, x, 1], h = Coeff[P2x, x, 2]}, Dist[b*c - a*d, Subst[Int[(b^2*f - a*b
*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g + c^2*h
)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b*x)/(c
+ d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x, 2] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{(a+b\log(x))^3}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}\\ &= -\frac{\text{Subst}\left(\int x^3\,dx, x, a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc}\\ &= -\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc}\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 37, normalized size = 1.00

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] -1/4*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^4/(b*c)
```

Maple [**F**]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^3}{-c^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x)
```

```
[Out] int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(31) = 62$.
time = 0.32, size = 526, normalized size = 14.22

$$\frac{1}{2}b^3\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3+\frac{3}{2}ab^2\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2+\frac{3}{2}a^2b\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+\frac{1}{64}\left(\frac{24\left(\log(cx+1)^2-2\log(cx+1)\log(cx-1)+\log(cx-1)^2\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2}{c}+\frac{8\left(\log(cx+1)^3-3\log(cx+1)^2\log(cx-1)+3\log(cx+1)\log(cx-1)^2-\log(cx-1)^3\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c}+\frac{\log(cx+1)^4-4\log(cx+1)^3\log(cx-1)+6\log(cx+1)^2\log(cx-1)^2-4\log(cx+1)\log(cx-1)^3+\log(cx-1)^4}{c}\right)b^3+\frac{1}{8}ab^2\left(\frac{6\left(\log(cx+1)^2-2\log(cx+1)\log(cx-1)+\log(cx-1)^2\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c}+\frac{\log(cx+1)^3-3\log(cx+1)^2\log(cx-1)+3\log(cx+1)\log(cx-1)^2-\log(cx-1)^3}{c}\right)+\frac{1}{2}a^3\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)+\frac{3\left(\log(cx+1)^2-2\log(cx+1)\log(cx-1)+\log(cx-1)^2\right)a^2b}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algorit
hm="maxima")
```

```
[Out] 1/2*b^3*(log(c*x + 1)/c - log(c*x - 1)/c)*log(sqrt(-c*x + 1)/sqrt(c*x + 1))
^3 + 3/2*a*b^2*(log(c*x + 1)/c - log(c*x - 1)/c)*log(sqrt(-c*x + 1)/sqrt(c*
x + 1))^2 + 3/2*a^2*b*(log(c*x + 1)/c - log(c*x - 1)/c)*log(sqrt(-c*x + 1)/
sqrt(c*x + 1)) + 1/64*(24*(log(c*x + 1)^2 - 2*log(c*x + 1)*log(c*x - 1) + l
og(c*x - 1)^2)*log(sqrt(-c*x + 1)/sqrt(c*x + 1))^2/c + 8*(log(c*x + 1)^3 -
3*log(c*x + 1)^2*log(c*x - 1) + 3*log(c*x + 1)*log(c*x - 1)^2 - log(c*x - 1
)^3)*log(sqrt(-c*x + 1)/sqrt(c*x + 1))/c + (log(c*x + 1)^4 - 4*log(c*x + 1)
^3*log(c*x - 1) + 6*log(c*x + 1)^2*log(c*x - 1)^2 - 4*log(c*x + 1)*log(c*x
- 1)^3 + log(c*x - 1)^4)/c)*b^3 + 1/8*a*b^2*(6*(log(c*x + 1)^2 - 2*log(c*x
+ 1)*log(c*x - 1) + log(c*x - 1)^2)*log(sqrt(-c*x + 1)/sqrt(c*x + 1))/c + (
log(c*x + 1)^3 - 3*log(c*x + 1)^2*log(c*x - 1) + 3*log(c*x + 1)*log(c*x - 1
)^2 - log(c*x - 1)^3)/c) + 1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) + 3/8*
(log(c*x + 1)^2 - 2*log(c*x + 1)*log(c*x - 1) + log(c*x - 1)^2)*a^2*b/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(31) = 62$.
time = 0.35, size = 101, normalized size = 2.73

$$-\frac{b^3\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^4+4\,ab^2\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3+6\,a^2b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2+4\,a^3\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algorit
hm="fricas")
```

```
[Out] -1/4*(b^3*log(sqrt(-c*x + 1)/sqrt(c*x + 1))^4 + 4*a*b^2*log(sqrt(-c*x + 1)/
sqrt(c*x + 1))^3 + 6*a^2*b*log(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 4*a^3*log(
sqrt(-c*x + 1)/sqrt(c*x + 1)))/c
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(29) = 58$.
time = 6.44, size = 65, normalized size = 1.76

$$\begin{cases} -\frac{a^3 \operatorname{atan}\left(\frac{x}{\sqrt{-\frac{1}{c^2}}}\right)}{c^2\sqrt{-\frac{1}{c^2}}} & \text{for } b = 0 \\ a^3x & \text{for } c = 0 \\ -\frac{\left(a+b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^4}{4bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

```
[Out] Piecewise((-a**3*atan(x/sqrt(-1/c**2))/(c**2*sqrt(-1/c**2)), Eq(b, 0)), (a*
*3*x, Eq(c, 0)), (-(a + b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)))**4/(4*b*c), Tr
ue))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algorit
hm="giac")
```

```
[Out] integrate(-(b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^3/(c^2*x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int -\frac{\left(a+b\,\ln\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^3}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1), x)
```

3.45 $\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

Optimal. Leaf size=37

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc}$$

```
[Out] -1/3*(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/b/c
```

Rubi [A]
time = 0.07, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2573, 2576, 12, 2339, 30}

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{3bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] -1/3*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
```

```
rQ[n]
```

Rule 2576

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g = Coef
f[P2x, x, 1], h = Coeff[P2x, x, 2]}, Dist[b*c - a*d, Subst[Int[(b^2*f - a*b
*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g + c^2*h
)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b*x)/(c
+ d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x, 2] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{(a+b\log(x))^2}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}\\ &= -\frac{\text{Subst}\left(\int x^2\,dx, x, a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc}\\ &= -\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 1.00

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
[Out] -1/3*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(b*c)
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^2}{-c^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x)
```

```
[Out] int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(31) = 62$.
time = 0.29, size = 268, normalized size = 7.24

$$\frac{1}{2}b^2\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2+ab\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+\frac{1}{24}b^2\left(\frac{6\left(\log(cx+1)^2-2\log(cx+1)\log(cx-1)+\log(cx-1)^2\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c}+\frac{\log(cx+1)^3-3\log(cx+1)^2\log(cx-1)+3\log(cx+1)\log(cx-1)^2-\log(cx-1)^3}{c}\right)+\frac{1}{2}a^2\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)+\frac{\left(\log(cx+1)^2-2\log(cx+1)\log(cx-1)+\log(cx-1)^2\right)ab}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algorit
hm="maxima")
```

```
[Out] 1/2*b^2*(log(c*x + 1)/c - log(c*x - 1)/c)*log(sqrt(-c*x + 1)/sqrt(c*x + 1))
^2 + a*b*(log(c*x + 1)/c - log(c*x - 1)/c)*log(sqrt(-c*x + 1)/sqrt(c*x + 1)
) + 1/24*b^2*(6*(log(c*x + 1)^2 - 2*log(c*x + 1)*log(c*x - 1) + log(c*x - 1
)^2)*log(sqrt(-c*x + 1)/sqrt(c*x + 1))/c + (log(c*x + 1)^3 - 3*log(c*x + 1)
^2*log(c*x - 1) + 3*log(c*x + 1)*log(c*x - 1)^2 - log(c*x - 1)^3)/c) + 1/2*
a^2*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/4*(log(c*x + 1)^2 - 2*log(c*x + 1
)*log(c*x - 1) + log(c*x - 1)^2)*a*b/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(31) = 62$.
time = 0.36, size = 74, normalized size = 2.00

$$-\frac{b^2\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3+3\,ab\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2+3\,a^2\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algorit
hm="fricas")
```

```
[Out] -1/3*(b^2*log(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b*log(sqrt(-c*x + 1)/sq
rt(c*x + 1))^2 + 3*a^2*log(sqrt(-c*x + 1)/sqrt(c*x + 1)))/c
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(29) = 58$.

time = 4.72, size = 65, normalized size = 1.76

$$\begin{cases} -\frac{a^2 \operatorname{atan}\left(\frac{x}{\sqrt{-\frac{1}{c^2}}}\right)}{c^2\sqrt{-\frac{1}{c^2}}} & \text{for } b = 0 \\ a^2 x & \text{for } c = 0 \\ -\frac{\left(a+b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^3}{3bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

```
[Out] Piecewise((-a**2*atan(x/sqrt(-1/c**2))/(c**2*sqrt(-1/c**2)), Eq(b, 0)), (a*
*2*x, Eq(c, 0)), (-(a + b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)))**3/(3*b*c), Tr
ue))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algorit
hm="giac")
```

```
[Out] integrate(-(b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2/(c^2*x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int -\frac{\left(a+b\,\ln\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1), x)
```

3.46 $\int \frac{a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

Optimal. Leaf size=37

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc}$$

```
[Out] -1/2*(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/b/c
```

Rubi [A]
time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2573, 2576, 12, 2338}

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] -1/2*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 2576

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g = Coef
```

```
f[P2x, x, 1], h = Coeff[P2x, x, 2]}, Dist[b*c - a*d, Subst[Int[(b^2*f - a*b
*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g + c^2*h
)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b*x)/(c
+ d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x, 2] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx &= -\frac{\mathrm{Subst}\left(\int \frac{a+b\log(x)}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}\\ &= -\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc}\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 37, normalized size = 1.00

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
[Out] -1/2*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(b*c)
```

Maple [**F**]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{a+b\ln\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{-c^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x)
[Out] int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(31) = 62$.
time = 0.29, size = 105, normalized size = 2.84

$$\frac{1}{2}\,b\left(\frac{\log\left(cx+1\right)}{c}-\frac{\log\left(cx-1\right)}{c}\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+\frac{1}{2}\,a\left(\frac{\log\left(cx+1\right)}{c}-\frac{\log\left(cx-1\right)}{c}\right)+\frac{\left(\log\left(cx+1\right)^2-2\,\log\left(cx+1\right)\log\left(cx-1\right)+\log\left(cx-1\right)^2\right)b}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algorithm
="maxima")
```

```
[Out] 1/2*b*(log(c*x + 1)/c - log(c*x - 1)/c)*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) +
 1/2*a*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/8*(log(c*x + 1)^2 - 2*log(c*x
+ 1)*log(c*x - 1) + log(c*x - 1)^2)*b/c
```

Fricas [A]
time = 0.35, size = 47, normalized size = 1.27

$$-\frac{b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2+2\,a\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algorithm
="fricas")
```

```
[Out] -1/2*(b*log(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*log(sqrt(-c*x + 1)/sqrt(c
*x + 1)))/c
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(29) = 58$.
time = 4.59, size = 61, normalized size = 1.65

$$\begin{cases} -\frac{a\,\mathrm{atan}\left(\frac{x}{\sqrt{-\frac{1}{c^2}}}\right)}{c^2\sqrt{-\frac{1}{c^2}}} & \text{for}\ b=0 \\ ax & \text{for}\ c=0 \\ -\frac{\left(a+b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^2}{2bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

```
[Out] Piecewise((-a*atan(x/sqrt(-1/c**2))/(c**2*sqrt(-1/c**2)), Eq(b, 0)), (a*x,
Eq(c, 0)), (-(a + b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)))**2/(2*b*c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(31) = 62$.
time = 5.02, size = 86, normalized size = 2.32

$$-\frac{b\log(cx+1)^2}{8\,c}+\frac{b\log(cx-1)^2}{8\,c}+\frac{1}{4}\left(\frac{b\log(cx+1)}{c}-\frac{b\log(cx-1)}{c}\right)\log(-cx+1)+\frac{a\log(cx+1)}{2\,c}-\frac{a\log(cx-1)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algorithm
="giac")
```

```
[Out] -1/8*b*log(c*x + 1)^2/c + 1/8*b*log(c*x - 1)^2/c + 1/4*(b*log(c*x + 1)/c -
b*log(c*x - 1)/c)*log(-c*x + 1) + 1/2*a*log(c*x + 1)/c - 1/2*a*log(c*x - 1)
/c
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int -\frac{a+b\,\ln\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1), x)
```

3.47 $\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

Optimal. Leaf size=34

$$-\frac{\log\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc}$$

```
[Out] -ln(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/b/c
```

Rubi [A]
time = 0.06, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2573, 6816}

$$-\frac{\log\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] -(Log[a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]]]/(b*c))
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 6816

```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x(a+b\log(x))}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc}$$

$$= -\frac{\log\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc}$$

Mathematica [A]
time = 0.03, size = 34, normalized size = 1.00

$$-\frac{\log\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
[Out] -(Log[a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]]]/(b*c))
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{(-c^2x^2+1)\left(a+b\ln\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
[Out] int(1/(-c^2*x^2+1)/(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

Maxima [A]
time = 0.34, size = 36, normalized size = 1.06

$$-\frac{\log\left(-\frac{b\log(cx+1)-b\log(-cx+1)-2\,a}{2\,b}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algorit
hm="maxima")
```

```
[Out] -log(-1/2*(b*log(c*x + 1) - b*log(-c*x + 1) - 2*a)/b)/(b*c)
```

Fricas [A]
time = 0.37, size = 30, normalized size = 0.88

$$-\frac{\log\left(b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algorit
hm="fricas")
```

```
[Out] -log(b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(b*c)
```

Sympy [A]
time = 8.29, size = 54, normalized size = 1.59

$$\begin{cases} \frac{x}{a} & \text{for}\: c = 0 \wedge \left(b = 0 \vee c = 0\right) \\ \frac{-\frac{\log\left(x-\frac{1}{c}\right)}{2c}+\frac{\log\left(x+\frac{1}{c}\right)}{2c}}{a} & \text{for}\: b = 0 \\ -\frac{\log\left(\frac{a}{b}+\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)}{bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
```

```
[Out] Piecewise((x/a, Eq(c, 0) & (Eq(b, 0) | Eq(c, 0))), ((-log(x - 1/c)/(2*c) +
log(x + 1/c)/(2*c))/a, Eq(b, 0)), (-log(a/b + log(sqrt(-c*x + 1)/sqrt(c*x +
 1)))/(b*c), True))
```

Giac [A]
time = 5.08, size = 31, normalized size = 0.91

$$-\frac{\log\left(-b\log\left(cx+1\right)+b\log\left(-cx+1\right)+2\,a\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algorit
hm="giac")
```

```
[Out] -log(-b*log(c*x + 1) + b*log(-c*x + 1) + 2*a)/(b*c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$-\int \frac{1}{\left(a+b\,\ln\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)\,\left(c^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)),x)
[Out] -int(1/((a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)), x)
```

3.48 $\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$

Optimal. Leaf size=34

$$\frac{1}{bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}$$

```
[Out] 1/b/c/(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))
```

Rubi [A]
time = 0.07, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2573, 6818}

$$\frac{1}{bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
```

```
[Out] 1/(b*c*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]]))
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 6818

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x(a+b\log(x))^2}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{x^2}\,dx, x, a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc}$$

$$= \frac{1}{bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}$$

Mathematica [**A**]
time = 0.01, size = 34, normalized size = 1.00

$$\frac{1}{bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
[Out] 1/(b*c*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]]))
```

Maple [**F**]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{(-c^2x^2+1)\left(a+b\ln\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
[Out] int(1/(-c^2*x^2+1)/(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

Maxima [**A**]
time = 0.35, size = 34, normalized size = 1.00

$$-\frac{2}{b^2c\log(cx+1)-b^2c\log(-cx+1)-2\,abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, algor
ithm="maxima")
```

```
[Out] -2/(b^2*c*log(c*x + 1) - b^2*c*log(-c*x + 1) - 2*a*b*c)
```

Fricas [A]
time = 0.33, size = 29, normalized size = 0.85

$$\frac{1}{b^2 c \log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, algor
ithm="fricas")
```

```
[Out] 1/(b^2*c*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a*b*c)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(26) = 52$.
time = 87.98, size = 61, normalized size = 1.79

$$\begin{cases} \frac{x}{a^2} & \text{for}\: b = 0 \wedge c = 0 \\ \frac{-\frac{\log\left(x - \frac{1}{c}\right)}{2c} + \frac{\log\left(x + \frac{1}{c}\right)}{2c}}{a^2} & \text{for}\: b = 0 \\ \frac{x}{a^2} & \text{for}\: c = 0 \\ \frac{1}{abc + b^2 c \log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2,x)
```

```
[Out] Piecewise((x/a**2, Eq(b, 0) & Eq(c, 0)), ((-log(x - 1/c)/(2*c) + log(x + 1/
c)/(2*c))/a**2, Eq(b, 0)), (x/a**2, Eq(c, 0)), (1/(a*b*c + b**2*c*log(sqrt(
-c*x + 1)/sqrt(c*x + 1))), True))
```

Giac [A]
time = 4.86, size = 34, normalized size = 1.00

$$-\frac{2}{b^2 c \log\left(cx+1\right) - b^2 c \log\left(-cx+1\right) - 2\,abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, algor
ithm="giac")
```

```
[Out] -2/(b^2*c*log(c*x + 1) - b^2*c*log(-c*x + 1) - 2*a*b*c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$-\int \frac{1}{\left(a+b\,\ln\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2\,\left(c^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)),x)
[Out] -int(1/((a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)), x)
```

3.49 $\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}\,dx$

Optimal. Leaf size=37

$$\frac{1}{2bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}$$

```
[Out] 1/2/b/c/(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2
```

Rubi [A]
time = 0.07, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2573, 6818}

$$\frac{1}{2bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3),x]
```

```
[Out] 1/(2*b*c*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 6818

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x(a+b\log(x))^3}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{x^3}\,dx, x, a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc}$$

$$= \frac{1}{2bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}$$

Mathematica [**A**]
time = 0.01, size = 37, normalized size = 1.00

$$\frac{1}{2bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3),x]
[Out] 1/(2*b*c*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)
```

Maple [**F**]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{(-c^2x^2+1)\left(a+b\ln\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3,x)
[Out] int(1/(-c^2*x^2+1)/(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3,x)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(31) = 62$.
time = 0.36, size = 80, normalized size = 2.16

$$\frac{2}{b^3c\log(cx+1)^2 + b^3c\log(-cx+1)^2 - 4\,ab^2c\log(cx+1) + 4\,a^2bc - 2\,(b^3c\log(cx+1) - 2\,ab^2c)\log(-cx+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3,x, algor
ithm="maxima")
```

```
[Out] 2/(b^3*c*log(c*x + 1)^2 + b^3*c*log(-c*x + 1)^2 - 4*a*b^2*c*log(c*x + 1) +
4*a^2*b*c - 2*(b^3*c*log(c*x + 1) - 2*a*b^2*c)*log(-c*x + 1))
```

Fricas [A]
time = 0.32, size = 59, normalized size = 1.59

$$\frac{1}{2\left(b^{3}c\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^{2}+2\,ab^{2}c\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a^{2}bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3,x, algor
ithm="fricas")
```

```
[Out] 1/2/(b^3*c*log(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b^2*c*log(sqrt(-c*x +
1)/sqrt(c*x + 1)) + a^2*b*c)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(31) = 62.
time = 2.48, size = 85, normalized size = 2.30

$$\frac{2}{b^{3}c\log\left(cx+1\right)^{2}-2\,b^{3}c\log\left(cx+1\right)\log\left(-cx+1\right)+b^{3}c\log\left(-cx+1\right)^{2}-4\,ab^{2}c\log\left(cx+1\right)+4\,ab^{2}c\log\left(-cx+1\right)+4\,a^{2}bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3,x, algor
ithm="giac")
```

```
[Out] 2/(b^3*c*log(c*x + 1)^2 - 2*b^3*c*log(c*x + 1)*log(-c*x + 1) + b^3*c*log(-c
*x + 1)^2 - 4*a*b^2*c*log(c*x + 1) + 4*a*b^2*c*log(-c*x + 1) + 4*a^2*b*c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$-\int \frac{1}{{\left(a+b\,\ln\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)}^3\,\left(c^2\,x^2-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3*(c^2*x^2 - 1)),x)
[Out] -int(1/((a + b*log((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3*(c^2*x^2 - 1)), x)
```

3.50 $\int \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=30

$$-\frac{\log^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$

```
[Out] -1/2*ln((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/a
```

Rubi [A]
time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2573, 2576, 12, 2338}

$$-\frac{\log^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] -1/2*Log[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/a
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 2576

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g = Coef
f[P2x, x, 1], h = Coeff[P2x, x, 2]}, Dist[b*c - a*d, Subst[Int[(b^2*f - a*b
```

```
*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g + c^2*h
)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b*x)/(c
+ d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x, 2] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\log^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 1.00

$$-\frac{\log^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
[Out] -1/2*Log[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/a
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
[Out] int(ln((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(24) = 48.
time = 0.52, size = 83, normalized size = 2.77

$$\frac{1}{2}\left(\frac{\log\left(ax+1\right)}{a}-\frac{\log\left(ax-1\right)}{a}\right)\log\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)+\frac{\log\left(ax-1\right)^2}{8\,a}+\frac{\log\left(ax+1\right)^2-2\,\log\left(ax+1\right)\log\left(ax-1\right)}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="maxi
ma")
```

```
[Out] 1/2*(log(a*x + 1)/a - log(a*x - 1)/a)*log(sqrt(-a*x + 1)/sqrt(a*x + 1)) + 1
/8*log(a*x - 1)^2/a + 1/8*(log(a*x + 1)^2 - 2*log(a*x + 1)*log(a*x - 1))/a
```

Fricas [A]
time = 0.35, size = 24, normalized size = 0.80

$$-\frac{\log\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="fric
as")
```

```
[Out] -1/2*log(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(24) = 48$.
time = 3.20, size = 65, normalized size = 2.17

$$-\frac{\operatorname{atan}^2\left(\frac{x}{\sqrt{-\frac{1}{a^2}}}\right)}{2a}-\frac{\log\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)\operatorname{atan}\left(\frac{x}{\sqrt{-\frac{1}{a^2}}}\right)}{a^2\sqrt{-\frac{1}{a^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

```
[Out] -atan(x/sqrt(-1/a**2))**2/(2*a) - log(sqrt(-a*x + 1)/sqrt(a*x + 1))*atan(x/
sqrt(-1/a**2))/(a**2*sqrt(-1/a**2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(24) = 48$.
time = 2.76, size = 58, normalized size = 1.93

$$\frac{1}{4}\left(\frac{\log\left(ax+1\right)}{a}-\frac{\log\left(ax-1\right)}{a}\right)\log\left(-ax+1\right)-\frac{\log\left(ax+1\right)^2}{8\,a}+\frac{\log\left(ax-1\right)^2}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="giac
")
```

```
[Out] 1/4*(log(a*x + 1)/a - log(a*x - 1)/a)*log(-a*x + 1) - 1/8*log(a*x + 1)^2/a
+ 1/8*log(a*x - 1)^2/a
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$-\int \frac{\ln\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-log((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1),x)
[Out] -int(log((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1), x)
```

3.51 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)(s+t\log(i(g+hx)^n))^2}{gk+hkx}\,dx$

Optimal. Leaf size=410

$$-\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^3}{3hknt}-\frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^3}{3hknt}+\frac{\log\left(e(f(a+bx)^p(\right.}{}$$

```
[Out] -1/3*p*r*ln(-h*(b*x+a)/(-a*h+b*g))*(s+t*ln(i*(h*x+g)^n))^3/h/k/n/t-1/3*q*r*
ln(-h*(d*x+c)/(-c*h+d*g))*(s+t*ln(i*(h*x+g)^n))^3/h/k/n/t+1/3*ln(e*(f*(b*x+
a)^p*(d*x+c)^q)^r)*(s+t*ln(i*(h*x+g)^n))^3/h/k/n/t-p*r*(s+t*ln(i*(h*x+g)^n)
)^2*polylog(2,b*(h*x+g)/(-a*h+b*g))/h/k-q*r*(s+t*ln(i*(h*x+g)^n))^2*polylog
(2,d*(h*x+g)/(-c*h+d*g))/h/k+2*n*p*r*t*(s+t*ln(i*(h*x+g)^n))*polylog(3,b*(h
*x+g)/(-a*h+b*g))/h/k+2*n*q*r*t*(s+t*ln(i*(h*x+g)^n))*polylog(3,d*(h*x+g)/(
-c*h+d*g))/h/k-2*n^2*p*r*t^2*polylog(4,b*(h*x+g)/(-a*h+b*g))/h/k-2*n^2*q*r*
t^2*polylog(4,d*(h*x+g)/(-c*h+d*g))/h/k
```

Rubi [A]
time = 0.33, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2585, 2443, 2481, 2421, 2430, 6724}

$$-\frac{pr\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)\left(t\log\left(i(g+hx)^n\right)+s\right)^2}{hk}+\frac{2nprt\text{PolyLog}\left(3,\frac{b(g+hx)}{bg-ah}\right)\left(t\log\left(i(g+hx)^n\right)+s\right)}{hk}-\frac{2n^2prt^2\text{PolyLog}\left(4,\frac{b(g+hx)}{bg-ah}\right)}{hk}-\frac{qr\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)\left(t\log\left(i(g+hx)^n\right)+s\right)^2}{hk}+\frac{2nqrt\text{PolyLog}\left(3,\frac{d(g+hx)}{dg-ch}\right)\left(t\log\left(i(g+hx)^n\right)+s\right)}{hk}-\frac{2n^2qrt^2\text{PolyLog}\left(4,\frac{d(g+hx)}{dg-ch}\right)}{hk}+\frac{\left(t\log\left(i(g+hx)^n\right)+s\right)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3hknt}-\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(t\log\left(i(g+hx)^n\right)+s\right)^3}{3hknt}-\frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\left(t\log\left(i(g+hx)^n\right)+s\right)^3}{3hknt}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*(s + t*Log[i*(g + h*x)^n])^2)/(g*
k + h*k*x),x]
```

```
[Out] -1/3*(p*r*Log[-((h*(a + b*x))/(b*g - a*h))]*(s + t*Log[i*(g + h*x)^n])^3)/(
h*k*n*t) - (q*r*Log[-((h*(c + d*x))/(d*g - c*h))]*(s + t*Log[i*(g + h*x)^n]
)^3)/(3*h*k*n*t) + (Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*(s + t*Log[i*(g +
h*x)^n])^3)/(3*h*k*n*t) - (p*r*(s + t*Log[i*(g + h*x)^n])^2*PolyLog[2, (b*(
g + h*x))/(b*g - a*h)])/(h*k) - (q*r*(s + t*Log[i*(g + h*x)^n])^2*PolyLog[2
, (d*(g + h*x))/(d*g - c*h)])/(h*k) + (2*n*p*r*t*(s + t*Log[i*(g + h*x)^n])
*PolyLog[3, (b*(g + h*x))/(b*g - a*h)])/(h*k) + (2*n*q*r*t*(s + t*Log[i*(g
+ h*x)^n])*PolyLog[3, (d*(g + h*x))/(d*g - c*h)])/(h*k) - (2*n^2*p*r*t^2*Po
lyLog[4, (b*(g + h*x))/(b*g - a*h)])/(h*k) - (2*n^2*q*r*t^2*PolyLog[4, (d*(
g + h*x))/(d*g - c*h)])/(h*k)
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2585

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))/((j_.
) + (k_.)*(x_)), x_Symbol] :> Simp[(s + t*Log[i*(g + h*x)^n])^(m + 1)*(Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(k*n*t*(m + 1))), x] + (-Dist[b*p*(r/(k*n*
t*(m + 1))), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Dis
t[d*q*(r/(k*n*t*(m + 1))), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*x)
, x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r},
x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\left(s+t\log \left(51(g+hx)^n\right)\right)^2}{gk+hkx}\,dx = \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\left(s+t\log (51(g\right.}{3hknt}$$

$$= -\frac{pr\log \left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log \left(51(g+hx)^n\right)\right)}{3hknt}$$

$$= -\frac{pr\log \left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log \left(51(g+hx)^n\right)\right)}{3hknt}$$

$$= -\frac{pr\log \left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log \left(51(g+hx)^n\right)\right)}{3hknt}$$

$$= -\frac{pr\log \left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log \left(51(g+hx)^n\right)\right)}{3hknt}$$

$$= -\frac{pr\log \left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log \left(51(g+hx)^n\right)\right)}{3hknt}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 958 vs. $2(410) = 820$.
time = 6.13, size = 958, normalized size = 2.34

Antiderivative was successfully verified.

```
[In] Integrate[(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*(s + t*Log[i*(g + h*x)^n])^
2)/(g*k + h*k*x),x]
```

```
[Out] -1/3*(3*p*r*s^2*Log[(h*(a + b*x))/(-(b*g) + a*h)]*Log[g + h*x] + 3*q*r*s^2*
Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[g + h*x] - 3*s^2*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]*Log[g + h*x] - 3*n*p*r*s*t*Log[(h*(a + b*x))/(-(b*g) + a*h
)]*Log[g + h*x]^2 - 3*n*q*r*s*t*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[g + h
*x]^2 + 3*n*s*t*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g + h*x]^2 + n^2*p
*r*t^2*Log[(h*(a + b*x))/(-(b*g) + a*h)]*Log[g + h*x]^3 + n^2*q*r*t^2*Log[(
h*(c + d*x))/(-(d*g) + c*h)]*Log[g + h*x]^3 - n^2*t^2*Log[e*(f*(a + b*x)^p*
(c + d*x)^q)^r]*Log[g + h*x]^3 + 6*p*r*s*t*Log[(h*(a + b*x))/(-(b*g) + a*h)
]*Log[g + h*x]*Log[i*(g + h*x)^n] + 6*q*r*s*t*Log[(h*(c + d*x))/(-(d*g) + c
*h)]*Log[g + h*x]*Log[i*(g + h*x)^n] - 6*s*t*Log[e*(f*(a + b*x)^p*(c + d*x)
^q)^r]*Log[g + h*x]*Log[i*(g + h*x)^n] - 3*n*p*r*t^2*Log[(h*(a + b*x))/(-(b
*g) + a*h)]*Log[g + h*x]^2*Log[i*(g + h*x)^n] - 3*n*q*r*t^2*Log[(h*(c + d*x
```

```
))/(-(d*g) + c*h)]*Log[g + h*x]^2*Log[i*(g + h*x)^n] + 3*n*t^2*Log[e*(f*(a
+ b*x)^p*(c + d*x)^q)^r]*Log[g + h*x]^2*Log[i*(g + h*x)^n] + 3*p*r*t^2*Log[
(h*(a + b*x))/(-(b*g) + a*h)]*Log[g + h*x]*Log[i*(g + h*x)^n]^2 + 3*q*r*t^2
*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[g + h*x]*Log[i*(g + h*x)^n]^2 - 3*t^
2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g + h*x]*Log[i*(g + h*x)^n]^2 +
3*p*r*(s + t*Log[i*(g + h*x)^n])^2*PolyLog[2, (b*(g + h*x))/(b*g - a*h)] +
3*q*r*(s + t*Log[i*(g + h*x)^n])^2*PolyLog[2, (d*(g + h*x))/(d*g - c*h)] -
6*n*p*r*s*t*PolyLog[3, (b*(g + h*x))/(b*g - a*h)] - 6*n*p*r*t^2*Log[i*(g +
h*x)^n]*PolyLog[3, (b*(g + h*x))/(b*g - a*h)] - 6*n*q*r*s*t*PolyLog[3, (d*(
g + h*x))/(d*g - c*h)] - 6*n*q*r*t^2*Log[i*(g + h*x)^n]*PolyLog[3, (d*(g +
h*x))/(d*g - c*h)] + 6*n^2*p*r*t^2*PolyLog[4, (b*(g + h*x))/(b*g - a*h)] +
6*n^2*q*r*t^2*PolyLog[4, (d*(g + h*x))/(d*g - c*h)])/(h*k)
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(e(f(bx+a)^{p}\left(dx+c\right)^{q})^{r}\right)\left(s+t\ln \left(i(hx+g)^{n}\right)\right)^{2}}{hkx+gk}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*ln(i*(h*x+g)^n))^2/(h*k*x+g*k),x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*ln(i*(h*x+g)^n))^2/(h*k*x+g*k),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*log(i*(h*x+g)^n))^2/(h*k*x+
g*k),x, algorithm="maxima")
```

```
[Out] 1/12*((4*n^2*t^2*log(h*x + g)^3 + 12*t^2*log(h*x + g)*log((h*x + g)^n)^2 -
6*(I*pi*n*t^2 + 2*n*s*t)*log(h*x + g)^2 - 3*(pi^2*t^2 - 4*I*pi*s*t - 4*s^2)
*log(h*x + g) - 12*(n*t^2*log(h*x + g)^2 + (-I*pi*t^2 - 2*s*t)*log(h*x + g)
)*log((h*x + g)^n))*log(((b*x + a)^p)^r) + (4*n^2*t^2*log(h*x + g)^3 + 12*t
^2*log(h*x + g)*log((h*x + g)^n)^2 - 6*(I*pi*n*t^2 + 2*n*s*t)*log(h*x + g)^
2 - 3*(pi^2*t^2 - 4*I*pi*s*t - 4*s^2)*log(h*x + g) - 12*(n*t^2*log(h*x + g)
^2 + (-I*pi*t^2 - 2*s*t)*log(h*x + g))*log((h*x + g)^n))*log(((d*x + c)^q)^
r))/(h*k) - integrate(1/12*(3*pi^2*(h*r*t^2*log(f) + h*t^2)*a*c - 12*I*pi*(
h*r*s*t*log(f) + h*s*t)*a*c + 4*((p*r + q*r)*b*d*h*n^2*t^2*x^2 + b*c*g*n^2*
p*r*t^2 + a*d*g*n^2*q*r*t^2 + (a*d*h*n^2*q*r*t^2 + (c*h*n^2*p*r*t^2 + (p*r
+ q*r)*d*g*n^2*t^2)*b)*x)*log(h*x + g)^3 - 12*(h*r*s^2*log(f) + h*s^2)*a*c
+ 3*(pi^2*(h*r*t^2*log(f) + h*t^2)*b*d - 4*I*pi*(h*r*s*t*log(f) + h*s*t)*b*
d - 4*(h*r*s^2*log(f) + h*s^2)*b*d)*x^2 - 6*(2*b*c*g*n*p*r*s*t + 2*a*d*g*n*
```

```
q*r*s*t + (I*pi*(p*r + q*r)*b*d*h*n*t^2 + 2*(p*r*s + q*r*s)*b*d*h*n*t)*x^2
+ I*pi*(b*c*g*n*p*r*t^2 + a*d*g*n*q*r*t^2) + (2*a*d*h*n*q*r*s*t + I*pi*(a*d
*h*n*q*r*t^2 + (c*h*n*p*r*t^2 + (p*r + q*r)*d*g*n*t^2)*b) + 2*(c*h*n*p*r*s*
t + (p*r*s + q*r*s)*d*g*n*t)*b)*x)*log(h*x + g)^2 - 12*((h*r*t^2*log(f) + h
*t^2)*b*d*x^2 + (h*r*t^2*log(f) + h*t^2)*a*c + ((h*r*t^2*log(f) + h*t^2)*b*
c + (h*r*t^2*log(f) + h*t^2)*a*d)*x - ((p*r + q*r)*b*d*h*t^2*x^2 + b*c*g*p*
r*t^2 + a*d*g*q*r*t^2 + (a*d*h*q*r*t^2 + (c*h*p*r*t^2 + (p*r + q*r)*d*g*t^2
)*b)*x)*log(h*x + g))*log((h*x + g)^n)^2 + 3*(pi^2*((h*r*t^2*log(f) + h*t^2
)*b*c + (h*r*t^2*log(f) + h*t^2)*a*d) - 4*(h*r*s^2*log(f) + h*s^2)*b*c - 4*
(h*r*s^2*log(f) + h*s^2)*a*d - 4*I*pi*((h*r*s*t*log(f) + h*s*t)*b*c + (h*r*
s*t*log(f) + h*s*t)*a*d))*x + 3*(4*b*c*g*p*r*s^2 + 4*a*d*g*q*r*s^2 - pi^2*(
b*c*g*p*r*t^2 + a*d*g*q*r*t^2) - (pi^2*(p*r + q*r)*b*d*h*t^2 - 4*I*pi*(p*r*
s + q*r*s)*b*d*h*t - 4*(p*r*s^2 + q*r*s^2)*b*d*h)*x^2 + 4*I*pi*(b*c*g*p*r*s
*t + a*d*g*q*r*s*t) + (4*a*d*h*q*r*s^2 - pi^2*(a*d*h*q*r*t^2 + (c*h*p*r*t^2
 + (p*r + q*r)*d*g*t^2)*b) + 4*I*pi*(a*d*h*q*r*s*t + (c*h*p*r*s*t + (p*r*s
+ q*r*s)*d*g*t)*b) + 4*(c*h*p*r*s^2 + (p*r*s^2 + q*r*s^2)*d*g)*b)*x)*log(h*
x + g) - 12*(I*pi*(h*r*t^2*log(f) + h*t^2)*a*c + 2*(h*r*s*t*log(f) + h*s*t)
*a*c + (I*pi*(h*r*t^2*log(f) + h*t^2)*b*d + 2*(h*r*s*t*log(f) + h*s*t)*b*d)
*x^2 + ((p*r + q*r)*b*d*h*n*t^2*x^2 + b*c*g*n*p*r*t^2 + a*d*g*n*q*r*t^2 + (
a*d*h*n*q*r*t^2 + (c*h*n*p*r*t^2 + (p*r + q*r)*d*g*n*t^2)*b)*x)*log(h*x + g
)^2 + (2*(h*r*s*t*log(f) + h*s*t)*b*c + 2*(h*r*s*t*log(f) + h*s*t)*a*d + I*
pi*((h*r*t^2*log(f) + h*t^2)*b*c + (h*r*t^2*log(f) + h*t^2)*a*d))*x - (2*b*
c*g*p*r*s*t + 2*a*d*g*q*r*s*t - (-I*pi*(p*r + q*r)*b*d*h*t^2 - 2*(p*r*s + q
*r*s)*b*d*h*t)*x^2 + I*pi*(b*c*g*p*r*t^2 + a*d*g*q*r*t^2) + (2*a*d*h*q*r*s*
t + I*pi*(a*d*h*q*r*t^2 + (c*h*p*r*t^2 + (p*r + q*r)*d*g*t^2)*b) + 2*(c*h*p
*r*s*t + (p*r*s + q*r*s)*d*g*t)*b)*x)*log(h*x + g))*log((h*x + g)^n))/(b*d*
h^2*k*x^3 + a*c*g*h*k + (a*d*h^2*k + (d*g*h*k + c*h^2*k)*b)*x^2 + (b*c*g*h*
k + (d*g*h*k + c*h^2*k)*a)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*log(i*(h*x+g)^n))^2/(h*k*x+
g*k),x, algorithm="fricas")
```

```
[Out] integral(-1/4*(pi^2*t^2 - 4*I*pi*s*t - 4*(n^2*p*r*t^2*log(b*x + a) + n^2*q*
r*t^2*log(d*x + c) + n^2*r*t^2*log(f) + n^2*t^2)*log(h*x + g)^2 - 4*s^2 + (
pi^2*p*r*t^2 - 4*I*pi*p*r*s*t - 4*p*r*s^2)*log(b*x + a) + (pi^2*q*r*t^2 - 4
*I*pi*q*r*s*t - 4*q*r*s^2)*log(d*x + c) - 4*(I*pi*n*t^2 + 2*n*s*t + (I*pi*n
*p*r*t^2 + 2*n*p*r*s*t)*log(b*x + a) + (I*pi*n*q*r*t^2 + 2*n*q*r*s*t)*log(d
*x + c) + (I*pi*n*r*t^2 + 2*n*r*s*t)*log(f))*log(h*x + g) + (pi^2*r*t^2 - 4
*I*pi*r*s*t - 4*r*s^2)*log(f))/(h*k*x + g*k), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)*(s+t*ln(i*(h*x+g)**n))**2/(h*k
*x+g*k),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*log(i*(h*x+g)^n))^2/(h*k*x+
g*k),x, algorithm="giac")
```

```
[Out] integrate((t*log(I*(h*x + g)^n) + s)^2*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)
/(h*k*x + g*k), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\,{\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)}^r\right)\,{\left(s+t\,\ln\left(i\,{\left(g+h\,x\right)}^n\right)\right)}^2}{g\,k+h\,k\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(s + t*log(i*(g + h*x)^n))^2)/(g*
k + h*k*x),x)
```

```
[Out] int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(s + t*log(i*(g + h*x)^n))^2)/(g*
k + h*k*x), x)
```

3.52 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)(s+t\log(i(g+hx)^n))}{gk+hkx}\,dx$

Optimal. Leaf size=306

$$-\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}{2hknt}-\frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}{2hknt}+\frac{\log\left(e(f(a+bx)^p(\right.}{}$$

```
[Out] -1/2*p*r*ln(-h*(b*x+a)/(-a*h+b*g))*(s+t*ln(i*(h*x+g)^n))^2/h/k/n/t-1/2*q*r*
ln(-h*(d*x+c)/(-c*h+d*g))*(s+t*ln(i*(h*x+g)^n))^2/h/k/n/t+1/2*ln(e*(f*(b*x+
a)^p*(d*x+c)^q)^r)*(s+t*ln(i*(h*x+g)^n))^2/h/k/n/t-p*r*(s+t*ln(i*(h*x+g)^n)
)*polylog(2,b*(h*x+g)/(-a*h+b*g))/h/k-q*r*(s+t*ln(i*(h*x+g)^n))*polylog(2,d
*(h*x+g)/(-c*h+d*g))/h/k+n*p*r*t*polylog(3,b*(h*x+g)/(-a*h+b*g))/h/k+n*q*r*
t*polylog(3,d*(h*x+g)/(-c*h+d*g))/h/k
```

Rubi [A]
time = 0.21, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {2585, 2443, 2481, 2421, 6724}

$$-\frac{pr\,\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)\left(t\log\left(i(g+hx)^n\right)+s\right)}{hk}+\frac{nprt\,\text{PolyLog}\left(3,\frac{b(g+hx)}{bg-ah}\right)}{hk}-\frac{qr\,\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)\left(t\log\left(i(g+hx)^n\right)+s\right)}{hk}+\frac{nqrt\,\text{PolyLog}\left(3,\frac{d(g+hx)}{dg-ch}\right)}{hk}+\frac{\left(t\log\left(i(g+hx)^n\right)+s\right)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2hknt}-\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(t\log\left(i(g+hx)^n\right)+s\right)^2}{2hknt}-\frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\left(t\log\left(i(g+hx)^n\right)+s\right)^2}{2hknt}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*(s + t*Log[i*(g + h*x)^n]))/(g*k
+ h*k*x),x]
```

```
[Out] -1/2*(p*r*Log[-((h*(a + b*x))/(b*g - a*h))]*(s + t*Log[i*(g + h*x)^n])^2)/(
h*k*n*t) - (q*r*Log[-((h*(c + d*x))/(d*g - c*h))]*(s + t*Log[i*(g + h*x)^n]
)^2)/(2*h*k*n*t) + (Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*(s + t*Log[i*(g +
h*x)^n])^2)/(2*h*k*n*t) - (p*r*(s + t*Log[i*(g + h*x)^n])*PolyLog[2, (b*(g
+ h*x))/(b*g - a*h)])/(h*k) - (q*r*(s + t*Log[i*(g + h*x)^n])*PolyLog[2, (d
*(g + h*x))/(d*g - c*h)])/(h*k) + (n*p*r*t*PolyLog[3, (b*(g + h*x))/(b*g -
a*h)])/(h*k) + (n*q*r*t*PolyLog[3, (d*(g + h*x))/(d*g - c*h)])/(h*k)
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
```

```
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2585

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))/((j_.
) + (k_.)*(x_)), x_Symbol] :> Simp[(s + t*Log[i*(g + h*x)^n])^(m + 1)*(Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(k*n*t*(m + 1))), x] + (-Dist[b*p*(r/(k*n*
t*(m + 1))), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Dis
t[d*q*(r/(k*n*t*(m + 1))), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*x)
, x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r},
x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\left(s+t\log \left(52(g+hx)^n\right)\right)}{gk+hkx}\,dx = \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\left(s+t\log \left(52(g\right.\right.}{2hknt}$$

$$= -\frac{pr\log \left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log \left(52(g+hx)^n\right)\right)^2}{2hknt}$$

$$= -\frac{pr\log \left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log \left(52(g+hx)^n\right)\right)^2}{2hknt}$$

$$= -\frac{pr\log \left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log \left(52(g+hx)^n\right)\right)^2}{2hknt}$$

$$= -\frac{pr\log \left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log \left(52(g+hx)^n\right)\right)^2}{2hknt}$$

Mathematica [A]
time = 0.95, size = 436, normalized size = 1.42

$$\frac{-2prs\log \left(\frac{h(a+bx)}{-bg+ah}\right)\log(g+hx)-2qrs\log \left(\frac{h(c+dx)}{-dg+ch}\right)\log(g+hx)+2s\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\log(g+hx)+nprt\log \left(\frac{h(a+bx)}{-bg+ah}\right)\log^2(g+hx)+nqrt\log \left(\frac{h(c+dx)}{-dg+ch}\right)\log^2(g+hx)-nt\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\log^2(g+hx)-2prt\log \left(\frac{h(a+bx)}{-bg+ah}\right)\log(g+hx)\log \left(i(g+hx)^n\right)-2qrt\log \left(\frac{h(c+dx)}{-dg+ch}\right)\log(g+hx)\log \left(i(g+hx)^n\right)+2t\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)\log(g+hx)\log \left(i(g+hx)^n\right)-2pr\left(s+t\log \left(i(g+hx)^n\right)\right)\mathrm{Li}_2\left(\frac{b(g+hx)}{bg-ah}\right)-2qr\left(s+t\log \left(i(g+hx)^n\right)\right)\mathrm{Li}_2\left(\frac{d(g+hx)}{dg-ch}\right)+2nprt\mathrm{Li}_3\left(\frac{b(g+hx)}{bg-ah}\right)+2nqrt\mathrm{Li}_3\left(\frac{d(g+hx)}{dg-ch}\right)}{2hk}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*(s + t*Log[i*(g + h*x)^n]))
/(g*k + h*k*x),x]
```

```
[Out] (-2*p*r*s*Log[(h*(a + b*x))/(-(b*g) + a*h)]*Log[g + h*x] - 2*q*r*s*Log[(h*(
c + d*x))/(-(d*g) + c*h)]*Log[g + h*x] + 2*s*Log[e*(f*(a + b*x)^p*(c + d*x)
^q)^r]*Log[g + h*x] + n*p*r*t*Log[(h*(a + b*x))/(-(b*g) + a*h)]*Log[g + h*x
]^2 + n*q*r*t*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[g + h*x]^2 - n*t*Log[e*
(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g + h*x]^2 - 2*p*r*t*Log[(h*(a + b*x))/(
-(b*g) + a*h)]*Log[g + h*x]*Log[i*(g + h*x)^n] - 2*q*r*t*Log[(h*(c + d*x))/
(-(d*g) + c*h)]*Log[g + h*x]*Log[i*(g + h*x)^n] + 2*t*Log[e*(f*(a + b*x)^p*
(c + d*x)^q)^r]*Log[g + h*x]*Log[i*(g + h*x)^n] - 2*p*r*(s + t*Log[i*(g + h
*x)^n])*PolyLog[2, (b*(g + h*x))/(b*g - a*h)] - 2*q*r*(s + t*Log[i*(g + h*x
)^n])*PolyLog[2, (d*(g + h*x))/(d*g - c*h)] + 2*n*p*r*t*PolyLog[3, (b*(g +
h*x))/(b*g - a*h)] + 2*n*q*r*t*PolyLog[3, (d*(g + h*x))/(d*g - c*h)])/(2*h*
k)
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(e(f(bx+a)^p(dx+c)^q)^r\right)\left(s+t\ln \left(i(hx+g)^n\right)\right)}{hkx+gk}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*ln(i*(h*x+g)^n))/(h*k*x+g*k),x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*ln(i*(h*x+g)^n))/(h*k*x+g*k),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*log(i*(h*x+g)^n))/(h*k*x+g*
k),x, algorithm="maxima")
```

```
[Out] -1/2*((n*t*log(h*x + g)^2 - 2*t*log(h*x + g)*log((h*x + g)^n) - (I*pi*t + 2
*s)*log(h*x + g))*log(((b*x + a)^p)^r) + (n*t*log(h*x + g)^2 - 2*t*log(h*x
+ g)*log((h*x + g)^n) - (I*pi*t + 2*s)*log(h*x + g))*log(((d*x + c)^q)^r))/
(h*k) - integrate(1/2*(-I*pi*(h*r*t*log(f) + h*t)*a*c - 2*(h*r*s*log(f) + h
*s)*a*c + (-I*pi*(h*r*t*log(f) + h*t)*b*d - 2*(h*r*s*log(f) + h*s)*b*d)*x^2
 - ((p*r + q*r)*b*d*h*n*t*x^2 + b*c*g*n*p*r*t + a*d*g*n*q*r*t + (a*d*h*n*q*
r*t + (c*h*n*p*r*t + (p*r + q*r)*d*g*n*t)*b)*x)*log(h*x + g)^2 - (2*(h*r*s*
log(f) + h*s)*b*c + 2*(h*r*s*log(f) + h*s)*a*d + I*pi*((h*r*t*log(f) + h*t)
*b*c + (h*r*t*log(f) + h*t)*a*d))*x + (2*b*c*g*p*r*s + 2*a*d*g*q*r*s + (I*p
i*(p*r + q*r)*b*d*h*t + 2*(p*r*s + q*r*s)*b*d*h)*x^2 + I*pi*(b*c*g*p*r*t +
a*d*g*q*r*t) + (2*a*d*h*q*r*s + I*pi*(a*d*h*q*r*t + (c*h*p*r*t + (p*r + q*r
)*d*g*t)*b) + 2*(c*h*p*r*s + (p*r*s + q*r*s)*d*g)*b)*x)*log(h*x + g) - 2*((
h*r*t*log(f) + h*t)*b*d*x^2 + (h*r*t*log(f) + h*t)*a*c + ((h*r*t*log(f) + h
*t)*b*c + (h*r*t*log(f) + h*t)*a*d)*x - ((p*r + q*r)*b*d*h*t*x^2 + b*c*g*p*
r*t + a*d*g*q*r*t + (a*d*h*q*r*t + (c*h*p*r*t + (p*r + q*r)*d*g*t)*b)*x)*lo
g(h*x + g))*log((h*x + g)^n))/(b*d*h^2*k*x^3 + a*c*g*h*k + (a*d*h^2*k + (d*
g*h*k + c*h^2*k)*b)*x^2 + (b*c*g*h*k + (d*g*h*k + c*h^2*k)*a)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*log(i*(h*x+g)^n))/(h*k*x+g*
k),x, algorithm="fricas")
```

```
[Out] integral(-1/2*(-I*pi*t + (-I*pi*p*r*t - 2*p*r*s)*log(b*x + a) + (-I*pi*q*r*
t - 2*q*r*s)*log(d*x + c) - 2*(n*p*r*t*log(b*x + a) + n*q*r*t*log(d*x + c)
+ n*r*t*log(f) + n*t)*log(h*x + g) + (-I*pi*r*t - 2*r*s)*log(f) - 2*s)/(h*k
*x + g*k), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)*(s+t*ln(i*(h*x+g)**n))/(h*k*x+
g*k),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*log(i*(h*x+g)^n))/(h*k*x+g*
k),x, algorithm="giac")
```

```
[Out] integrate((t*log(I*(h*x + g)^n) + s)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(
h*k*x + g*k), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\,{\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)}^r\right)\,\left(s+t\,\ln\left(i\,{\left(g+h\,x\right)}^n\right)\right)}{g\,k+h\,k\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(s + t*log(i*(g + h*x)^n)))/(g*k
+ h*k*x),x)
```

```
[Out] int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*(s + t*log(i*(g + h*x)^n)))/(g*k
+ h*k*x), x)
```

3.53 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{gk+hkx}\,dx$

Optimal. Leaf size=172

$$-\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(gk+hkx)}{hk}-\frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(gk+hkx)}{hk}+\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\log(gk$$

```
[Out] -p*r*ln(-h*(b*x+a)/(-a*h+b*g))*ln(h*k*x+g*k)/h/k-q*r*ln(-h*(d*x+c)/(-c*h+d*
g))*ln(h*k*x+g*k)/h/k+ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*ln(h*k*x+g*k)/h/k-p*r
*polylog(2,b*(h*x+g)/(-a*h+b*g))/h/k-q*r*polylog(2,d*(h*x+g)/(-c*h+d*g))/h/
k
```

Rubi [A]
time = 0.09, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2580, 2441, 2440, 2438}

$$-\frac{pr\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)}{hk}-\frac{qr\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)}{hk}+\frac{\log(gk+hkx)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{hk}-\frac{pr\log(gk+hkx)\log\left(-\frac{h(a+bx)}{bg-ah}\right)}{hk}-\frac{qr\log(gk+hkx)\log\left(-\frac{h(c+dx)}{dg-ch}\right)}{hk}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g*k + h*k*x),x]
```

```
[Out] -((p*r*Log[-((h*(a + b*x))/(b*g - a*h))]*Log[g*k + h*k*x])/(h*k)) - (q*r*Lo
g[-((h*(c + d*x))/(d*g - c*h))]*Log[g*k + h*k*x])/(h*k) + (Log[e*(f*(a + b*
x)^p*(c + d*x)^q)^r]*Log[g*k + h*k*x])/(h*k) - (p*r*PolyLog[2, (b*(g + h*x)
)/(b*g - a*h)])/(h*k) - (q*r*PolyLog[2, (d*(g + h*x))/(d*g - c*h)])/(h*k)
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2580

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/h), x] + (-Dist[b*p*(r/h), Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[d*q*(r/h), Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{gk+hkx}\,dx = \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\log(gk+hkx)}{hk} - \frac{(bpr)\int \frac{\log(gk+hkx)}{a+bx}\,dx}{hk} -$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(gk+hkx)}{hk} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(gk+hkx)}{hk}$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(gk+hkx)}{hk} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(gk+hkx)}{hk}$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(gk+hkx)}{hk} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(gk+hkx)}{hk}$$

Mathematica [A]
time = 0.07, size = 166, normalized size = 0.97

$$\frac{-pr\log(a+bx)\log(g+hx) - qr\log(c+dx)\log(g+hx) + \log\left(e(f(a+bx)^p(c+dx)^q)^r\right)\log(g+hx) + pr\log(a+bx)\log\left(\frac{b(g+hx)}{bg-ah}\right) + qr\log(c+dx)\log\left(\frac{d(g+hx)}{dg-ch}\right) + pr\mathrm{Li}_2\left(\frac{h(a+bx)}{-bg+ah}\right) + qr\mathrm{Li}_2\left(\frac{h(c+dx)}{-dg+ch}\right)}{hk}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g*k + h*k*x),x]
```

```
[Out] (-(p*r*Log[a + b*x]*Log[g + h*x]) - q*r*Log[c + d*x]*Log[g + h*x] + Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g + h*x] + p*r*Log[a + b*x]*Log[(b*(g + h
*x))/(b*g - a*h)] + q*r*Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)] + p*r*P
olyLog[2, (h*(a + b*x))/(-(b*g) + a*h)] + q*r*PolyLog[2, (h*(c + d*x))/(-(d
*g) + c*h)])/(h*k)
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\left(bx+a\right)^{p}\left(dx+c\right)^{q}\right)^{r}\right)}{hkx+gk}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k),x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k),x)
```

Maxima [A]
time = 0.29, size = 205, normalized size = 1.19

$$\frac{\left(\frac{\left(\log(bx+a)\log\left(\frac{bhx+ah}{bg-ah}+1\right)+\mathrm{Li}_2\left(-\frac{bhx+ah}{bg-ah}\right)\right)fp}{hk}+\frac{\left(\log(dx+c)\log\left(\frac{dhx+ch}{dg-ch}+1\right)+\mathrm{Li}_2\left(-\frac{dhx+ch}{dg-ch}\right)\right)fq}{hk}\right)r}{f}-\frac{(fp\log(bx+a)+fq\log(dx+c))r\log(hkx+gk)}{fhk}+\frac{\log(hkx+gk)\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)}{hk}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k),x, algorithm="maxima
")
```

```
[Out] ((log(b*x + a)*log((b*h*x + a*h)/(b*g - a*h) + 1) + dilog(-(b*h*x + a*h)/(b
*g - a*h)))*f*p/(h*k) + (log(d*x + c)*log((d*h*x + c*h)/(d*g - c*h) + 1) +
dilog(-(d*h*x + c*h)/(d*g - c*h)))*f*q/(h*k))*r/f - (f*p*log(b*x + a) + f*q
*log(d*x + c))*r*log(h*k*x + g*k)/(f*h*k) + log(h*k*x + g*k)*log(((b*x + a)
^p*(d*x + c)^q*f)^r*e)/(h*k)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k),x, algorithm="fricas
")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*k*x + g*k), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*k*x+g*k),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k),x, algorithm="giac")
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*k*x + g*k), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(e\,{\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)}^r\right)}{g\,k+h\,k\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(g*k + h*k*x),x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(g*k + h*k*x), x)
```

3.54 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(gk+hkx)(s+t\log(i(g+hx)^n))}\,dx$

Optimal. Leaf size=51

$$\operatorname{Int}\left(\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)},x\right)$$

```
[Out] Unintegrable(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)^
n)),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i*(g + h
*x)^n])),x]
```

```
[Out] Defer[Int][Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i
*(g + h*x)^n])), x]
```

Rubi steps

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(54(g+hx)^n\right)\right)}\,dx = \int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(54(g+hx)^n\right)\right)}\,dx$$

Mathematica [A]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i*
(g + h*x)^n])),x]
```

```
[Out] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i*
(g + h*x)^n])), x]
```

Maple [A]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(e(f(bx+a)^{p} (dx+c)^{q})^{r}\right)}{(hkx+gk)\left(s+t\ln \left(i\left(hx+g\right)^{n}\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)^n)),x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)^n)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^n
)),x, algorithm="maxima")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((h*k*x + g*k)*(t*log(I*(h*x
 + g)^n) + s)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^n
)),x, algorithm="fricas")
```

```
[Out] integral(2*(p*r*log(b*x + a) + q*r*log(d*x + c) + r*log(f) + 1)/(2*h*k*s*x
+ 2*g*k*s + I*pi*(h*k*t*x + g*k*t) + 2*(h*k*n*t*x + g*k*n*t)*log(h*x + g)),
 x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)*
*n)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^n
)),x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((h*k*x + g*k)*(t*log(I*(h*x
 + g)^n) + s)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(e\,{\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)}^r\right)}{\left(g\,k+h\,k\,x\right)\,\left(s+t\,\ln\left(i\,{\left(g+h\,x\right)}^n\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/((g*k + h*k*x)*(s + t*log(i*(g + h
*x)^n))),x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/((g*k + h*k*x)*(s + t*log(i*(g + h
*x)^n))), x)
```

3.55 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{(gk+hkx)(s+t\log(i(g+hx)^n))^2}\,dx$

Optimal. Leaf size=51

$$\text{Int}\left(\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)^
n))^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i*(g + h
*x)^n])^2),x]
```

```
[Out] Defer[Int][Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i
*(g + h*x)^n])^2), x]
```

Rubi steps

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(55(g+hx)^n\right)\right)^2}\,dx=\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(55(g+hx)^n\right)\right)^2}\,dx$$

Mathematica [A]
time = 1.40, size = 0, normalized size = 0.00

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i*
(g + h*x)^n])^2),x]
```

```
[Out] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i*
(g + h*x)^n])^2), x]
```

Maple [A]
time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(e(f(bx+a)^{p} (dx+c)^{q})^{r}\right)}{(hkx+gk) \left(s+t\ln \left(i \left(hx+g\right)^{n}\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)^n))^2,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)^n))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^n
))^2,x, algorithm="maxima")
```

```
[Out] -(r*log(f) + log(((b*x + a)^p)^r) + log(((d*x + c)^q)^r) + 1)/(1/2*I*pi*h*k
*n*t^2 + h*k*n*t^2*log((h*x + g)^n) + h*k*n*s*t) + integrate((b*c*p*r + a*d
*q*r + (p*r + q*r)*b*d*x)/(1/2*I*pi*a*c*h*k*n*t^2 + a*c*h*k*n*s*t - 1/2*(-I
*pi*b*d*h*k*n*t^2 - 2*b*d*h*k*n*s*t)*x^2 + 1/2*(2*b*c*h*k*n*s*t + 2*a*d*h*k
*n*s*t + I*pi*(b*c*h*k*n*t^2 + a*d*h*k*n*t^2))*x + (b*d*h*k*n*t^2*x^2 + a*c
*h*k*n*t^2 + (b*c*h*k*n*t^2 + a*d*h*k*n*t^2)*x)*log((h*x + g)^n)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^n
))^2,x, algorithm="fricas")
```

```
[Out] -(2*p*r*log(b*x + a) + 2*q*r*log(d*x + c) - (2*h*k*n^2*t^2*log(h*x + g) + I
*pi*h*k*n*t^2 + 2*h*k*n*s*t)*integral(2*((b*d*p + b*d*q)*r*x + (b*c*p + a*d
*q)*r)/(2*b*d*h*k*n*s*t*x^2 + 2*a*c*h*k*n*s*t + 2*(b*c + a*d)*h*k*n*s*t*x +
 I*pi*(b*d*h*k*n*t^2*x^2 + a*c*h*k*n*t^2 + (b*c + a*d)*h*k*n*t^2*x) + 2*(b*
d*h*k*n^2*t^2*x^2 + a*c*h*k*n^2*t^2 + (b*c + a*d)*h*k*n^2*t^2*x)*log(h*x +
g)), x) + 2*r*log(f) + 2)/(2*h*k*n^2*t^2*log(h*x + g) + I*pi*h*k*n*t^2 + 2*
h*k*n*s*t)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)*
*n))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3436 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^n
))^2,x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((h*k*x + g*k)*(t*log(I*(h*x
 + g)^n) + s)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(e\,{\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)}^r\right)}{\left(g\,k+h\,k\,x\right)\,{\left(s+t\,\ln\left(i\,{\left(g+h\,x\right)}^n\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/((g*k + h*k*x)*(s + t*log(i*(g + h
*x)^n))^2),x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/((g*k + h*k*x)*(s + t*log(i*(g + h
*x)^n))^2), x)
```

3.56 $\int \frac{\log^3\left(i\left(j(hx)^t\right)^u\right)\log(e(f(a+bx)^p(c+dx)^q)^r)}{x}\,dx$

Optimal. Leaf size=328

$$-\frac{pr\log^4\left(i(j(hx)^t)^u\right)\log\left(1+\frac{bx}{a}\right)}{4tu}+\frac{\log^4\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4tu}-\frac{qr\log^4\left(i(j(hx)^t)^u\right)\log\left(\right.}{4tu}$$

```
[Out] -1/4*p*r*ln(i*(j*(h*x)^t)^u)^4*ln(1+b*x/a)/t/u+1/4*ln(i*(j*(h*x)^t)^u)^4*ln
(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/t/u-1/4*q*r*ln(i*(j*(h*x)^t)^u)^4*ln(1+d*x/c)
/t/u-p*r*ln(i*(j*(h*x)^t)^u)^3*polylog(2,-b*x/a)-q*r*ln(i*(j*(h*x)^t)^u)^3*
polylog(2,-d*x/c)+3*p*r*t*u*ln(i*(j*(h*x)^t)^u)^2*polylog(3,-b*x/a)+3*q*r*t
*u*ln(i*(j*(h*x)^t)^u)^2*polylog(3,-d*x/c)-6*p*r*t^2*u^2*ln(i*(j*(h*x)^t)^u
)*polylog(4,-b*x/a)-6*q*r*t^2*u^2*ln(i*(j*(h*x)^t)^u)*polylog(4,-d*x/c)+6*p
*r*t^3*u^3*polylog(5,-b*x/a)+6*q*r*t^3*u^3*polylog(5,-d*x/c)
```

Rubi [A]
time = 0.87, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2585, 2354, 2421, 2430, 6724, 2495}

$$-6prt^2u^2\text{PolyLog}\left(4,-\frac{bx}{a}\right)\log\left(i(j(hx)^t)^u\right)-pr\text{PolyLog}\left(2,-\frac{bx}{a}\right)\log^3\left(i(j(hx)^t)^u\right)+3prtu\text{PolyLog}\left(3,-\frac{bx}{a}\right)\log^2\left(i(j(hx)^t)^u\right)+6prt^3u^3\text{PolyLog}\left(5,-\frac{bx}{a}\right)-6qrt^2u^2\text{PolyLog}\left(4,-\frac{dx}{c}\right)\log\left(i(j(hx)^t)^u\right)-qr\text{PolyLog}\left(2,-\frac{dx}{c}\right)\log^3\left(i(j(hx)^t)^u\right)+3qrtu\text{PolyLog}\left(3,-\frac{dx}{c}\right)\log^2\left(i(j(hx)^t)^u\right)+6qrt^3u^3\text{PolyLog}\left(5,-\frac{dx}{c}\right)+\frac{\log^4\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4tu}-\frac{pr\log\left(\frac{bx}{a}+1\right)\log^4\left(i(j(hx)^t)^u\right)}{4tu}-\frac{qr\log\left(\frac{dx}{c}+1\right)\log^4\left(i(j(hx)^t)^u\right)}{4tu}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[i*(j*(h*x)^t)^u]^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/x,x]
```

```
[Out] -1/4*(p*r*Log[i*(j*(h*x)^t)^u]^4*Log[1 + (b*x)/a])/(t*u) + (Log[i*(j*(h*x)^
t)^u]^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*t*u) - (q*r*Log[i*(j*(h*x)
^t)^u]^4*Log[1 + (d*x)/c])/(4*t*u) - p*r*Log[i*(j*(h*x)^t)^u]^3*PolyLog[2,
-((b*x)/a)] - q*r*Log[i*(j*(h*x)^t)^u]^3*PolyLog[2, -((d*x)/c)] + 3*p*r*t*u
*Log[i*(j*(h*x)^t)^u]^2*PolyLog[3, -((b*x)/a)] + 3*q*r*t*u*Log[i*(j*(h*x)^t
)^u]^2*PolyLog[3, -((d*x)/c)] - 6*p*r*t^2*u^2*Log[i*(j*(h*x)^t)^u]*PolyLog[
4, -((b*x)/a)] - 6*q*r*t^2*u^2*Log[i*(j*(h*x)^t)^u]*PolyLog[4, -((d*x)/c)]
+ 6*p*r*t^3*u^3*PolyLog[5, -((b*x)/a)] + 6*q*r*t^3*u^3*PolyLog[5, -((d*x)/c
)]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
```

```
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 2585

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))/((j_.
) + (k_.)*(x_)), x_Symbol] :> Simp[(s + t*Log[i*(g + h*x)^n])^(m + 1)*(Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(k*n*t*(m + 1))), x] + (-Dist[b*p*(r/(k*n*
t*(m + 1))), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Dis
t[d*q*(r/(k*n*t*(m + 1))), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*x)
, x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r},
x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log^3\left(56(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = \text{Subst}\left(\int \frac{\log^3\left(56j^u(hx)^{tu}\right)\log\left(e(f(a+bx)^p(c+\right.}{x}\right.$$

$$= \text{Subst}\left(\text{Subst}\left(\int \frac{\log^3\left(56h^{tu}j^ux^{tu}\right)\log\left(e(f(a+bx\right.}{x}\right.\right.$$

$$= \frac{\log^4\left(56(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{4tu} -$$

$$= -\frac{pr\log^4\left(56(j(hx)^t)^u\right)\log\left(1+\frac{bx}{a}\right)}{4tu} + \frac{\log^4\left(56(j(h\right.}{}$$

$$= -\frac{pr\log^4\left(56(j(hx)^t)^u\right)\log\left(1+\frac{bx}{a}\right)}{4tu} + \frac{\log^4\left(56(j(h\right.}{}$$

$$= -\frac{pr\log^4\left(56(j(hx)^t)^u\right)\log\left(1+\frac{bx}{a}\right)}{4tu} + \frac{\log^4\left(56(j(h\right.}{}$$

$$= -\frac{pr\log^4\left(56(j(hx)^t)^u\right)\log\left(1+\frac{bx}{a}\right)}{4tu} + \frac{\log^4\left(56(j(h\right.}{}$$

$$= -\frac{pr\log^4\left(56(j(hx)^t)^u\right)\log\left(1+\frac{bx}{a}\right)}{4tu} + \frac{\log^4\left(56(j(h\right.}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1241 vs. $2(328) = 656$.
time = 1.17, size = 1241, normalized size = 3.78

Antiderivative was successfully verified.

```
[In] Integrate[(Log[i*(j*(h*x)^t)^u]^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/x,x
]
```

```
[Out] p*r*t^3*u^3*Log[x]*Log[h*x]^3*Log[a + b*x] - p*r*t^3*u^3*Log[h*x]^4*Log[a +
 b*x] - 3*p*r*t^2*u^2*Log[x]*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]*Log[a + b*x] +
 3*p*r*t^2*u^2*Log[h*x]^3*Log[i*(j*(h*x)^t)^u]*Log[a + b*x] + 3*p*r*t*u*Log
[x]*Log[h*x]*Log[i*(j*(h*x)^t)^u]^2*Log[a + b*x] - 3*p*r*t*u*Log[h*x]^2*Log
[i*(j*(h*x)^t)^u]^2*Log[a + b*x] - p*r*Log[x]*Log[i*(j*(h*x)^t)^u]^3*Log[a
+ b*x] + p*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^3*Log[a + b*x] + (p*r*t^3*u^3*Lo
g[h*x]^4*Log[1 + (b*x)/a])/4 - p*r*t^2*u^2*Log[h*x]^3*Log[i*(j*(h*x)^t)^u]*
Log[1 + (b*x)/a] + (3*p*r*t*u*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]^2*Log[1 + (b*
x)/a])/2 - p*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^3*Log[1 + (b*x)/a] + q*r*t^3*u
^3*Log[x]*Log[h*x]^3*Log[c + d*x] - q*r*t^3*u^3*Log[h*x]^4*Log[c + d*x] - 3
```

```
*q*r*t^2*u^2*Log[x]*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]*Log[c + d*x] + 3*q*r*t^
2*u^2*Log[h*x]^3*Log[i*(j*(h*x)^t)^u]*Log[c + d*x] + 3*q*r*t*u*Log[x]*Log[h
*x]*Log[i*(j*(h*x)^t)^u]^2*Log[c + d*x] - 3*q*r*t*u*Log[h*x]^2*Log[i*(j*(h*
x)^t)^u]^2*Log[c + d*x] - q*r*Log[x]*Log[i*(j*(h*x)^t)^u]^3*Log[c + d*x] +
q*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^3*Log[c + d*x] - t^3*u^3*Log[x]*Log[h*x]^
3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (3*t^3*u^3*Log[h*x]^4*Log[e*(f*(a
+ b*x)^p*(c + d*x)^q)^r])/4 + 3*t^2*u^2*Log[x]*Log[h*x]^2*Log[i*(j*(h*x)^t)
^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*t^2*u^2*Log[h*x]^3*Log[i*(j*(h
*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 3*t*u*Log[x]*Log[h*x]*Log[
i*(j*(h*x)^t)^u]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (3*t*u*Log[h*x]^2
*Log[i*(j*(h*x)^t)^u]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/2 + Log[x]*Lo
g[i*(j*(h*x)^t)^u]^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (q*r*t^3*u^3*Lo
g[h*x]^4*Log[1 + (d*x)/c])/4 - q*r*t^2*u^2*Log[h*x]^3*Log[i*(j*(h*x)^t)^u]*
Log[1 + (d*x)/c] + (3*q*r*t*u*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]^2*Log[1 + (d*
x)/c])/2 - q*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^3*Log[1 + (d*x)/c] - p*r*Log[i
*(j*(h*x)^t)^u]^3*PolyLog[2, -((b*x)/a)] - q*r*Log[i*(j*(h*x)^t)^u]^3*PolyL
og[2, -((d*x)/c)] + 3*p*r*t*u*Log[i*(j*(h*x)^t)^u]^2*PolyLog[3, -((b*x)/a)]
 + 3*q*r*t*u*Log[i*(j*(h*x)^t)^u]^2*PolyLog[3, -((d*x)/c)] - 6*p*r*t^2*u^2*
Log[i*(j*(h*x)^t)^u]*PolyLog[4, -((b*x)/a)] - 6*q*r*t^2*u^2*Log[i*(j*(h*x)^
t)^u]*PolyLog[4, -((d*x)/c)] + 6*p*r*t^3*u^3*PolyLog[5, -((b*x)/a)] + 6*q*r
*t^3*u^3*PolyLog[5, -((d*x)/c)]
```

Maple [F]
time = 1.13, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(i\left(j\left(hx\right)^{t}\right)^{u}\right)^{3} \ln \left(e\left(f\left(bx+a\right)^{p} \left(dx+c\right)^{q}\right)^{r}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(i*(j*(h*x)^t)^u)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

```
[Out] int(ln(i*(j*(h*x)^t)^u)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(i*(j*(h*x)^t)^u)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algo
rithm="maxima")
```

```
[Out] -1/8*(2*t^3*u^3*log(x)^4 - 4*(2*t^3*u^3*log(h) + 2*t^2*u^3*log(j) + I*pi*t^
2*u^2)*log(x)^3 - 8*log(x)*log((x^t)^u)^3 + 3*(4*t^3*u^3*log(h)^2 + 8*t^2*u
^3*log(h)*log(j) + 4*t*u^3*log(j)^2 - pi^2*t*u + 4*I*pi*(t^2*u^2*log(h) + t
```

```
*u^2*log(j)))*log(x)^2 + 12*(t*u*log(x)^2 + (-I*pi - 2*t*u*log(h) - 2*u*log
(j))*log(x))*log((x^t)^u)^2 - (8*t^3*u^3*log(h)^3 + 24*t^2*u^3*log(h)^2*log
(j) + 24*t*u^3*log(h)*log(j)^2 + 8*u^3*log(j)^3 - I*pi^3 - 6*pi^2*(t*u*log(
h) + u*log(j)) + 12*I*pi*(t^2*u^2*log(h)^2 + 2*t*u^2*log(h)*log(j) + u^2*lo
g(j)^2))*log(x) - 2*(4*t^2*u^2*log(x)^3 - 6*(2*t^2*u^2*log(h) + 2*t*u^2*log
(j) + I*pi*t*u)*log(x)^2 + 3*(4*t^2*u^2*log(h)^2 + 8*t*u^2*log(h)*log(j) +
4*u^2*log(j)^2 - pi^2 + 4*I*pi*(t*u*log(h) + u*log(j)))*log(x))*log((x^t)^u
))*log(((b*x + a)^p)^r) - 1/8*(2*t^3*u^3*log(x)^4 - 4*(2*t^3*u^3*log(h) + 2
*t^2*u^3*log(j) + I*pi*t^2*u^2)*log(x)^3 - 8*log(x)*log((x^t)^u)^3 + 3*(4*t
^3*u^3*log(h)^2 + 8*t^2*u^3*log(h)*log(j) + 4*t*u^3*log(j)^2 - pi^2*t*u + 4
*I*pi*(t^2*u^2*log(h) + t*u^2*log(j)))*log(x)^2 + 12*(t*u*log(x)^2 + (-I*pi
 - 2*t*u*log(h) - 2*u*log(j))*log(x))*log((x^t)^u)^2 - (8*t^3*u^3*log(h)^3
+ 24*t^2*u^3*log(h)^2*log(j) + 24*t*u^3*log(h)*log(j)^2 + 8*u^3*log(j)^3 -
I*pi^3 - 6*pi^2*(t*u*log(h) + u*log(j)) + 12*I*pi*(t^2*u^2*log(h)^2 + 2*t*u
^2*log(h)*log(j) + u^2*log(j)^2))*log(x) - 2*(4*t^2*u^2*log(x)^3 - 6*(2*t^2
*u^2*log(h) + 2*t*u^2*log(j) + I*pi*t*u)*log(x)^2 + 3*(4*t^2*u^2*log(h)^2 +
 8*t*u^2*log(h)*log(j) + 4*u^2*log(j)^2 - pi^2 + 4*I*pi*(t*u*log(h) + u*log
(j)))*log(x))*log((x^t)^u))*log(((d*x + c)^q)^r) - integrate(-1/8*(-I*pi^3*
(r*log(f) + 1)*a*c - 6*pi^2*(t*u*log(h) + (r*t*u*log(h) + r*u*log(j))*log(f
) + u*log(j))*a*c + 2*((p*r*t^3*u^3 + q*r*t^3*u^3)*b*d*x^2 + (b*c*p*r*t^3*u
^3 + a*d*q*r*t^3*u^3)*x)*log(x)^4 + 12*I*pi*(t^2*u^2*log(h)^2 + 2*t*u^2*log
(h)*log(j) + u^2*log(j)^2 + (r*t^2*u^2*log(h)^2 + 2*r*t*u^2*log(h)*log(j) +
 r*u^2*log(j)^2)*log(f))*a*c + 8*((r*log(f) + 1)*b*d*x^2 + (r*log(f) + 1)*a
*c + ((r*log(f) + 1)*b*c + (r*log(f) + 1)*a*d)*x - ((p*r + q*r)*b*d*x^2 + (
b*c*p*r + a*d*q*r)*x)*log(x))*log((x^t)^u)^3 + 4*((-I*pi*(p*r*t^2*u^2 + q*r
*t^2*u^2)*b*d - 2*((p*r*t^3*u^3 + q*r*t^3*u^3)*log(h) + (p*r*t^2*u^3 + q*r*
t^2*u^3)*log(j))*b*d)*x^2 - (2*(p*r*t^3*u^3*log(h) + p*r*t^2*u^3*log(j))*b*
c + 2*(q*r*t^3*u^3*log(h) + q*r*t^2*u^3*log(j))*a*d + I*pi*(b*c*p*r*t^2*u^2
 + a*d*q*r*t^2*u^2))*x)*log(x)^3 + 8*(t^3*u^3*log(h)^3 + 3*t^2*u^3*log(h)^2
*log(j) + 3*t*u^3*log(h)*log(j)^2 + u^3*log(j)^3 + (r*t^3*u^3*log(h)^3 + 3*
r*t^2*u^3*log(h)^2*log(j) + 3*r*t*u^3*log(h)*log(j)^2 + r*u^3*log(j)^3)*log
(f))*a*c + (-I*pi^3*(r*log(f) + 1)*b*d - 6*pi^2*(t*u*log(h) + (r*t*u*log(h)
 + r*u*log(j))*log(f) + u*log(j))*b*d + 12*I*pi*(t^2*u^2*log(h)^2 + 2*t*u^2
*log(h)*log(j) + u^2*log(j)^2 + (r*t^2*u^2*log(h)^2 + 2*r*t*u^2*log(h)*log(
j) + r*u^2*log(j)^2)*log(f))*b*d + 8*(t^3*u^3*log(h)^3 + 3*t^2*u^3*log(h)^2
*log(j) + 3*t*u^3*log(h)*log(j)^2 + u^3*log(j)^3 + (r*t^3*u^3*log(h)^3 + 3*
r*t^2*u^3*log(h)^2*log(j) + 3*r*t*u^3*log(h)*log(j)^2 + r*u^3*log(j)^3)*log
(f))*b*d)*x^2 + 12*(I*pi*(r*log(f) + 1)*a*c + 2*(t*u*log(h) + (r*t*u*log(h)
 + r*u*log(j))*log(f) + u*log(j))*a*c + (I*pi*(r*log(f) + 1)*b*d + 2*(t*u*l
og(h) + (r*t*u*log(h) + r*u*log(j))*log(f) + u*log(j))*b*d)*x^2 + ((p*r*t*u
 + q*r*t*u)*b*d*x^2 + (b*c*p*r*t*u + a*d*q*r*t*u)*x)*log(x)^2 + (2*(t*u*log
(h) + (r*t*u*log(h) + r*u*log(j))*log(f) + u*log(j))*b*c + 2*(t*u*log(h) +
(r*t*u*log(h) + r*u*log(j))*log(f) + u*log(j))*a*d + I*pi*((r*log(f) + 1)*b
*c + (r*log(f) + 1)*a*d))*x + ((-I*pi*(p*r + q*r)*b*d - 2*((p*r*t*u + q*r*t
*u)*log(h) + (p*r*u + q*r*u)*log(j))*b*d)*x^2 - (2*(p*r*t*u*log(h) + p*r*u*
```

```
log(j))*b*c + 2*(q*r*t*u*log(h) + q*r*u*log(j))*a*d + I*pi*(b*c*p*r + a*d*q
*r))*x)*log(x))*log((x^t)^u)^2 - 3*((pi^2*(p*r*t*u + q*r*t*u)*b*d - 4*I*pi*
((p*r*t^2*u^2 + q*r*t^2*u^2)*log(h) + (p*r*t*u^2 + q*r*t*u^2)*log(j))*b*d -
 4*((p*r*t^3*u^3 + q*r*t^3*u^3)*log(h)^2 + 2*(p*r*t^2*u^3 + q*r*t^2*u^3)*lo
g(h)*log(j) + (p*r*t*u^3 + q*r*t*u^3)*log(j)^2)*b*d)*x^2 + (pi^2*(b*c*p*r*t
*u + a*d*q*r*t*u) - 4*(p*r*t^3*u^3*log(h)^2 + 2*p*r*t^2*u^3*log(h)*log(j) +
 p*r*t*u^3*log(j)^2)*b*c - 4*(q*r*t^3*u^3*log(h)^2 + 2*q*r*t^2*u^3*log(h)*l
og(j) + q*r*t*u^3*log(j)^2)*a*d - 4*I*pi*((p*r*t^2*u^2*log(h) + p*r*t*u^2*l
og(j))*b*c + (q*r*t^2*u^2*log(h) + q*r*t*u^2*log(j))*a*d))*x)*log(x)^2 + (-
I*pi^3*((r*log(f) + 1)*b*c + (r*log(f) + 1)*a*d) - 6*pi^2*((t*u*log(h) + (r
*t*u*log(h) + r*u*log(j))*log(f) + u*log(j))*b*c + (t*u*log(h) + (r*t*u*log
(h) + r*u*log(j))*log(f) + u*log(j))*a*d) + 8*(t^3*u^3*log(h)^3 + 3*t^2*u^3
*log(h)^2*log(j) + 3*t*u^3*log(h)*log(j)^2 + u^3*log(j)^3 + (r*t^3*u^3*log(
h)^3 + 3*r*t^2*u^3*log(h)^2*log(j) + 3*r*t*u^3*log(h)*log(j)^2 + r*u^3*log(
j)^3)*log(f))*b*c + 8*(t^3*u^3*log(h)^3 + 3*t^2*u^3*log(h)^2*log(j) + 3*t*u
^3*log(h)*log(j)^2 + u^3*log(j)^3 + (r*t^3*u^3*log(h)^3 + 3*r*t^2*u^3*log(h
)^2*log(j) + 3*r*t*u^3*log(h)*log(j)^2 + r*u^3*log(j)^3)*log(f))*a*d + 12*I
*pi*((t^2*u^2*log(h)^2 + 2*t*u^2*log(h)*log(j) + u^2*log(j)^2 + (r*t^2*u^2*
log(h)^2 + 2*r*t*u^2*log(h)*log(j) + r*u^2*log(...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(i*(j*(h*x)^t)^u)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algo
rithm="fricas")
```

```
[Out] integral(-1/8*(I*pi^3*r*log(f) - 8*(p*r*t^3*u^3*log(b*x + a) + q*r*t^3*u^3*
log(d*x + c) + r*t^3*u^3*log(f) + t^3*u^3)*log(h*x)^3 - 8*(r*u^3*log(f) + u
^3)*log(j)^3 + I*pi^3 + 12*(-I*pi*r*t^2*u^2*log(f) - I*pi*t^2*u^2 - (2*p*r*
t^2*u^3*log(j) + I*pi*p*r*t^2*u^2)*log(b*x + a) - (2*q*r*t^2*u^3*log(j) + I
*pi*q*r*t^2*u^2)*log(d*x + c) - 2*(r*t^2*u^3*log(f) + t^2*u^3)*log(j))*log(
h*x)^2 + 12*(-I*pi*r*u^2*log(f) - I*pi*u^2)*log(j)^2 - (8*p*r*u^3*log(j)^3
+ 12*I*pi*p*r*u^2*log(j)^2 - 6*pi^2*p*r*u*log(j) - I*pi^3*p*r)*log(b*x + a)
 - (8*q*r*u^3*log(j)^3 + 12*I*pi*q*r*u^2*log(j)^2 - 6*pi^2*q*r*u*log(j) - I
*pi^3*q*r)*log(d*x + c) + 6*(pi^2*r*t*u*log(f) + pi^2*t*u - 4*(r*t*u^3*log(
f) + t*u^3)*log(j)^2 - (4*p*r*t*u^3*log(j)^2 + 4*I*pi*p*r*t*u^2*log(j) - pi
^2*p*r*t*u)*log(b*x + a) - (4*q*r*t*u^3*log(j)^2 + 4*I*pi*q*r*t*u^2*log(j)
- pi^2*q*r*t*u)*log(d*x + c) + 4*(-I*pi*r*t*u^2*log(f) - I*pi*t*u^2)*log(j)
)*log(h*x) + 6*(pi^2*r*u*log(f) + pi^2*u)*log(j))/x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(i*(j*(h*x)**t)**u)**3*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/x,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(i*(j*(h*x)^t)^u)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algo
rithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(I*((h*x)^t*j)^u)^3/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)^r\right)\,{\ln\left(i\,{\left(j\,{\left(h\,x\right)}^t\right)}^u\right)}^3}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*log(i*(j*(h*x)^t)^u)^3)/x,x)
```

```
[Out] int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*log(i*(j*(h*x)^t)^u)^3)/x, x)
```

3.57 $\int \frac{\log^2\left(i\left(j(hx)^t\right)^u\right)\log(e(f(a+bx)^p(c+dx)^q)^r)}{x}\,dx$

Optimal. Leaf size=262

$$-\frac{pr\log^3\left(i(j(hx)^t)^u\right)\log\left(1+\frac{bx}{a}\right)}{3tu}+\frac{\log^3\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3tu}-\frac{qr\log^3\left(i(j(hx)^t)^u\right)\log\left(1\right.}{3tu}$$

```
[Out] -1/3*p*r*ln(i*(j*(h*x)^t)^u)^3*ln(1+b*x/a)/t/u+1/3*ln(i*(j*(h*x)^t)^u)^3*ln
(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/t/u-1/3*q*r*ln(i*(j*(h*x)^t)^u)^3*ln(1+d*x/c)
/t/u-p*r*ln(i*(j*(h*x)^t)^u)^2*polylog(2,-b*x/a)-q*r*ln(i*(j*(h*x)^t)^u)^2*
polylog(2,-d*x/c)+2*p*r*t*u*ln(i*(j*(h*x)^t)^u)*polylog(3,-b*x/a)+2*q*r*t*u
*ln(i*(j*(h*x)^t)^u)*polylog(3,-d*x/c)-2*p*r*t^2*u^2*polylog(4,-b*x/a)-2*q*
r*t^2*u^2*polylog(4,-d*x/c)
```

Rubi [A]
time = 0.65, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2585, 2354, 2421, 2430, 6724, 2495}

$$-pr\,\text{PolyLog}\left(2,-\frac{bx}{a}\right)\log^2\left(i(j(hx)^t)^u\right)+2prtu\,\text{PolyLog}\left(3,-\frac{bx}{a}\right)\log\left(i(j(hx)^t)^u\right)-2prt^2u^2\,\text{PolyLog}\left(4,-\frac{bx}{a}\right)-qr\,\text{PolyLog}\left(2,-\frac{dx}{c}\right)\log^2\left(i(j(hx)^t)^u\right)+2qrtu\,\text{PolyLog}\left(3,-\frac{dx}{c}\right)\log\left(i(j(hx)^t)^u\right)-2qrt^2u^2\,\text{PolyLog}\left(4,-\frac{dx}{c}\right)+\frac{\log^3\left(i(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3tu}-\frac{pr\log\left(\frac{bx}{a}+1\right)\log^3\left(i(j(hx)^t)^u\right)}{3tu}-\frac{qr\log\left(\frac{dx}{c}+1\right)\log^3\left(i(j(hx)^t)^u\right)}{3tu}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[i*(j*(h*x)^t)^u]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/x,x]
```

```
[Out] -1/3*(p*r*Log[i*(j*(h*x)^t)^u]^3*Log[1 + (b*x)/a])/(t*u) + (Log[i*(j*(h*x)^
t)^u]^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*t*u) - (q*r*Log[i*(j*(h*x)
^t)^u]^3*Log[1 + (d*x)/c])/(3*t*u) - p*r*Log[i*(j*(h*x)^t)^u]^2*PolyLog[2,
-((b*x)/a)] - q*r*Log[i*(j*(h*x)^t)^u]^2*PolyLog[2, -((d*x)/c)] + 2*p*r*t*u
*Log[i*(j*(h*x)^t)^u]*PolyLog[3, -((b*x)/a)] + 2*q*r*t*u*Log[i*(j*(h*x)^t)^
u]*PolyLog[3, -((d*x)/c)] - 2*p*r*t^2*u^2*PolyLog[4, -((b*x)/a)] - 2*q*r*t^
2*u^2*PolyLog[4, -((d*x)/c)]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
```

```
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 2585

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))/((j_.
) + (k_.)*(x_)), x_Symbol] :> Simp[(s + t*Log[i*(g + h*x)^n])^(m + 1)*(Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(k*n*t*(m + 1))), x] + (-Dist[b*p*(r/(k*n*
t*(m + 1))), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Dis
t[d*q*(r/(k*n*t*(m + 1))), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*x)
, x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r},
x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log^2\left(57(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = \text{Subst}\Bigg(\int \frac{\log^2\left(57j^u(hx)^{tu}\right)\log\left(e(f(a+bx)^p(c+dx\right.}{x}$$

$$= \text{Subst}\Bigg(\text{Subst}\Bigg(\int \frac{\log^2\left(57h^{tu}j^ux^{tu}\right)\log\left(e(f(a+bx)^p\right.}{x}$$

$$= \frac{\log^3\left(57(j(hx)^t)^u\right)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{3tu} - \text{S}$$

$$= -\frac{pr\log^3\left(57(j(hx)^t)^u\right)\log\left(1+\frac{bx}{a}\right)}{3tu} + \frac{\log^3\left(57(j(hx\right.}{}$$

$$= -\frac{pr\log^3\left(57(j(hx)^t)^u\right)\log\left(1+\frac{bx}{a}\right)}{3tu} + \frac{\log^3\left(57(j(hx\right.}{}$$

$$= -\frac{pr\log^3\left(57(j(hx)^t)^u\right)\log\left(1+\frac{bx}{a}\right)}{3tu} + \frac{\log^3\left(57(j(hx\right.}{}$$

$$= -\frac{pr\log^3\left(57(j(hx)^t)^u\right)\log\left(1+\frac{bx}{a}\right)}{3tu} + \frac{\log^3\left(57(j(hx\right.}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 839 vs. $2(262) = 524$.
time = 0.60, size = 839, normalized size = 3.20

Antiderivative was successfully verified.

```
[In] Integrate[(Log[i*(j*(h*x)^t)^u]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/x,x
]
```

```
[Out] -(p*r*t^2*u^2*Log[x]*Log[h*x]^2*Log[a + b*x]) + p*r*t^2*u^2*Log[h*x]^3*Log[
a + b*x] + 2*p*r*t*u*Log[x]*Log[h*x]*Log[i*(j*(h*x)^t)^u]*Log[a + b*x] - 2*
p*r*t*u*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]*Log[a + b*x] - p*r*Log[x]*Log[i*(j*
(h*x)^t)^u]^2*Log[a + b*x] + p*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^2*Log[a + b*
x] - (p*r*t^2*u^2*Log[h*x]^3*Log[1 + (b*x)/a])/3 + p*r*t*u*Log[h*x]^2*Log[i
*(j*(h*x)^t)^u]*Log[1 + (b*x)/a] - p*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^2*Log[
1 + (b*x)/a] - q*r*t^2*u^2*Log[x]*Log[h*x]^2*Log[c + d*x] + q*r*t^2*u^2*Log
[h*x]^3*Log[c + d*x] + 2*q*r*t*u*Log[x]*Log[h*x]*Log[i*(j*(h*x)^t)^u]*Log[c
 + d*x] - 2*q*r*t*u*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]*Log[c + d*x] - q*r*Log[
x]*Log[i*(j*(h*x)^t)^u]^2*Log[c + d*x] + q*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^
2*Log[c + d*x] + t^2*u^2*Log[x]*Log[h*x]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q
)^r] - (2*t^2*u^2*Log[h*x]^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/3 - 2*t*
```

```
u*Log[x]*Log[h*x]*Log[i*(j*(h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
 + t*u*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
 + Log[x]*Log[i*(j*(h*x)^t)^u]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - (q*
r*t^2*u^2*Log[h*x]^3*Log[1 + (d*x)/c])/3 + q*r*t*u*Log[h*x]^2*Log[i*(j*(h*x
)^t)^u]*Log[1 + (d*x)/c] - q*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^2*Log[1 + (d*x
)/c] - p*r*Log[i*(j*(h*x)^t)^u]^2*PolyLog[2, -((b*x)/a)] - q*r*Log[i*(j*(h*
x)^t)^u]^2*PolyLog[2, -((d*x)/c)] + 2*p*r*t*u*Log[i*(j*(h*x)^t)^u]*PolyLog[
3, -((b*x)/a)] + 2*q*r*t*u*Log[i*(j*(h*x)^t)^u]*PolyLog[3, -((d*x)/c)] - 2*
p*r*t^2*u^2*PolyLog[4, -((b*x)/a)] - 2*q*r*t^2*u^2*PolyLog[4, -((d*x)/c)]
```

Maple [F]
time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(i\left(j(hx)^{t}\right)^{u}\right)^{2} \ln \left(e(f(bx+a)^{p}\left(dx+c\right)^{q})^{r}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(i*(j*(h*x)^t)^u)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

```
[Out] int(ln(i*(j*(h*x)^t)^u)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(i*(j*(h*x)^t)^u)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algo
rithm="maxima")
```

```
[Out] 1/12*(4*t^2*u^2*log(x)^3 - 6*(2*t^2*u^2*log(h) + 2*t*u^2*log(j) + I*pi*t*u)
*log(x)^2 + 12*log(x)*log((x^t)^u)^2 + 3*(4*t^2*u^2*log(h)^2 + 8*t*u^2*log(
h)*log(j) + 4*u^2*log(j)^2 - pi^2 + 4*I*pi*(t*u*log(h) + u*log(j)))*log(x)
- 12*(t*u*log(x)^2 + (-I*pi - 2*t*u*log(h) - 2*u*log(j))*log(x))*log((x^t)^
u))*log(((b*x + a)^p)^r) + 1/12*(4*t^2*u^2*log(x)^3 - 6*(2*t^2*u^2*log(h) +
 2*t*u^2*log(j) + I*pi*t*u)*log(x)^2 + 12*log(x)*log((x^t)^u)^2 + 3*(4*t^2*
u^2*log(h)^2 + 8*t*u^2*log(h)*log(j) + 4*u^2*log(j)^2 - pi^2 + 4*I*pi*(t*u*
log(h) + u*log(j)))*log(x) - 12*(t*u*log(x)^2 + (-I*pi - 2*t*u*log(h) - 2*u
*log(j))*log(x))*log((x^t)^u))*log(((d*x + c)^q)^r) - integrate(1/12*(3*pi^
2*(r*log(f) + 1)*a*c - 12*I*pi*(t*u*log(h) + (r*t*u*log(h) + r*u*log(j))*lo
g(f) + u*log(j))*a*c + 4*((p*r*t^2*u^2 + q*r*t^2*u^2)*b*d*x^2 + (b*c*p*r*t^
2*u^2 + a*d*q*r*t^2*u^2)*x)*log(x)^3 - 12*(t^2*u^2*log(h)^2 + 2*t*u^2*log(h
)*log(j) + u^2*log(j)^2 + (r*t^2*u^2*log(h)^2 + 2*r*t*u^2*log(h)*log(j) + r
*u^2*log(j)^2)*log(f))*a*c + 3*(pi^2*(r*log(f) + 1)*b*d - 4*I*pi*(t*u*log(h
) + (r*t*u*log(h) + r*u*log(j))*log(f) + u*log(j))*b*d - 4*(t^2*u^2*log(h)^
```

```
2 + 2*t*u^2*log(h)*log(j) + u^2*log(j)^2 + (r*t^2*u^2*log(h)^2 + 2*r*t*u^2*
log(h)*log(j) + r*u^2*log(j)^2)*log(f))*b*d)*x^2 - 12*((r*log(f) + 1)*b*d*x
^2 + (r*log(f) + 1)*a*c + ((r*log(f) + 1)*b*c + (r*log(f) + 1)*a*d)*x - ((p
*r + q*r)*b*d*x^2 + (b*c*p*r + a*d*q*r)*x)*log(x))*log((x^t)^u)^2 - 6*((I*p
i*(p*r*t*u + q*r*t*u)*b*d + 2*((p*r*t^2*u^2 + q*r*t^2*u^2)*log(h) + (p*r*t*
u^2 + q*r*t*u^2)*log(j))*b*d)*x^2 + (2*(p*r*t^2*u^2*log(h) + p*r*t*u^2*log(
j))*b*c + 2*(q*r*t^2*u^2*log(h) + q*r*t*u^2*log(j))*a*d + I*pi*(b*c*p*r*t*u
 + a*d*q*r*t*u))*x)*log(x)^2 + 3*(pi^2*((r*log(f) + 1)*b*c + (r*log(f) + 1)
*a*d) - 4*(t^2*u^2*log(h)^2 + 2*t*u^2*log(h)*log(j) + u^2*log(j)^2 + (r*t^2
*u^2*log(h)^2 + 2*r*t*u^2*log(h)*log(j) + r*u^2*log(j)^2)*log(f))*b*c - 4*(
t^2*u^2*log(h)^2 + 2*t*u^2*log(h)*log(j) + u^2*log(j)^2 + (r*t^2*u^2*log(h)
^2 + 2*r*t*u^2*log(h)*log(j) + r*u^2*log(j)^2)*log(f))*a*d - 4*I*pi*((t*u*l
og(h) + (r*t*u*log(h) + r*u*log(j))*log(f) + u*log(j))*b*c + (t*u*log(h) +
(r*t*u*log(h) + r*u*log(j))*log(f) + u*log(j))*a*d))*x - 12*(I*pi*(r*log(f)
 + 1)*a*c + 2*(t*u*log(h) + (r*t*u*log(h) + r*u*log(j))*log(f) + u*log(j))*
a*c + (I*pi*(r*log(f) + 1)*b*d + 2*(t*u*log(h) + (r*t*u*log(h) + r*u*log(j)
)*log(f) + u*log(j))*b*d)*x^2 + ((p*r*t*u + q*r*t*u)*b*d*x^2 + (b*c*p*r*t*u
 + a*d*q*r*t*u)*x)*log(x)^2 + (2*(t*u*log(h) + (r*t*u*log(h) + r*u*log(j))*
log(f) + u*log(j))*b*c + 2*(t*u*log(h) + (r*t*u*log(h) + r*u*log(j))*log(f)
 + u*log(j))*a*d + I*pi*((r*log(f) + 1)*b*c + (r*log(f) + 1)*a*d))*x + ((-I
*pi*(p*r + q*r)*b*d - 2*((p*r*t*u + q*r*t*u)*log(h) + (p*r*u + q*r*u)*log(j
))*b*d)*x^2 - (2*(p*r*t*u*log(h) + p*r*u*log(j))*b*c + 2*(q*r*t*u*log(h) +
q*r*u*log(j))*a*d + I*pi*(b*c*p*r + a*d*q*r))*x)*log(x))*log((x^t)^u) - 3*(
(pi^2*(p*r + q*r)*b*d - 4*I*pi*((p*r*t*u + q*r*t*u)*log(h) + (p*r*u + q*r*u
)*log(j))*b*d - 4*((p*r*t^2*u^2 + q*r*t^2*u^2)*log(h)^2 + 2*(p*r*t*u^2 + q*
r*t*u^2)*log(h)*log(j) + (p*r*u^2 + q*r*u^2)*log(j)^2)*b*d)*x^2 + (pi^2*(b*
c*p*r + a*d*q*r) - 4*(p*r*t^2*u^2*log(h)^2 + 2*p*r*t*u^2*log(h)*log(j) + p*
r*u^2*log(j)^2)*b*c - 4*(q*r*t^2*u^2*log(h)^2 + 2*q*r*t*u^2*log(h)*log(j) +
 q*r*u^2*log(j)^2)*a*d - 4*I*pi*((p*r*t*u*log(h) + p*r*u*log(j))*b*c + (q*r
*t*u*log(h) + q*r*u*log(j))*a*d))*x)*log(x))/(b*d*x^3 + a*c*x + (b*c + a*d)
*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(i*(j*(h*x)^t)^u)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algo
rithm="fricas")
```

```
[Out] integral(-1/4*(pi^2*r*log(f) - 4*(p*r*t^2*u^2*log(b*x + a) + q*r*t^2*u^2*lo
g(d*x + c) + r*t^2*u^2*log(f) + t^2*u^2)*log(h*x)^2 - 4*(r*u^2*log(f) + u^2
)*log(j)^2 + pi^2 - (4*p*r*u^2*log(j)^2 + 4*I*pi*p*r*u*log(j) - pi^2*p*r)*l
og(b*x + a) - (4*q*r*u^2*log(j)^2 + 4*I*pi*q*r*u*log(j) - pi^2*q*r)*log(d*x
```

```
 + c) - 4*(I*pi*r*t*u*log(f) + I*pi*t*u + (2*p*r*t*u^2*log(j) + I*pi*p*r*t*
u)*log(b*x + a) + (2*q*r*t*u^2*log(j) + I*pi*q*r*t*u)*log(d*x + c) + 2*(r*t
*u^2*log(f) + t*u^2)*log(j))*log(h*x) - 4*(I*pi*r*u*log(f) + I*pi*u)*log(j)
)/x, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(i*(j*(h*x)**t)**u)**2*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/x,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6438 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(i*(j*(h*x)^t)^u)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algo
rithm="giac")
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(I*((h*x)^t*j)^u)^2/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\,{\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)}^r\right)\,{\ln\left(i\,{\left(j\,{\left(h\,x\right)}^t\right)}^u\right)}^2}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*log(i*(j*(h*x)^t)^u)^2)/x,x)
[Out] int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*log(i*(j*(h*x)^t)^u)^2)/x, x)
```

3.58 $\int \frac{\log\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,dx$

Optimal. Leaf size=194

$$-\frac{pr\log^2\left(i\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right)}{2tu}+\frac{\log^2\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2tu}-\frac{qr\log^2\left(i\left(j(hx)^t\right)^u\right)\log\left(1\right.}{2tu}$$

```
[Out] -1/2*p*r*ln(i*(j*(h*x)^t)^u)^2*ln(1+b*x/a)/t/u+1/2*ln(i*(j*(h*x)^t)^u)^2*ln
(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/t/u-1/2*q*r*ln(i*(j*(h*x)^t)^u)^2*ln(1+d*x/c)
/t/u-p*r*ln(i*(j*(h*x)^t)^u)*polylog(2,-b*x/a)-q*r*ln(i*(j*(h*x)^t)^u)*poly
log(2,-d*x/c)+p*r*t*u*polylog(3,-b*x/a)+q*r*t*u*polylog(3,-d*x/c)
```

Rubi [A]
time = 0.43, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {2585, 2354, 2421, 6724, 2495}

$$-pr\text{PolyLog}\left(2,-\frac{bx}{a}\right)\log\left(i\left(j(hx)^t\right)^u\right)+prtu\text{PolyLog}\left(3,-\frac{bx}{a}\right)-qr\text{PolyLog}\left(2,-\frac{dx}{c}\right)\log\left(i\left(j(hx)^t\right)^u\right)+qrtu\text{PolyLog}\left(3,-\frac{dx}{c}\right)+\frac{\log^2\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2tu}-\frac{pr\log\left(\frac{bx}{a}+1\right)\log^2\left(i\left(j(hx)^t\right)^u\right)}{2tu}-\frac{qr\log\left(\frac{dx}{c}+1\right)\log^2\left(i\left(j(hx)^t\right)^u\right)}{2tu}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[i*(j*(h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/x,x]
```

```
[Out] -1/2*(p*r*Log[i*(j*(h*x)^t)^u]^2*Log[1 + (b*x)/a])/(t*u) + (Log[i*(j*(h*x)^
t)^u]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*t*u) - (q*r*Log[i*(j*(h*x)
^t)^u]^2*Log[1 + (d*x)/c])/(2*t*u) - p*r*Log[i*(j*(h*x)^t)^u]*PolyLog[2, -(
(b*x)/a)] - q*r*Log[i*(j*(h*x)^t)^u]*PolyLog[2, -((d*x)/c)] + p*r*t*u*PolyL
og[3, -((b*x)/a)] + q*r*t*u*PolyLog[3, -((d*x)/c)]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2495

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
```

```
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rule 2585

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))/((j_.
) + (k_.)*(x_)), x_Symbol] :> Simp[(s + t*Log[i*(g + h*x)^n])^(m + 1)*(Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(k*n*t*(m + 1))), x] + (-Dist[b*p*(r/(k*n*
t*(m + 1))), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Dis
t[d*q*(r/(k*n*t*(m + 1))), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*x)
, x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r},
x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log \left(58(j(hx)^t)^u\right) \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x} \, dx = \text{Subst}\left(\int \frac{\log \left(58j^u(hx)^{tu}\right) \log \left(e(f(a+bx)^p(c+dx\right.}{x}\right.$$

$$= \text{Subst}\left(\text{Subst}\left(\int \frac{\log \left(58h^{tu}j^u x^{tu}\right) \log \left(e(f(a+bx)^p\right.}{x}\right.\right.$$

$$= \frac{\log^2 \left(58(j(hx)^t)^u\right) \log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{2tu} - \text{S}$$

$$= -\frac{pr \log^2 \left(58(j(hx)^t)^u\right) \log \left(1+\frac{bx}{a}\right)}{2tu} + \frac{\log^2 \left(58(j(hx\right.}{}$$

$$= -\frac{pr \log^2 \left(58(j(hx)^t)^u\right) \log \left(1+\frac{bx}{a}\right)}{2tu} + \frac{\log^2 \left(58(j(hx\right.}{}$$

$$= -\frac{pr \log^2 \left(58(j(hx)^t)^u\right) \log \left(1+\frac{bx}{a}\right)}{2tu} + \frac{\log^2 \left(58(j(hx\right.}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 451 vs. 2(194) = 388.
time = 0.29, size = 451, normalized size = 2.32

$$prtu\log(x)\log(hx)\log(a+bx) - prtu\log^2(hx)\log(a+bx) - pr\log(x)\log\left(i\left(j(hx)^t\right)^u\right)\log(a+bx) + pr\log(hx)\log\left(i\left(j(hx)^t\right)^u\right)\log(a+bx) + \frac{1}{2}prtu\log^2(hx)\log\left(1+\frac{bx}{a}\right) - pr\log(hx)\log\left(i\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right) + qrtu\log(x)\log(hx)\log(c+dx) - qrtu\log^2(hx)\log(c+dx) - qr\log(x)\log\left(i\left(j(hx)^t\right)^u\right)\log(c+dx) + qr\log(hx)\log\left(i\left(j(hx)^t\right)^u\right)\log(c+dx) - tu\log(x)\log(hx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) + \frac{1}{2}tu\log^2(hx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) + \log(x)\log\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) + \frac{1}{2}qrtu\log^2(hx)\log\left(1+\frac{dx}{c}\right) - qr\log(hx)\log\left(i\left(j(hx)^t\right)^u\right)\log\left(1+\frac{dx}{c}\right) - pr\log\left(i\left(j(hx)^t\right)^u\right)\text{Li}_2\left(-\frac{bx}{a}\right) - qr\log\left(i\left(j(hx)^t\right)^u\right)\text{Li}_2\left(-\frac{dx}{c}\right) + prtu\text{Li}_3\left(-\frac{bx}{a}\right) + qrtu\text{Li}_3\left(-\frac{dx}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[i*(j*(h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/x,x]
```

```
[Out] p*r*t*u*Log[x]*Log[h*x]*Log[a + b*x] - p*r*t*u*Log[h*x]^2*Log[a + b*x] - p*
r*Log[x]*Log[i*(j*(h*x)^t)^u]*Log[a + b*x] + p*r*Log[h*x]*Log[i*(j*(h*x)^t)
^u]*Log[a + b*x] + (p*r*t*u*Log[h*x]^2*Log[1 + (b*x)/a])/2 - p*r*Log[h*x]*L
og[i*(j*(h*x)^t)^u]*Log[1 + (b*x)/a] + q*r*t*u*Log[x]*Log[h*x]*Log[c + d*x]
 - q*r*t*u*Log[h*x]^2*Log[c + d*x] - q*r*Log[x]*Log[i*(j*(h*x)^t)^u]*Log[c
+ d*x] + q*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]*Log[c + d*x] - t*u*Log[x]*Log[h*
x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (t*u*Log[h*x]^2*Log[e*(f*(a + b*x
)^p*(c + d*x)^q)^r])/2 + Log[x]*Log[i*(j*(h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(
c + d*x)^q)^r] + (q*r*t*u*Log[h*x]^2*Log[1 + (d*x)/c])/2 - q*r*Log[h*x]*Log
[i*(j*(h*x)^t)^u]*Log[1 + (d*x)/c] - p*r*Log[i*(j*(h*x)^t)^u]*PolyLog[2, -(
(b*x)/a)] - q*r*Log[i*(j*(h*x)^t)^u]*PolyLog[2, -((d*x)/c)] + p*r*t*u*PolyL
og[3, -((b*x)/a)] + q*r*t*u*PolyLog[3, -((d*x)/c)]
```

Maple [**F**]
time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(i\left(j(hx)^{t}\right)^{u}\right) \ln \left(e(f(bx+a)^{p}\,(dx+c)^{q})^{r}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(i*(j*(h*x)^t)^u)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

```
[Out] int(ln(i*(j*(h*x)^t)^u)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(i*(j*(h*x)^t)^u)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algori
thm="maxima")
```

```
[Out] -1/2*(t*u*log(x)^2 - (I*pi + 2*t*u*log(h) + 2*u*log(j))*log(x) - 2*log(x)*l
og((x^t)^u))*log(((b*x + a)^p)^r) - 1/2*(t*u*log(x)^2 - (I*pi + 2*t*u*log(h
) + 2*u*log(j))*log(x) - 2*log(x)*log((x^t)^u))*log(((d*x + c)^q)^r) - inte
grate(1/2*(-I*pi*(r*log(f) + 1)*a*c - 2*(t*u*log(h) + (r*t*u*log(h) + r*u*l
og(j))*log(f) + u*log(j))*a*c + (-I*pi*(r*log(f) + 1)*b*d - 2*(t*u*log(h) +
 (r*t*u*log(h) + r*u*log(j))*log(f) + u*log(j))*b*d)*x^2 - ((p*r*t*u + q*r*
t*u)*b*d*x^2 + (b*c*p*r*t*u + a*d*q*r*t*u)*x)*log(x)^2 - (2*(t*u*log(h) + (
r*t*u*log(h) + r*u*log(j))*log(f) + u*log(j))*b*c + 2*(t*u*log(h) + (r*t*u*
log(h) + r*u*log(j))*log(f) + u*log(j))*a*d + I*pi*((r*log(f) + 1)*b*c + (r
```

```
*log(f) + 1)*a*d))*x - 2*((r*log(f) + 1)*b*d*x^2 + (r*log(f) + 1)*a*c + ((r
*log(f) + 1)*b*c + (r*log(f) + 1)*a*d)*x - ((p*r + q*r)*b*d*x^2 + (b*c*p*r
+ a*d*q*r)*x)*log(x))*log((x^t)^u) + ((I*pi*(p*r + q*r)*b*d + 2*((p*r*t*u +
 q*r*t*u)*log(h) + (p*r*u + q*r*u)*log(j))*b*d)*x^2 + (2*(p*r*t*u*log(h) +
p*r*u*log(j))*b*c + 2*(q*r*t*u*log(h) + q*r*u*log(j))*a*d + I*pi*(b*c*p*r +
 a*d*q*r))*x)*log(x))/(b*d*x^3 + a*c*x + (b*c + a*d)*x^2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(i*(j*(h*x)^t)^u)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algori
thm="fricas")
```

```
[Out] integral(-1/2*(-I*pi - I*pi*r*log(f) - (2*p*r*u*log(j) + I*pi*p*r)*log(b*x
+ a) - (2*q*r*u*log(j) + I*pi*q*r)*log(d*x + c) - 2*(p*r*t*u*log(b*x + a) +
 q*r*t*u*log(d*x + c) + r*t*u*log(f) + t*u)*log(h*x) - 2*(r*u*log(f) + u)*l
og(j))/x, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(i*(j*(h*x)**t)**u)*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/x,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(i*(j*(h*x)^t)^u)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algori
thm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(I*((h*x)^t*j)^u)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(e\left(f\left(a+b\,x\right)^p\left(c+d\,x\right)^q\right)^r\right)\,\ln\left(i\left(j\left(h\,x\right)^t\right)^u\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*log(i*(j*(h*x)^t)^u))/x,x)
[Out] int((log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)*log(i*(j*(h*x)^t)^u))/x, x)
```

3.59 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{x}\,dx$

Optimal. Leaf size=81

$$-pr\log(x)\log\left(1+\frac{bx}{a}\right)+\log(x)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)-qr\log(x)\log\left(1+\frac{dx}{c}\right)-pr\mathrm{Li}_2\left(-\frac{bx}{a}\right)-qr$$

```
[Out] -p*r*ln(x)*ln(1+b*x/a)+ln(x)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)-q*r*ln(x)*ln(1
+d*x/c)-p*r*polylog(2,-b*x/a)-q*r*polylog(2,-d*x/c)
```

Rubi [A]
time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2580, 2354, 2438}

$$-pr\text{PolyLog}\left(2,-\frac{bx}{a}\right)-qr\text{PolyLog}\left(2,-\frac{dx}{c}\right)+\log(x)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)-pr\log(x)\log\left(\frac{bx}{a}+1\right)-qr\log(x)\log\left(\frac{dx}{c}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/x,x]
```

```
[Out] -(p*r*Log[x]*Log[1 + (b*x)/a]) + Log[x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r
] - q*r*Log[x]*Log[1 + (d*x)/c] - p*r*PolyLog[2, -((b*x)/a)] - q*r*PolyLog[
2, -((d*x)/c)]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2580

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[Log[g + h*x]*(Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/h), x] + (-Dist[b*p*(r/h), Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[d*q*(r/h), Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\log \left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x}\,dx = \log(x)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right) - (bpr)\int \frac{\log(x)}{a+bx}\,dx - (dqr)\int \frac{\mathrm{lo}}{c}$$
$$= -pr\log(x)\log\left(1+\frac{bx}{a}\right) + \log(x)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right) - qr\,\mathrm{lo}$$
$$= -pr\log(x)\log\left(1+\frac{bx}{a}\right) + \log(x)\log\left(e(f(a+bx)^p(c+dx)^q)^r\right) - qr\,\mathrm{lo}$$

Mathematica [A]
time = 0.05, size = 78, normalized size = 0.96

$$\log(x)\left(-pr\log\left(1+\frac{bx}{a}\right)+\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)-qr\log\left(1+\frac{dx}{c}\right)\right)-pr\mathrm{Li}_2\left(-\frac{bx}{a}\right)-qr\mathrm{Li}_2\left(-\frac{dx}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/x,x]
```

```
[Out] Log[x]*(-(p*r*Log[1 + (b*x)/a]) + Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - q*
r*Log[1 + (d*x)/c]) - p*r*PolyLog[2, -((b*x)/a)] - q*r*PolyLog[2, -((d*x)/c
)]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f(bx+a)^p\,(dx+c)^q\right)^r\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

Maxima [A]
time = 0.29, size = 127, normalized size = 1.57

$$-\frac{(fp\log(bx+a)+fq\log(dx+c))r\log(x)}{f}+\log\left(((bx+a)^p(dx+c)^qf)^re\right)\log(x)+\frac{\left(\left(\log(bx+a)\log\left(-\frac{bx+a}{a}+1\right)+\mathrm{Li}_2\left(\frac{bx+a}{a}\right)\right)fp+\left(\log(dx+c)\log\left(-\frac{dx+c}{c}+1\right)+\mathrm{Li}_2\left(\frac{dx+c}{c}\right)\right)fq\right)r}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algorithm="maxima")
```

```
[Out] -(f*p*log(b*x + a) + f*q*log(d*x + c))*r*log(x)/f + log(((b*x + a)^p*(d*x +
 c)^q*f)^r*e)*log(x) + ((log(b*x + a)*log(-(b*x + a)/a + 1) + dilog((b*x +
a)/a))*f*p + (log(d*x + c)*log(-(d*x + c)/c + 1) + dilog((d*x + c)/c))*f*q)
*r/f
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algorithm="fricas")
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(e(f(a + bx)^{p} (c + dx)^{q})^{r}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/x,x)
[Out] Integral(log(e*(f*(a + b*x)**p*(c + d*x)**q)**r)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algorithm="giac")
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(e\,{\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)}^r\right)}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/x,x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/x, x)
```

3.60 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{x\log(i(j(hx)^t)^u)}\,dx$

Optimal. Leaf size=42

$$\text{Int}\left(\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log\left(i\left(j(hx)^t\right)^u\right)},x\right)$$

[Out] CannotIntegrate(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/ln(i*(j*(h*x)^t)^u),x)

Rubi [A]
time = 0.37, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log\left(i\left(j(hx)^t\right)^u\right)}\,dx$$

Verification is not applicable to the result.

[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]),x]

[Out] Defer[Int][Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]), x
]

Rubi steps

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log\left(60\left(j(hx)^t\right)^u\right)}\,dx = \int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log\left(60\left(j(hx)^t\right)^u\right)}\,dx$$

Mathematica [A]
time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log\left(i\left(j(hx)^t\right)^u\right)}\,dx$$

Verification is not applicable to the result.

[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]),x]

[Out] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]), x]

Maple [A]
time = 1.06, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(f\left(bx+a\right)^{p}\left(dx+c\right)^{q}\right)^{r}\right)}{x\ln\left(i\left(j\left(hx\right)^{t}\right)^{u}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/ln(i*(j*(h*x)^t)^u),x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/ln(i*(j*(h*x)^t)^u),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/log(i*(j*(h*x)^t)^u),x, algori
thm="maxima")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(x*log(I*((h*x)^t*j)^u)), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/log(i*(j*(h*x)^t)^u),x, algori
thm="fricas")
```

```
[Out] integral(2*(p*r*log(b*x + a) + q*r*log(d*x + c) + r*log(f) + 1)/(2*t*u*x*lo
g(h*x) + 2*u*x*log(j) + I*pi*x), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/x/ln(i*(j*(h*x)**t)**u),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/log(i*(j*(h*x)^t)^u),x, algori
thm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(x*log(I*((h*x)^t*j)^u)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(e\left(f\left(a+b\,x\right)^p\,\left(c+d\,x\right)^q\right)^r\right)}{x\,\ln\left(i\,\left(j\,\left(h\,x\right)^t\right)^u\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(x*log(i*(j*(h*x)^t)^u)),x)
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(x*log(i*(j*(h*x)^t)^u)), x)
```

3.61 $\int \frac{\log(e(f(a+bx)^p(c+dx)^q)^r)}{x\log^2\left(i(j(hx)^t)^u\right)}\,dx$

Optimal. Leaf size=42

$$\mathrm{Int}\left(\frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log^2\left(i\left(j(hx)^t\right)^u\right)},x\right)$$

```
[Out] CannotIntegrate(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/ln(i*(j*(h*x)^t)^u)^2,x)
```

Rubi [A]
time = 0.30, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log^2\left(i\left(j(hx)^t\right)^u\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]^2),x]
```

```
[Out] Defer[Int][Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]^2),
 x]
```

Rubi steps

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log^2\left(61\left(j(hx)^t\right)^u\right)}\,dx=\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log^2\left(61\left(j(hx)^t\right)^u\right)}\,dx$$

Mathematica [A]
time = 1.51, size = 0, normalized size = 0.00

$$\int \frac{\log\left(e(f(a+bx)^p(c+dx)^q)^r\right)}{x\log^2\left(i\left(j(hx)^t\right)^u\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]^2),x
]
```

```
[Out] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]^2),
x]
```

Maple [A]
time = 1.06, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(e(f(bx+a)^p (dx+c)^q)^r\right)}{x \ln \left(i \left(j \left(hx\right)^t\right)^u\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/ln(i*(j*(h*x)^t)^u)^2,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/ln(i*(j*(h*x)^t)^u)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/log(i*(j*(h*x)^t)^u)^2,x, algo
rithm="maxima")
```

```
[Out] -2*(r*log(f) + log(((b*x + a)^p)^r) + log(((d*x + c)^q)^r) + 1)/(2*t^2*u^2*
log(h) + 2*t*u^2*log(j) + I*pi*t*u + 2*t*u*log((x^t)^u)) + integrate((b*c*p
*r + a*d*q*r + (p*r + q*r)*b*d*x)/(1/2*I*pi*a*c*t*u + (t^2*u^2*log(h) + t*u
^2*log(j))*a*c - 1/2*(-I*pi*b*d*t*u - 2*(t^2*u^2*log(h) + t*u^2*log(j))*b*d
)*x^2 + 1/2*(2*(t^2*u^2*log(h) + t*u^2*log(j))*b*c + 2*(t^2*u^2*log(h) + t*
u^2*log(j))*a*d + I*pi*(b*c*t*u + a*d*t*u))*x + (b*d*t*u*x^2 + a*c*t*u + (b
*c*t*u + a*d*t*u)*x)*log((x^t)^u)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/log(i*(j*(h*x)^t)^u)^2,x, algo
rithm="fricas")
```

```
[Out] -(2*p*r*log(b*x + a) + 2*q*r*log(d*x + c) - (2*t^2*u^2*log(h*x) + 2*t*u^2*l
og(j) + I*pi*t*u)*integral(((b*d*p + b*d*q)*r*x + (b*c*p + a*d*q)*r)/(1/2*I
*pi*(b*d*t*u*x^2 + a*c*t*u + (b*c + a*d)*t*u*x) + (b*d*t^2*u^2*x^2 + a*c*t^
2*u^2 + (b*c + a*d)*t^2*u^2*x)*log(h*x) + (b*d*t*u^2*x^2 + a*c*t*u^2 + (b*c
 + a*d)*t*u^2*x)*log(j)), x) + 2*r*log(f) + 2)/(2*t^2*u^2*log(h*x) + 2*t*u^
2*log(j) + I*pi*t*u)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/x/ln(i*(j*(h*x)**t)**u)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6439 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/log(i*(j*(h*x)^t)^u)^2,x, algo
rithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(x*log(I*((h*x)^t*j)^u)^2),
x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(e\,{\left(f\,{\left(a+b\,x\right)}^p\,{\left(c+d\,x\right)}^q\right)}^r\right)}{x\,{\ln\left(i\,{\left(j\,{\left(h\,x\right)}^t\right)}^u\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(x*log(i*(j*(h*x)^t)^u)^2),x)
```

```
[Out] int(log(e*(f*(a + b*x)^p*(c + d*x)^q)^r)/(x*log(i*(j*(h*x)^t)^u)^2), x)
```

3.62 $\int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$

Optimal. Leaf size=31

$$\operatorname{Int}\left(\frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x},x\right)$$

```
[Out] Unintegrable(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)^3/x,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^3)/x,x]
[Out] Defer[Int][(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^3)/x, x]
```

Rubi steps

$$\int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$$

Mathematica [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^3)/x,x]
[Out] Integrate[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^3)/x, x]
```

Maple [A]
time = 2.07, size = 0, normalized size = 0.00

$$\int \frac{\ln(x)\ln\left(\frac{bx+a}{(-ad+cb)x}\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)^3/x,x)
```

```
[Out] int(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)^3/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)^3/x,x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x + a)^3*log(x)^2 - integrate(1/2*(2*(b*x + a)*log(x)^4 + 6*(b*x*
log(b*c - a*d) + a*log(b*c - a*d))*log(x)^3 + 3*((3*b*x + 2*a)*log(x)^2 + 2
*(b*x*log(b*c - a*d) + a*log(b*c - a*d))*log(x))*log(b*x + a)^2 + 6*(b*x*lo
g(b*c - a*d)^2 + a*log(b*c - a*d)^2)*log(x)^2 - 6*((b*x + a)*log(x)^3 + 2*(
b*x*log(b*c - a*d) + a*log(b*c - a*d))*log(x)^2 + (b*x*log(b*c - a*d)^2 + a
*log(b*c - a*d)^2)*log(x))*log(b*x + a) + 2*(b*x*log(b*c - a*d)^3 + a*log(b
*c - a*d)^3)*log(x))/(b*x^2 + a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)^3/x,x, algorithm="fricas")
```

```
[Out] integral(log(x)*log((b*x + a)/((b*c - a*d)*x))^3/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{3a\int\frac{\log{(x)}^2\log{\left(\frac{a}{-adx+bcx}+\frac{bx}{-adx+bcx}\right)}^2}{ax+bx^2}\,dx}{2}+\frac{\log{(x)}^2\log{\left(\frac{a+bx}{x(-ad+bc)}\right)}^3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)**3/x,x)
```

```
[Out] 3*a*Integral(log(x)**2*log(a/(-a*d*x + b*c*x) + b*x/(-a*d*x + b*c*x))**2/(a
*x + b*x**2), x)/2 + log(x)**2*log((a + b*x)/(x*(-a*d + b*c)))**3/2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)^3/x,x, algorithm="giac")
```

```
[Out] integrate(log(x)*log((b*x + a)/((b*c - a*d)*x))^3/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(-\frac{a+b\,x}{x\,(a\,d-b\,c)}\right)^3\,\ln\left(x\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(-(a + b*x)/(x*(a*d - b*c)))^3*log(x))/x,x)
```

```
[Out] int((log(-(a + b*x)/(x*(a*d - b*c)))^3*log(x))/x, x)
```

3.63 $\int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x},x\right)$$

```
[Out] Unintegrable(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)^2/x,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^2)/x,x]
[Out] Defer[Int][(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^2)/x, x]
```

Rubi steps

$$\int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$$

Mathematica [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^2)/x,x]
[Out] Integrate[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^2)/x, x]
```

Maple [A]
time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{\ln(x)\ln\left(\frac{bx+a}{(-ad+cb)x}\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)^2/x,x)
```

```
[Out] int(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)^2/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)^2/x,x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x + a)^2*log(x)^2 - integrate(-((b*x + a)*log(x)^3 + 2*(b*x*log(b
*c - a*d) + a*log(b*c - a*d))*log(x)^2 - ((3*b*x + 2*a)*log(x)^2 + 2*(b*x*l
og(b*c - a*d) + a*log(b*c - a*d))*log(x))*log(b*x + a) + (b*x*log(b*c - a*d
)^2 + a*log(b*c - a*d)^2)*log(x))/(b*x^2 + a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)^2/x,x, algorithm="fricas")
```

```
[Out] integral(log(x)*log((b*x + a)/((b*c - a*d)*x))^2/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$a\int\frac{\log{(x)}^2\log{\left(\frac{a}{-adx+bcx}+\frac{bx}{-adx+bcx}\right)}}{ax+bx^2}\,dx+\frac{\log{(x)}^2\log{\left(\frac{a+bx}{x(-ad+bc)}\right)}^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)**2/x,x)
```

```
[Out] a*Integral(log(x)**2*log(a/(-a*d*x + b*c*x) + b*x/(-a*d*x + b*c*x))/(a*x +
b*x**2), x) + log(x)**2*log((a + b*x)/(x*(-a*d + b*c)))**2/2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)^2/x,x, algorithm="giac")
```

```
[Out] integrate(log(x)*log((b*x + a)/((b*c - a*d)*x))^2/x, x)
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(-\frac{a+b\,x}{x\,\left(a\,d-b\,c\right)}\right)^2\,\ln\left(x\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(-(a + b*x)/(x*(a*d - b*c)))^2*log(x))/x,x)
```

```
[Out] int((log(-(a + b*x)/(x*(a*d - b*c)))^2*log(x))/x, x)
```

3.64 $\int \frac{\log(x)\log\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$

Optimal. Leaf size=82

$$-\frac{1}{2}\log\left(1+\frac{a}{bx}\right)\log^2(x)+\frac{1}{2}\log\left(\frac{b}{bc-ad}+\frac{a}{(bc-ad)x}\right)\log^2(x)+\log(x)\text{Li}_2\left(-\frac{a}{bx}\right)+\text{Li}_3\left(-\frac{a}{bx}\right)$$

```
[Out] -1/2*ln(1+a/b/x)*ln(x)^2+1/2*ln(b/(-a*d+b*c)+a/(-a*d+b*c)/x)*ln(x)^2+ln(x)*
polylog(2,-a/b/x)+polylog(3,-a/b/x)
```

Rubi [A]
time = 0.12, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2427, 2422, 2375, 2421, 6724}

$$\text{PolyLog}\left(3,-\frac{a}{bx}\right)+\log(x)\text{PolyLog}\left(2,-\frac{a}{bx}\right)+\frac{1}{2}\log^2(x)\log\left(\frac{a}{x(bc-ad)}+\frac{b}{bc-ad}\right)-\frac{1}{2}\log^2(x)\log\left(\frac{a}{bx}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)])/x,x]
```

```
[Out] -1/2*(Log[1 + a/(b*x)]*Log[x]^2) + (Log[b/(b*c - a*d) + a/((b*c - a*d)*x)]*
Log[x]^2)/2 + Log[x]*PolyLog[2, -(a/(b*x))] + PolyLog[3, -(a/(b*x))]
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
```

```
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2427

```
Int[Log[(d_.)*(u_)^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((g_.
)*(x_))^(q_.), x_Symbol] :> Int[(g*x)^q*Log[d*ExpandToSum[u, x]^r]*(a + b*L
og[c*x^n])^p, x] /; FreeQ[{a, b, c, d, g, r, n, p, q}, x] && BinomialQ[u, x
] &&  !BinomialMatchQ[u, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log(x)\log\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx &= \int \frac{\log\left(\frac{b}{bc-ad}+\frac{a}{(bc-ad)x}\right)\log(x)}{x}\,dx \\
&= \frac{1}{2}\log\left(\frac{b}{bc-ad}+\frac{a}{(bc-ad)x}\right)\log^2(x)+\frac{a\int \frac{\log^2(x)}{\left(\frac{b}{bc-ad}+\frac{a}{(bc-ad)x}\right)x^2}\,dx}{2(bc-ad)} \\
&= -\frac{1}{2}\log\left(1+\frac{a}{bx}\right)\log^2(x)+\frac{1}{2}\log\left(\frac{b}{bc-ad}+\frac{a}{(bc-ad)x}\right)\log^2(x)+\int \frac{\log(1
\\
&= -\frac{1}{2}\log\left(1+\frac{a}{bx}\right)\log^2(x)+\frac{1}{2}\log\left(\frac{b}{bc-ad}+\frac{a}{(bc-ad)x}\right)\log^2(x)+\log(x)\mathrm{Li}
\\
&= -\frac{1}{2}\log\left(1+\frac{a}{bx}\right)\log^2(x)+\frac{1}{2}\log\left(\frac{b}{bc-ad}+\frac{a}{(bc-ad)x}\right)\log^2(x)+\log(x)\mathrm{Li}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 0.80

$$
\frac{1}{6}\log^2(x)\left(\log(x)-3\log\left(1+\frac{bx}{a}\right)+3\log\left(\frac{a+bx}{bcx-adx}\right)\right)-\log(x)\mathrm{Li}_2\left(-\frac{bx}{a}\right)+\mathrm{Li}_3\left(-\frac{bx}{a}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)])/x,x]
```

```
[Out] (Log[x]^2*(Log[x] - 3*Log[1 + (b*x)/a] + 3*Log[(a + b*x)/(b*c*x - a*d*x)]))
/6 - Log[x]*PolyLog[2, -((b*x)/a)] + PolyLog[3, -((b*x)/a)]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(78) = 156$.
time = 0.40, size = 213, normalized size = 2.60

method	result
default	$\frac{\ln(x)^2\ln\left(-\frac{bx+a}{(ad-cb)x}\right)}{2}+\frac{ad\ln(x)^3}{6ad-6cb}-\frac{ad\ln(x)^2\ln\left(1+\frac{bx}{a}\right)}{2(ad-cb)}-\frac{ad\ln(x)\,\mathrm{polylog}\left(2,-\frac{bx}{a}\right)}{ad-cb}+\frac{ad\,\mathrm{polylog}\left(3,-\frac{bx}{a}\right)}{ad-cb}-\frac{cb\ln(x)^3}{6(ad-cb)}+$
risch	$\frac{\ln(x)^2\ln(bx+a)}{2}-\frac{\ln(x)^3}{3}+\frac{i\ln(x)^2\pi\mathrm{csgn}\left(\frac{i(bx+a)}{x(ad-cb)}\right)^3}{4}+\frac{i\ln(x)^2\pi}{2}-\frac{i\ln(x)^2\pi\,\mathrm{csgn}\left(\frac{i}{ad-cb}\right)\mathrm{csgn}\left(\frac{i(bx+a)}{ad-cb}\right)\mathrm{csgn}(i(bx+a))}{4}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(x)^2*ln(-(b*x+a)/(a*d-b*c)/x)+1/6/(a*d-b*c)*a*d*ln(x)^3-1/2/(a*d-b*c
)*a*d*ln(x)^2*ln(1+b*x/a)-1/(a*d-b*c)*a*d*ln(x)*polylog(2,-b*x/a)+1/(a*d-b*
c)*a*d*polylog(3,-b*x/a)-1/6/(a*d-b*c)*c*b*ln(x)^3+1/2/(a*d-b*c)*c*b*ln(x)^
2*ln(1+b*x/a)+1/(a*d-b*c)*c*b*ln(x)*polylog(2,-b*x/a)-1/(a*d-b*c)*c*b*polyl
og(3,-b*x/a)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: TypeError >> unable to make sense of Maxima expression 'l
i[2]' in Sage
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)/x,x, algorithm="fricas")
```

```
[Out] integral(log(x)*log((b*x + a)/((b*c - a*d)*x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a\int\frac{\log(x)^2}{ax+bx^2}\,dx}{2}+\frac{\log(x)^2\log\left(\frac{a+bx}{x(-ad+bc)}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)/x,x)
```

```
[Out] a*Integral(log(x)**2/(a*x + b*x**2), x)/2 + log(x)**2*log((a + b*x)/(x*(-a*
d + b*c)))/2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)/x,x, algorithm="giac")
```

```
[Out] integrate(log(x)*log((b*x + a)/((b*c - a*d)*x))/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(-\frac{a+b\,x}{x\,(a\,d-b\,c)}\right)\,\ln\left(x\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(-(a + b*x)/(x*(a*d - b*c)))*log(x))/x,x)
```

```
[Out] int((log(-(a + b*x)/(x*(a*d - b*c)))*log(x))/x, x)
```

3.65 $\int \frac{\log(x)}{x \log\left(\frac{a+bx}{(bc-ad)x}\right)} \, dx$

Optimal. Leaf size=31

$$\operatorname{Int}\left(\frac{\log(x)}{x \log\left(\frac{a+bx}{(bc-ad)x}\right)}, x\right)$$

`[Out] Unintegrable(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x),x)`

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log(x)}{x \log\left(\frac{a+bx}{(bc-ad)x}\right)} \, dx$$

Verification is not applicable to the result.

`[In] Int[Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]),x]`

`[Out] Defer[Int][Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]), x]`

Rubi steps

$$\int \frac{\log(x)}{x \log\left(\frac{a+bx}{(bc-ad)x}\right)} \, dx = \int \frac{\log(x)}{x \log\left(\frac{a+bx}{(bc-ad)x}\right)} \, dx$$

Mathematica [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\log(x)}{x \log\left(\frac{a+bx}{(bc-ad)x}\right)} \, dx$$

Verification is not applicable to the result.

`[In] Integrate[Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]),x]`

`[Out] Integrate[Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]), x]`

Maple [A]
time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\ln(x)}{x \ln\left(\frac{bx+a}{(-ad+cb)x}\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x),x)
[Out] int(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/log((b*x+a)/(-a*d+b*c)/x),x, algorithm="maxima")
[Out] integrate(log(x)/(x*log((b*x + a)/((b*c - a*d)*x))), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/log((b*x+a)/(-a*d+b*c)/x),x, algorithm="fricas")
[Out] integral(log(x)/(x*log((b*x + a)/((b*c - a*d)*x))), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log{\left(x \right)}}{x \log{\left(\frac{a}{- a d x + b c x} + \frac{b x}{- a d x + b c x} \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x),x)
[Out] Integral(log(x)/(x*log(a/(-a*d*x + b*c*x) + b*x/(-a*d*x + b*c*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/log((b*x+a)/(-a*d+b*c)/x),x, algorithm="giac")
[Out] integrate(log(x)/(x*log((b*x + a)/((b*c - a*d)*x))), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln(x)}{x \ln\left(-\frac{a+bx}{x(ad-bc)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/(x*log(-(a + b*x)/(x*(a*d - b*c)))),x)
[Out] int(log(x)/(x*log(-(a + b*x)/(x*(a*d - b*c)))), x)
```

3.66 $\int \frac{\log(x)}{x \log^2\left(\frac{a+bx}{(bc-ad)x}\right)} dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\log(x)}{x \log^2\left(\frac{a+bx}{(bc-ad)x}\right)}, x\right)$$

```
[Out] Unintegrable(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x)^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log(x)}{x \log^2\left(\frac{a+bx}{(bc-ad)x}\right)} dx$$

Verification is not applicable to the result.

```
[In] Int[Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]^2),x]
[Out] Defer[Int][Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]^2), x]
```

Rubi steps

$$\int \frac{\log(x)}{x \log^2\left(\frac{a+bx}{(bc-ad)x}\right)} dx = \int \frac{\log(x)}{x \log^2\left(\frac{a+bx}{(bc-ad)x}\right)} dx$$

Mathematica [A]
time = 1.16, size = 0, normalized size = 0.00

$$\int \frac{\log(x)}{x \log^2\left(\frac{a+bx}{(bc-ad)x}\right)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]^2),x]
[Out] Integrate[Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]^2), x]
```

Maple [A]
time = 19.79, size = 0, normalized size = 0.00

$$\int \frac{\ln(x)}{x \ln\left(\frac{bx+a}{(-ad+cb)x}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x)^2,x)
```

```
[Out] int(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/log((b*x+a)/(-a*d+b*c)/x)^2,x, algorithm="maxima")
```

```
[Out] -(b*x + a)*log(x)/(a*log(b*c - a*d) - a*log(b*x + a) + a*log(x)) - integrat
e(-(b*x*log(x) + b*x + a)/(a*x*log(b*c - a*d) - a*x*log(b*x + a) + a*x*log(
x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/log((b*x+a)/(-a*d+b*c)/x)^2,x, algorithm="fricas")
```

```
[Out] integral(log(x)/(x*log((b*x + a)/((b*c - a*d)*x))^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a\log(x)+bx\log(x)}{a\log\left(\frac{a+bx}{x(-ad+bc)}\right)}-\frac{\int\frac{b}{\log\left(\frac{a}{-adx+bcx}+\frac{bx}{-adx+bcx}\right)}\,dx+\int\frac{a}{x\log\left(\frac{a}{-adx+bcx}+\frac{bx}{-adx+bcx}\right)}\,dx+\int\frac{b\log(x)}{\log\left(\frac{a}{-adx+bcx}+\frac{bx}{-adx+bcx}\right)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x)**2,x)
```

```
[Out] (a*log(x) + b*x*log(x))/(a*log((a + b*x)/(x*(-a*d + b*c)))) - (Integral(b/l
og(a/(-a*d*x + b*c*x) + b*x/(-a*d*x + b*c*x)), x) + Integral(a/(x*log(a/(-a
*d*x + b*c*x) + b*x/(-a*d*x + b*c*x))), x) + Integral(b*log(x)/log(a/(-a*d*
x + b*c*x) + b*x/(-a*d*x + b*c*x)), x))/a
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/log((b*x+a)/(-a*d+b*c)/x)^2,x, algorithm="giac")
[Out] integrate(log(x)/(x*log((b*x + a)/((b*c - a*d)*x))^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(x\right)}{x\,{\ln\left(-\frac{a+b\,x}{x\,\left(a\,d-b\,c\right)}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/(x*log(-(a + b*x)/(x*(a*d - b*c)))^2),x)
[Out] int(log(x)/(x*log(-(a + b*x)/(x*(a*d - b*c)))^2), x)
```

3.67 $\int \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log(h(f+gx)^m)}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=620

$$\frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{4(bc-ad)n}+\frac{\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{4(bc-ad)n}-\frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{4(bc-ad)n}$$

```
[Out] 1/4*m*ln(e*((b*x+a)/(d*x+c))^n)^4*ln((-a*d+b*c)/b/(d*x+c))/(-a*d+b*c)/n+1/4
*ln(e*((b*x+a)/(d*x+c))^n)^4*ln(h*(g*x+f)^m)/(-a*d+b*c)/n-1/4*m*ln(e*((b*x+
a)/(d*x+c))^n)^4*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*d+b*c)/n+m
*ln(e*((b*x+a)/(d*x+c))^n)^3*polylog(2,d*(b*x+a)/b/(d*x+c))/(-a*d+b*c)-m*ln
(e*((b*x+a)/(d*x+c))^n)^3*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/
(-a*d+b*c)-3*m*n*ln(e*((b*x+a)/(d*x+c))^n)^2*polylog(3,d*(b*x+a)/b/(d*x+c))
/(-a*d+b*c)+3*m*n*ln(e*((b*x+a)/(d*x+c))^n)^2*polylog(3,(-c*g+d*f)*(b*x+a)/
(-a*g+b*f)/(d*x+c))/(-a*d+b*c)+6*m*n^2*ln(e*((b*x+a)/(d*x+c))^n)*polylog(4,
d*(b*x+a)/b/(d*x+c))/(-a*d+b*c)-6*m*n^2*ln(e*((b*x+a)/(d*x+c))^n)*polylog(4
,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*d+b*c)-6*m*n^3*polylog(5,d*(b*x
+a)/b/(d*x+c))/(-a*d+b*c)+6*m*n^3*polylog(5,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(
d*x+c))/(-a*d+b*c)
```

Rubi [A]
time = 0.56, antiderivative size = 620, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2589, 2553, 2404, 2354, 2421, 2430, 6724}

$$-\frac{6mn^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{PolyLog}\left(4,\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{bc-ad}-\frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{PolyLog}\left(2,\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{bc-ad}+\frac{3mn\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{PolyLog}\left(3,\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{bc-ad}+\frac{6mn^2\text{PolyLog}\left(4,\frac{d(a+bx)}{b(c+dx)}\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bc-ad}+\frac{m\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bc-ad}-\frac{3mn\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bc-ad}+\frac{6mn^3\text{PolyLog}\left(5,\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{bc-ad}-\frac{6mn^3\text{PolyLog}\left(5,\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad}+\frac{\log\left(h(f+gx)^m\right)\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4n(bc-ad)}-\frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{4n(bc-ad)}+\frac{m\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4n(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[e*((a + b*x)/(c + d*x))^n]^3*Log[h*(f + g*x)^m])/((a + b*x)*(c + d
*x)),x]
```

```
[Out] (m*Log[e*((a + b*x)/(c + d*x))^n]^4*Log[(b*c - a*d)/(b*(c + d*x))])/(4*(b*c
 - a*d)*n) + (Log[e*((a + b*x)/(c + d*x))^n]^4*Log[h*(f + g*x)^m])/(4*(b*c
- a*d)*n) - (m*Log[e*((a + b*x)/(c + d*x))^n]^4*Log[1 - ((d*f - c*g)*(a + b
*x))/((b*f - a*g)*(c + d*x))])/(4*(b*c - a*d)*n) + (m*Log[e*((a + b*x)/(c +
 d*x))^n]^3*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*c - a*d) - (m*Log[e
*((a + b*x)/(c + d*x))^n]^3*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)
*(c + d*x))])/(b*c - a*d) - (3*m*n*Log[e*((a + b*x)/(c + d*x))^n]^2*PolyLog
[3, (d*(a + b*x))/(b*(c + d*x))])/(b*c - a*d) + (3*m*n*Log[e*((a + b*x)/(c
+ d*x))^n]^2*PolyLog[3, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/(
b*c - a*d) + (6*m*n^2*Log[e*((a + b*x)/(c + d*x))^n]*PolyLog[4, (d*(a + b*x
))/(b*(c + d*x))])/(b*c - a*d) - (6*m*n^2*Log[e*((a + b*x)/(c + d*x))^n]*Po
lyLog[4, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/(b*c - a*d) - (6
```

```
*m*n^3*PolyLog[5, (d*(a + b*x))/(b*(c + d*x))])/(b*c - a*d) + (6*m*n^3*Poly
Log[5, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/(b*c - a*d)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2589

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*Log[(i_.)*((j_.)*((g_.) + (h_.)*(x_))^(t_.))^(u_.)]*(v_), x_S
ymbol] :> With[{k = Simplify[v*(a + b*x)*(c + d*x)]}, Simp[k*Log[i*(j*(g +
h*x)^t)^u]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s + 1)/(p*r*(s + 1)*(b*c
- a*d))), x] - Dist[k*h*t*(u/(p*r*(s + 1)*(b*c - a*d))), Int[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^(s + 1)/(g + h*x), x], x] /; FreeQ[k, x]] /; FreeQ[{
```

```
a, b, c, d, e, f, g, h, i, j, p, q, r, s, t, u}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[p + q, 0] && NeQ[s, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx &= \frac{\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{4(bc-ad)n} - \frac{(gm)\int \frac{\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx}{4(bc-ad)n} \\
&= \frac{\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{4(bc-ad)n} - \frac{(dm)\int \frac{\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx}\,dx}{4(bc-ad)n} + \frac{((d
\\
&= \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{4(bc-ad)n} - \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f+}{(bf-ag)(c+}\right.}{4(bc-ad)n} \\
&= \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{4(bc-ad)n} - \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f+}{(bf-ag)(c+}\right.}{4(bc-ad)n} \\
&= \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{4(bc-ad)n} - \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f+}{(bf-ag)(c+}\right.}{4(bc-ad)n} \\
&= \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{4(bc-ad)n} - \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f+}{(bf-ag)(c+}\right.}{4(bc-ad)n} \\
&= \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{4(bc-ad)n} - \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f+}{(bf-ag)(c+}\right.}{4(bc-ad)n}
\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 19087 vs. $2(620) = 1240$.
time = 21.35, size = 19087, normalized size = 30.79

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Log[e*((a + b*x)/(c + d*x))^n]^3*Log[h*(f + g*x)^m])/((a + b*x)*
(c + d*x)),x]
```

[Out] Result too large to show

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{3}\ln\left(h(gx+f)^{m}\right)}{(bx+a)\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)^3*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)^3*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^3*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
algorithm="maxima")
```

```
[Out] -1/4*(n^3*log(b*x + a)^4 + n^3*log(d*x + c)^4 - 4*n^2*log(b*x + a)^3 - 4*(n
^3*log(b*x + a) - n^2)*log(d*x + c)^3 - 4*(log(b*x + a) - log(d*x + c))*log
((b*x + a)^n)^3 + 4*(log(b*x + a) - log(d*x + c))*log((d*x + c)^n)^3 + 6*n*
log(b*x + a)^2 + 6*(n^3*log(b*x + a)^2 - 2*n^2*log(b*x + a) + n)*log(d*x +
c)^2 + 6*(n*log(b*x + a)^2 + n*log(d*x + c)^2 - 2*(n*log(b*x + a) - 1)*log(
d*x + c) - 2*log(b*x + a))*log((b*x + a)^n)^2 + 6*(n*log(b*x + a)^2 + n*log
(d*x + c)^2 - 2*(n*log(b*x + a) - 1)*log(d*x + c) - 2*(log(b*x + a) - log(d
*x + c))*log((b*x + a)^n) - 2*log(b*x + a))*log((d*x + c)^n)^2 - 4*(n^3*log
(b*x + a)^3 - 3*n^2*log(b*x + a)^2 + 3*n*log(b*x + a) - 1)*log(d*x + c) - 4
*(n^2*log(b*x + a)^3 - n^2*log(d*x + c)^3 - 3*n*log(b*x + a)^2 + 3*(n^2*log
(b*x + a) - n)*log(d*x + c)^2 - 3*(n^2*log(b*x + a)^2 - 2*n*log(b*x + a) +
1)*log(d*x + c) + 3*log(b*x + a))*log((b*x + a)^n) + 4*(n^2*log(b*x + a)^3
- n^2*log(d*x + c)^3 - 3*n*log(b*x + a)^2 + 3*(n^2*log(b*x + a) - n)*log(d*
x + c)^2 + 3*(log(b*x + a) - log(d*x + c))*log((b*x + a)^n)^2 - 3*(n^2*log(
b*x + a)^2 - 2*n*log(b*x + a) + 1)*log(d*x + c) - 3*(n*log(b*x + a)^2 + n*l
og(d*x + c)^2 - 2*(n*log(b*x + a) - 1)*log(d*x + c) - 2*log(b*x + a))*log((
b*x + a)^n) + 3*log(b*x + a))*log((d*x + c)^n) - 4*log(b*x + a))*log((g*x +
 f)^m)/(b*c - a*d) + integrate(1/4*((b*d*g*m*n^3*x^2 + a*c*g*m*n^3 + (b*c*g
*m*n^3 + a*d*g*m*n^3)*x)*log(b*x + a)^4 + (b*d*g*m*n^3*x^2 + a*c*g*m*n^3 +
(b*c*g*m*n^3 + a*d*g*m*n^3)*x)*log(d*x + c)^4 - 4*(b*d*g*m*n^2*x^2 + a*c*g*
m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(b*x + a)^3 + 4*(b*d*g*m*n^2*x^2
+ a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x - (b*d*g*m*n^3*x^2 + a*c*g*m*
n^3 + (b*c*g*m*n^3 + a*d*g*m*n^3)*x)*log(b*x + a))*log(d*x + c)^3 + 4*(b*c*
```

```
f*log(h) - a*d*f*log(h) + (b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2 +
a*c*g*m + (b*c*g*m + a*d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (b
*c*g*m + a*d*g*m)*x)*log(d*x + c))*log((b*x + a)^n)^3 - 4*(b*c*f*log(h) - a
*d*f*log(h) + (b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2 + a*c*g*m + (b
*c*g*m + a*d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d
*g*m)*x)*log(d*x + c))*log((d*x + c)^n)^3 + 4*b*c*f*log(h) - 4*a*d*f*log(h)
 + 6*(b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a)^2
 + 6*(b*d*g*m*n*x^2 + a*c*g*m*n + (b*d*g*m*n^3*x^2 + a*c*g*m*n^3 + (b*c*g*m
*n^3 + a*d*g*m*n^3)*x)*log(b*x + a)^2 + (b*c*g*m*n + a*d*g*m*n)*x - 2*(b*d*
g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(b*x + a))*lo
g(d*x + c)^2 + 6*(2*b*c*f*log(h) - 2*a*d*f*log(h) + (b*d*g*m*n*x^2 + a*c*g*
m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a)^2 + (b*d*g*m*n*x^2 + a*c*g*m*
n + (b*c*g*m*n + a*d*g*m*n)*x)*log(d*x + c)^2 + 2*(b*c*g*log(h) - a*d*g*log
(h))*x - 2*(b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*log(b*x + a) + 2
*(b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x - (b*d*g*m*n*x^2 + a*c*g*m*
n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a))*log(d*x + c))*log((b*x + a)^n)
^2 + 6*(2*b*c*f*log(h) - 2*a*d*f*log(h) + (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c
*g*m*n + a*d*g*m*n)*x)*log(b*x + a)^2 + (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g
*m*n + a*d*g*m*n)*x)*log(d*x + c)^2 + 2*(b*c*g*log(h) - a*d*g*log(h))*x - 2
*(b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*log(b*x + a) + 2*(b*d*g*m*
x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x - (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g
*m*n + a*d*g*m*n)*x)*log(b*x + a))*log(d*x + c) + 2*(b*c*f*log(h) - a*d*f*l
og(h) + (b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m
 + a*d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*
x)*log(d*x + c))*log((b*x + a)^n))*log((d*x + c)^n)^2 + 4*(b*c*g*log(h) - a
*d*g*log(h))*x - 4*(b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*log(b*x
+ a) + 4*(b*d*g*m*x^2 + a*c*g*m - (b*d*g*m*n^3*x^2 + a*c*g*m*n^3 + (b*c*g*m
*n^3 + a*d*g*m*n^3)*x)*log(b*x + a)^3 + 3*(b*d*g*m*n^2*x^2 + a*c*g*m*n^2 +
(b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(b*x + a)^2 + (b*c*g*m + a*d*g*m)*x - 3*(
b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a))*log(d*
x + c) - 4*((b*d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)
*log(b*x + a)^3 - (b*d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n
^2)*x)*log(d*x + c)^3 - 3*b*c*f*log(h) + 3*a*d*f*log(h) - 3*(b*d*g*m*n*x^2
+ a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a)^2 - 3*(b*d*g*m*n*x^2
+ a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x - (b*d*g*m*n^2*x^2 + a*c*g*m*n^2 +
(b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(b*x + a))*log(d*x + c)^2 - 3*(b*c*g*log(
h) - a*d*g*log(h))*x + 3*(b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*lo
g(b*x + a) - 3*(b*d*g*m*x^2 + a*c*g*m + (b*d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b
*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(b*x + a)^2 + (b*c*g*m + a*d*g*m)*x - 2*(b*
d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a))*log(d*x
+ c))*log((b*x + a)^n) + 4*((b*d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 +
 a*d*g*m*n^2)*x)*log(b*x + a)^3 - (b*d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m
*n^2 + a*d*g*m*n^2)*x)*log(d*x + c)^3 - 3*b*c*f*log(h) + 3*a*d*f*log(h) - 3
*(b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a)^2 - 3
*(b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^3*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
algorithm="fricas")
```

```
[Out] integral(log((g*x + f)^m*h)*log(((b*x + a)/(d*x + c))^n*e)^3/(b*d*x^2 + a*c
 + (b*c + a*d)*x), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)**3*ln(h*(g*x+f)**m)/(b*x+a)/(d*x+c),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^3*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
algorithm="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)*log(((b*x + a)/(d*x + c))^n*e)^3/((b*x + a)*(d
*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(h\,{\left(f+g\,x\right)}^m\right)\,{\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}^3}{\left(a+b\,x\right)\,\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(h*(f + g*x)^m)*log(e*((a + b*x)/(c + d*x))^n)^3)/((a + b*x)*(c + d
*x)),x)
```

```
[Out] int((log(h*(f + g*x)^m)*log(e*((a + b*x)/(c + d*x))^n)^3)/((a + b*x)*(c + d
*x)), x)
```

3.68 $\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log(h(f+gx)^m)}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=496

$$\frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3(bc-ad)n}+\frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{3(bc-ad)n}-\frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{3(bc-ad)n}$$

```
[Out] 1/3*m*ln(e*((b*x+a)/(d*x+c))^n)^3*ln((-a*d+b*c)/b/(d*x+c))/(-a*d+b*c)/n+1/3
*ln(e*((b*x+a)/(d*x+c))^n)^3*ln(h*(g*x+f)^m)/(-a*d+b*c)/n-1/3*m*ln(e*((b*x+
a)/(d*x+c))^n)^3*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*d+b*c)/n+m
*ln(e*((b*x+a)/(d*x+c))^n)^2*polylog(2,d*(b*x+a)/b/(d*x+c))/(-a*d+b*c)-m*ln
(e*((b*x+a)/(d*x+c))^n)^2*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/
(-a*d+b*c)-2*m*n*ln(e*((b*x+a)/(d*x+c))^n)*polylog(3,d*(b*x+a)/b/(d*x+c))/(
-a*d+b*c)+2*m*n*ln(e*((b*x+a)/(d*x+c))^n)*polylog(3,(-c*g+d*f)*(b*x+a)/(-a*
g+b*f)/(d*x+c))/(-a*d+b*c)+2*m*n^2*polylog(4,d*(b*x+a)/b/(d*x+c))/(-a*d+b*c
)-2*m*n^2*polylog(4,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*d+b*c)
```

Rubi [A]
time = 0.39, antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}=0.156$, Rules used = {2589, 2553, 2404, 2354, 2421, 2430, 6724}

$$-\frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{PolyLog}\left(2,\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{bc-ad}+\frac{2mn\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{PolyLog}\left(3,\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{bc-ad}+\frac{m\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bc-ad}-\frac{2mn\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bc-ad}-\frac{2mn^2\text{PolyLog}\left(4,\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{bc-ad}+\frac{2mn^2\text{PolyLog}\left(4,\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad}+\frac{\log\left(h(f+gx)^m\right)\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3n(bc-ad)}-\frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{3n(bc-ad)}+\frac{m\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3n(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[e*((a + b*x)/(c + d*x))^n]^2*Log[h*(f + g*x)^m])/((a + b*x)*(c + d
*x)),x]
```

```
[Out] (m*Log[e*((a + b*x)/(c + d*x))^n]^3*Log[(b*c - a*d)/(b*(c + d*x))])/(3*(b*c
 - a*d)*n) + (Log[e*((a + b*x)/(c + d*x))^n]^3*Log[h*(f + g*x)^m])/(3*(b*c
- a*d)*n) - (m*Log[e*((a + b*x)/(c + d*x))^n]^3*Log[1 - ((d*f - c*g)*(a + b
*x))/((b*f - a*g)*(c + d*x))])/(3*(b*c - a*d)*n) + (m*Log[e*((a + b*x)/(c +
 d*x))^n]^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*c - a*d) - (m*Log[e
*((a + b*x)/(c + d*x))^n]^2*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)
*(c + d*x))])/(b*c - a*d) - (2*m*n*Log[e*((a + b*x)/(c + d*x))^n]*PolyLog[3
, (d*(a + b*x))/(b*(c + d*x))])/(b*c - a*d) + (2*m*n*Log[e*((a + b*x)/(c +
d*x))^n]*PolyLog[3, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/(b*c
- a*d) + (2*m*n^2*PolyLog[4, (d*(a + b*x))/(b*(c + d*x))])/(b*c - a*d) - (2
*m*n^2*PolyLog[4, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/(b*c -
a*d)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
```

```
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2589

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*Log[(i_.)*((j_.)*((g_.) + (h_.)*(x_))^(t_.))^(u_.)]*(v_), x_S
ymbol] :> With[{k = Simplify[v*(a + b*x)*(c + d*x)]}, Simp[k*Log[i*(j*(g +
h*x)^t)^u]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s + 1)/(p*r*(s + 1)*(b*c
- a*d))), x] - Dist[k*h*t*(u/(p*r*(s + 1)*(b*c - a*d))), Int[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^(s + 1)/(g + h*x), x], x] /; FreeQ[k, x]] /; FreeQ[{
a, b, c, d, e, f, g, h, i, j, p, q, r, s, t, u}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[p + q, 0] && NeQ[s, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx = \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{3(bc-ad)n} - \frac{(gm)\int \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx}{3(bc-ad)n}$$

$$= \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{3(bc-ad)n} - \frac{(dm)\int \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx}\,dx}{3(bc-ad)n} + \frac{((d}{}$$

$$= \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3(bc-ad)n} - \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f+}{(bf-ag)(c+}\right.}{3(bc-ad)n}$$

$$= \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3(bc-ad)n} - \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f+}{(bf-ag)(c+}\right.}{3(bc-ad)n}$$

$$= \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3(bc-ad)n} - \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f+}{(bf-ag)(c+}\right.}{3(bc-ad)n}$$

$$= \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3(bc-ad)n} - \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f+}{(bf-ag)(c+}\right.}{3(bc-ad)n}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 3463 vs. $2(496) = 992$.
time = 6.76, size = 3463, normalized size = 6.98

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(Log[e*((a + b*x)/(c + d*x))^n]^2*Log[h*(f + g*x)^m])/((a + b*x)*
(c + d*x)),x]
```

```
[Out] (3*m*n*Log[e*((a + b*x)/(c + d*x))^n]*Log[(a + b*x)/(c + d*x)]^2*Log[(b*c -
 a*d)/(b*c + b*d*x)] - 2*m*n^2*Log[(a + b*x)/(c + d*x)]^3*Log[(b*c - a*d)/(
b*c + b*d*x)] - 3*m*n*Log[a/b + x]^2*Log[e*((a + b*x)/(c + d*x))^n]*Log[f +
 g*x] - 3*m*n*Log[c/d + x]^2*Log[e*((a + b*x)/(c + d*x))^n]*Log[f + g*x] +
6*m*n*Log[a/b + x]*Log[a + b*x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[f + g*x]
 - 6*m*n*Log[c/d + x]*Log[a + b*x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[f + g
*x] + 6*m*n*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[e*((a + b*x)
/(c + d*x))^n]*Log[f + g*x] + 3*m*Log[a/b + x]*Log[e*((a + b*x)/(c + d*x))^
n]^2*Log[f + g*x] - 3*m*Log[c/d + x]*Log[e*((a + b*x)/(c + d*x))^n]^2*Log[f
 + g*x] - 3*m*Log[a + b*x]*Log[e*((a + b*x)/(c + d*x))^n]^2*Log[f + g*x] +
```

3*m*n^2*Log[a/b + x]^2*Log[(a + b*x)/(c + d*x)]*Log[f + g*x] + 3*m*n^2*Log[
c/d + x]^2*Log[(a + b*x)/(c + d*x)]*Log[f + g*x] - 6*m*n^2*Log[a/b + x]*Log
[a + b*x]*Log[(a + b*x)/(c + d*x)]*Log[f + g*x] + 6*m*n^2*Log[c/d + x]*Log[
a + b*x]*Log[(a + b*x)/(c + d*x)]*Log[f + g*x] - 6*m*n^2*Log[c/d + x]*Log[(
d*(a + b*x))/(-(b*c) + a*d)]*Log[(a + b*x)/(c + d*x)]*Log[f + g*x] - 6*m*n*
Log[a/b + x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[(a + b*x)/(c + d*x)]*Log[f
+ g*x] + 6*m*n*Log[c/d + x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[(a + b*x)/(c
 + d*x)]*Log[f + g*x] + 3*m*n^2*Log[a/b + x]*Log[(a + b*x)/(c + d*x)]^2*Log
[f + g*x] - 3*m*n^2*Log[c/d + x]*Log[(a + b*x)/(c + d*x)]^2*Log[f + g*x] +
3*m*n^2*Log[a + b*x]*Log[(a + b*x)/(c + d*x)]^2*Log[f + g*x] + 3*m*n*Log[e*
((a + b*x)/(c + d*x))^n]*Log[(a + b*x)/(c + d*x)]^2*Log[f + g*x] - 3*m*n^2*
Log[(a + b*x)/(c + d*x)]^3*Log[f + g*x] - 6*m*n*Log[a/b + x]*Log[e*((a + b*
x)/(c + d*x))^n]*Log[c + d*x]*Log[f + g*x] + 6*m*n*Log[c/d + x]*Log[e*((a +
 b*x)/(c + d*x))^n]*Log[c + d*x]*Log[f + g*x] + 3*m*Log[e*((a + b*x)/(c + d
*x))^n]^2*Log[c + d*x]*Log[f + g*x] + 6*m*n^2*Log[a/b + x]*Log[(a + b*x)/(c
 + d*x)]*Log[c + d*x]*Log[f + g*x] - 6*m*n^2*Log[c/d + x]*Log[(a + b*x)/(c
+ d*x)]*Log[c + d*x]*Log[f + g*x] - 3*m*n^2*Log[(a + b*x)/(c + d*x)]^2*Log[
c + d*x]*Log[f + g*x] + 6*m*n*Log[a/b + x]*Log[e*((a + b*x)/(c + d*x))^n]*L
og[(b*(c + d*x))/(b*c - a*d)]*Log[f + g*x] - 6*m*n^2*Log[a/b + x]*Log[(a +
b*x)/(c + d*x)]*Log[(b*(c + d*x))/(b*c - a*d)]*Log[f + g*x] - 3*m*Log[a/b +
 x]*Log[e*((a + b*x)/(c + d*x))^n]^2*Log[(b*(f + g*x))/(b*f - a*g)] + 6*m*n
*Log[a/b + x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[(a + b*x)/(c + d*x)]*Log[(
b*(f + g*x))/(b*f - a*g)] - 3*m*n^2*Log[a/b + x]*Log[(a + b*x)/(c + d*x)]^2
Log[(b(f + g*x))/(b*f - a*g)] + 3*m*Log[c/d + x]*Log[e*((a + b*x)/(c + d*
x))^n]^2*Log[(d*(f + g*x))/(d*f - c*g)] - 6*m*n*Log[c/d + x]*Log[e*((a + b*
x)/(c + d*x))^n]*Log[(a + b*x)/(c + d*x)]*Log[(d*(f + g*x))/(d*f - c*g)] +
3*m*n^2*Log[c/d + x]*Log[(a + b*x)/(c + d*x)]^2*Log[(d*(f + g*x))/(d*f - c*
g)] - 3*m*n*Log[e*((a + b*x)/(c + d*x))^n]*Log[(a + b*x)/(c + d*x)]^2*Log[(
(b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))] + 2*m*n^2*Log[(a + b*x)/(c
+ d*x)]^3*Log[((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))] + 3*n*Log[a/
b + x]^2*Log[e*((a + b*x)/(c + d*x))^n]*Log[h*(f + g*x)^m] + 3*n*Log[c/d +
x]^2*Log[e*((a + b*x)/(c + d*x))^n]*Log[h*(f + g*x)^m] - 6*n*Log[a/b + x]*L
og[a + b*x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[h*(f + g*x)^m] + 6*n*Log[c/d
 + x]*Log[a + b*x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[h*(f + g*x)^m] - 6*n*
Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[e*((a + b*x)/(c + d*x))^
n]*Log[h*(f + g*x)^m] + 3*Log[a + b*x]*Log[e*((a + b*x)/(c + d*x))^n]^2*Log
[h*(f + g*x)^m] - 3*n^2*Log[a/b + x]^2*Log[(a + b*x)/(c + d*x)]*Log[h*(f +
g*x)^m] - 3*n^2*Log[c/d + x]^2*Log[(a + b*x)/(c + d*x)]*Log[h*(f + g*x)^m]
+ 6*n^2*Log[a/b + x]*Log[a + b*x]*Log[(a + b*x)/(c + d*x)]*Log[h*(f + g*x)^
m] - 6*n^2*Log[c/d + x]*Log[a + b*x]*Log[(a + b*x)/(c + d*x)]*Log[h*(f + g*
x)^m] + 6*n^2*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[(a + b*x)/
(c + d*x)]*Log[h*(f + g*x)^m] - 3*n^2*Log[a + b*x]*Log[(a + b*x)/(c + d*x)]
^2*Log[h*(f + g*x)^m] + n^2*Log[(a + b*x)/(c + d*x)]^3*Log[h*(f + g*x)^m] +
 6*n*Log[a/b + x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[c + d*x]*Log[h*(f + g*
x)^m] - 6*n*Log[c/d + x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[c + d*x]*Log[h*

```
(f + g*x)^m] - 3*Log[e*((a + b*x)/(c + d*x))^n]^2*Log[c + d*x]*Log[h*(f + g
*x)^m] - 6*n^2*Log[a/b + x]*Log[(a + b*x)/(c + d*x)]*Log[c + d*x]*Log[h*(f
+ g*x)^m] + 6*n^2*Log[c/d + x]*Log[(a + b*x)/(c + d*x)]*Log[c + d*x]*Log[h*
(f + g*x)^m] + 3*n^2*Log[(a + b*x)/(c + d*x)]^2*Log[c + d*x]*Log[h*(f + g*x
)^m] - 6*n*Log[a/b + x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[(b*(c + d*x))/(b
*c - a*d)]*Log[h*(f + g*x)^m] + 6*n^2*Log[a/b + x]*Log[(a + b*x)/(c + d*x)]
*Log[(b*(c + d*x))/(b*c - a*d)]*Log[h*(f + g*x)^m] - 6*n*(-Log[e*((a + b*x)
/(c + d*x))^n] + n*Log[(a + b*x)/(c + d*x)])*(m*Log[f + g*x] - Log[h*(f + g
*x)^m])*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 3*m*(Log[e*((a + b*x)/(c
 + d*x))^n] - n*Log[(a + b*x)/(c + d*x)])^2*PolyLog[2, (g*(a + b*x))/(-(b*f
) + a*g)] + 6*m*n*Log[e*((a + b*x)/(c + d*x))^n]*Log[(a + b*x)/(c + d*x)]*P
olyLog[2, (d*(a + b*x))/(b*(c + d*x))] - 3*m*n^...
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(e \left(\frac{bx+a}{dx+c}\right)^{n}\right)^{2} \ln \left(h (gx+f)^{m}\right)}{(bx+a)\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)^2*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)^2*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
algorithm="maxima")
```

```
[Out] 1/3*(n^2*log(b*x + a)^3 - n^2*log(d*x + c)^3 - 3*n*log(b*x + a)^2 + 3*(n^2*
log(b*x + a) - n)*log(d*x + c)^2 + 3*(log(b*x + a) - log(d*x + c))*log((b*x
 + a)^n)^2 + 3*(log(b*x + a) - log(d*x + c))*log((d*x + c)^n)^2 - 3*(n^2*lo
g(b*x + a)^2 - 2*n*log(b*x + a) + 1)*log(d*x + c) - 3*(n*log(b*x + a)^2 + n
*log(d*x + c)^2 - 2*(n*log(b*x + a) - 1)*log(d*x + c) - 2*log(b*x + a))*log
((b*x + a)^n) + 3*(n*log(b*x + a)^2 + n*log(d*x + c)^2 - 2*(n*log(b*x + a)
- 1)*log(d*x + c) - 2*(log(b*x + a) - log(d*x + c))*log((b*x + a)^n) - 2*lo
g(b*x + a))*log((d*x + c)^n) + 3*log(b*x + a))*log((g*x + f)^m)/(b*c - a*d)
 - integrate(1/3*((b*d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n
^2)*x)*log(b*x + a)^3 - (b*d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d
*g*m*n^2)*x)*log(d*x + c)^3 - 3*b*c*f*log(h) + 3*a*d*f*log(h) - 3*(b*d*g*m*
n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a)^2 - 3*(b*d*g*m*
```

```
n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x - (b*d*g*m*n^2*x^2 + a*c*g*m*
n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(b*x + a))*log(d*x + c)^2 - 3*(b*c*
f*log(h) - a*d*f*log(h) + (b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2 +
a*c*g*m + (b*c*g*m + a*d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (b
*c*g*m + a*d*g*m)*x)*log(d*x + c))*log((b*x + a)^n)^2 - 3*(b*c*f*log(h) - a
*d*f*log(h) + (b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2 + a*c*g*m + (b
*c*g*m + a*d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d
*g*m)*x)*log(d*x + c))*log((d*x + c)^n)^2 - 3*(b*c*g*log(h) - a*d*g*log(h))
*x + 3*(b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*log(b*x + a) - 3*(b*
d*g*m*x^2 + a*c*g*m + (b*d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g
*m*n^2)*x)*log(b*x + a)^2 + (b*c*g*m + a*d*g*m)*x - 2*(b*d*g*m*n*x^2 + a*c*
g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a))*log(d*x + c) - 3*(2*b*c*f*
log(h) - 2*a*d*f*log(h) + (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m
*n)*x)*log(b*x + a)^2 + (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n
)*x)*log(d*x + c)^2 + 2*(b*c*g*log(h) - a*d*g*log(h))*x - 2*(b*d*g*m*x^2 +
a*c*g*m + (b*c*g*m + a*d*g*m)*x)*log(b*x + a) + 2*(b*d*g*m*x^2 + a*c*g*m +
(b*c*g*m + a*d*g*m)*x - (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n
)*x)*log(b*x + a))*log(d*x + c))*log((b*x + a)^n) + 3*(2*b*c*f*log(h) - 2*a
*d*f*log(h) + (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b
*x + a)^2 + (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(d*x
 + c)^2 + 2*(b*c*g*log(h) - a*d*g*log(h))*x - 2*(b*d*g*m*x^2 + a*c*g*m + (b
*c*g*m + a*d*g*m)*x)*log(b*x + a) + 2*(b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a
*d*g*m)*x - (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x
 + a))*log(d*x + c) + 2*(b*c*f*log(h) - a*d*f*log(h) + (b*c*g*log(h) - a*d*
g*log(h))*x - (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*log(b*x + a)
+ (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*log(d*x + c))*log((b*x +
a)^n))*log((d*x + c)^n))/(a*b*c^2*f - a^2*c*d*f + (b^2*c*d*g - a*b*d^2*g)*x
^3 - (a*b*d^2*f + a^2*d^2*g - (c*d*f + c^2*g)*b^2)*x^2 + (b^2*c^2*f + a*b*c
^2*g - (d^2*f + c*d*g)*a^2)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
algorithm="fricas")
```

```
[Out] integral(log((g*x + f)^m*h)*log(((b*x + a)/(d*x + c))^n*e)^2/(b*d*x^2 + a*c
 + (b*c + a*d)*x), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)**2*ln(h*(g*x+f)**m)/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
algorithm="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)*log(((b*x + a)/(d*x + c))^n*e)^2/((b*x + a)*(d
*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(h\,{\left(f+g\,x\right)}^m\right)\,{\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}^2}{\left(a+b\,x\right)\,\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(h*(f + g*x)^m)*log(e*((a + b*x)/(c + d*x))^n)^2)/((a + b*x)*(c + d
*x)),x)
```

```
[Out] int((log(h*(f + g*x)^m)*log(e*((a + b*x)/(c + d*x))^n)^2)/((a + b*x)*(c + d
*x)), x)
```

3.69 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log(h(f+gx)^m)}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=371

$$\frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{2(bc-ad)n}+\frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{2(bc-ad)n}-\frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{2(bc-ad)n}$$

```
[Out] 1/2*m*ln(e*((b*x+a)/(d*x+c))^n)^2*ln((-a*d+b*c)/b/(d*x+c))/(-a*d+b*c)/n+1/2
*ln(e*((b*x+a)/(d*x+c))^n)^2*ln(h*(g*x+f)^m)/(-a*d+b*c)/n-1/2*m*ln(e*((b*x+
a)/(d*x+c))^n)^2*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*d+b*c)/n+m
*ln(e*((b*x+a)/(d*x+c))^n)*polylog(2,d*(b*x+a)/b/(d*x+c))/(-a*d+b*c)-m*ln(e
*((b*x+a)/(d*x+c))^n)*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*
d+b*c)-m*n*polylog(3,d*(b*x+a)/b/(d*x+c))/(-a*d+b*c)+m*n*polylog(3,(-c*g+d*
f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*d+b*c)
```

Rubi [A]

time = 0.31, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {2589, 2553, 2404, 2354, 2421, 6724}

$$-\frac{m\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{PolyLog}\left(2,\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{bc-ad}+\frac{m\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bc-ad}+\frac{mn\text{PolyLog}\left(3,\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{bc-ad}-\frac{mn\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{bc-ad}+\frac{\log\left(h(f+gx)^m\right)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2n(bc-ad)}-\frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{2n(bc-ad)}+\frac{m\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2n(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[e*((a + b*x)/(c + d*x))^n]*Log[h*(f + g*x)^m])/((a + b*x)*(c + d*x
)),x]
```

```
[Out] (m*Log[e*((a + b*x)/(c + d*x))^n]^2*Log[(b*c - a*d)/(b*(c + d*x))])/(2*(b*c
 - a*d)*n) + (Log[e*((a + b*x)/(c + d*x))^n]^2*Log[h*(f + g*x)^m])/(2*(b*c
- a*d)*n) - (m*Log[e*((a + b*x)/(c + d*x))^n]^2*Log[1 - ((d*f - c*g)*(a + b
*x))/((b*f - a*g)*(c + d*x))])/(2*(b*c - a*d)*n) + (m*Log[e*((a + b*x)/(c +
 d*x))^n]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*c - a*d) - (m*Log[e*(
(a + b*x)/(c + d*x))^n]*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c
+ d*x))])/(b*c - a*d) - (m*n*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/(b*c
- a*d) + (m*n*PolyLog[3, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/
(b*c - a*d)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2589

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*Log[(i_.)*((j_.)*((g_.) + (h_.)*(x_))^(t_.))^(u_.)]*(v_), x_S
ymbol] :> With[{k = Simplify[v*(a + b*x)*(c + d*x)]}, Simp[k*Log[i*(j*(g +
h*x)^t)^u]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s + 1)/(p*r*(s + 1)*(b*c
- a*d))), x] - Dist[k*h*t*(u/(p*r*(s + 1)*(b*c - a*d))), Int[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^(s + 1)/(g + h*x), x], x] /; FreeQ[k, x]] /; FreeQ[{
a, b, c, d, e, f, g, h, i, j, p, q, r, s, t, u}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[p + q, 0] && NeQ[s, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx = \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{2(bc-ad)n} - \frac{(gm)\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx}{2(bc-ad)n}$$

$$= \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{2(bc-ad)n} - \frac{(dm)\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx}\,dx}{2(bc-ad)n} + \frac{((c$$

$$= \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{2(bc-ad)n} - \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f}{(bf-ag)(c}\right.}{2(bc-ad)n}$$

$$= \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{2(bc-ad)n} - \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f}{(bf-ag)(c}\right.}{2(bc-ad)n}$$

$$= \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{2(bc-ad)n} - \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f}{(bf-ag)(c}\right.}{2(bc-ad)n}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1408 vs. $2(371) = 742$.
time = 1.71, size = 1408, normalized size = 3.80

Antiderivative was successfully verified.

```
[In] Integrate[(Log[e*((a + b*x)/(c + d*x))^n]*Log[h*(f + g*x)^m])/((a + b*x)*(c
 + d*x)),x]
```

```
[Out] (m*n*Log[(a + b*x)/(c + d*x)]^2*Log[(b*c - a*d)/(b*c + b*d*x)] - m*n*Log[a/
b + x]^2*Log[f + g*x] - m*n*Log[c/d + x]^2*Log[f + g*x] + 2*m*n*Log[a/b + x
]*Log[a + b*x]*Log[f + g*x] - 2*m*n*Log[c/d + x]*Log[a + b*x]*Log[f + g*x]
+ 2*m*n*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[f + g*x] + 2*m*L
og[a/b + x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[f + g*x] - 2*m*Log[c/d + x]*
Log[e*((a + b*x)/(c + d*x))^n]*Log[f + g*x] - 2*m*Log[a + b*x]*Log[e*((a +
b*x)/(c + d*x))^n]*Log[f + g*x] - 2*m*n*Log[a/b + x]*Log[(a + b*x)/(c + d*x
)]*Log[f + g*x] + 2*m*n*Log[c/d + x]*Log[(a + b*x)/(c + d*x)]*Log[f + g*x]
+ m*n*Log[(a + b*x)/(c + d*x)]^2*Log[f + g*x] - 2*m*n*Log[a/b + x]*Log[c +
d*x]*Log[f + g*x] + 2*m*n*Log[c/d + x]*Log[c + d*x]*Log[f + g*x] + 2*m*Log[
e*((a + b*x)/(c + d*x))^n]*Log[c + d*x]*Log[f + g*x] + 2*m*n*Log[a/b + x]*L
og[(b*(c + d*x))/(b*c - a*d)]*Log[f + g*x] - 2*m*Log[a/b + x]*Log[e*((a + b
*x)/(c + d*x))^n]*Log[(b*(f + g*x))/(b*f - a*g)] + 2*m*n*Log[a/b + x]*Log[(
a + b*x)/(c + d*x)]*Log[(b*(f + g*x))/(b*f - a*g)] + 2*m*Log[c/d + x]*Log[e
*((a + b*x)/(c + d*x))^n]*Log[(d*(f + g*x))/(d*f - c*g)] - 2*m*n*Log[c/d +
x]*Log[(a + b*x)/(c + d*x)]*Log[(d*(f + g*x))/(d*f - c*g)] - m*n*Log[(a + b
```

```
*x)/(c + d*x)]^2*Log[((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))] + n*L
og[a/b + x]^2*Log[h*(f + g*x)^m] + n*Log[c/d + x]^2*Log[h*(f + g*x)^m] - 2*
n*Log[a/b + x]*Log[a + b*x]*Log[h*(f + g*x)^m] + 2*n*Log[c/d + x]*Log[a + b
*x]*Log[h*(f + g*x)^m] - 2*n*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]
*Log[h*(f + g*x)^m] + 2*Log[a + b*x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[h*(
f + g*x)^m] + 2*n*Log[a/b + x]*Log[c + d*x]*Log[h*(f + g*x)^m] - 2*n*Log[c/
d + x]*Log[c + d*x]*Log[h*(f + g*x)^m] - 2*Log[e*((a + b*x)/(c + d*x))^n]*L
og[c + d*x]*Log[h*(f + g*x)^m] - 2*n*Log[a/b + x]*Log[(b*(c + d*x))/(b*c -
a*d)]*Log[h*(f + g*x)^m] + 2*n*(m*Log[f + g*x] - Log[h*(f + g*x)^m])*PolyLo
g[2, (d*(a + b*x))/(-(b*c) + a*d)] - 2*m*(Log[e*((a + b*x)/(c + d*x))^n] -
n*Log[(a + b*x)/(c + d*x)])*PolyLog[2, (g*(a + b*x))/(-(b*f) + a*g)] + 2*m*
n*Log[(a + b*x)/(c + d*x)]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))] - 2*m*n*
Log[(a + b*x)/(c + d*x)]*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c
 + d*x))] + 2*m*n*Log[f + g*x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 2*n*
Log[h*(f + g*x)^m]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] + 2*m*Log[e*((a +
b*x)/(c + d*x))^n]*PolyLog[2, (g*(c + d*x))/(-(d*f) + c*g)] - 2*m*n*Log[(a
+ b*x)/(c + d*x)]*PolyLog[2, (g*(c + d*x))/(-(d*f) + c*g)] - 2*m*n*PolyLog[
3, (d*(a + b*x))/(b*(c + d*x))] + 2*m*n*PolyLog[3, ((d*f - c*g)*(a + b*x))/
((b*f - a*g)*(c + d*x))])/(2*b*c - 2*a*d)
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right) \ln \left(h(gx+f)^{m}\right)}{(bx+a)\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x, al
gorithm="maxima")
```

```
[Out] -1/2*(n*log(b*x + a)^2 + n*log(d*x + c)^2 - 2*(n*log(b*x + a) - 1)*log(d*x
+ c) - 2*(log(b*x + a) - log(d*x + c))*log((b*x + a)^n) + 2*(log(b*x + a) -
 log(d*x + c))*log((d*x + c)^n) - 2*log(b*x + a))*log((g*x + f)^m)/(b*c - a
*d) + integrate(1/2*(2*b*c*f*log(h) - 2*a*d*f*log(h) + (b*d*g*m*n*x^2 + a*c
*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a)^2 + (b*d*g*m*n*x^2 + a*c*g
*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(d*x + c)^2 + 2*(b*c*g*log(h) - a*d*g*
```

```
log(h))*x - 2*(b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*log(b*x + a)
+ 2*(b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x - (b*d*g*m*n*x^2 + a*c*g
*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a))*log(d*x + c) + 2*(b*c*f*log
(h) - a*d*f*log(h) + (b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2 + a*c*g
*m + (b*c*g*m + a*d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (b*c*g*
m + a*d*g*m)*x)*log(d*x + c))*log((b*x + a)^n) - 2*(b*c*f*log(h) - a*d*f*lo
g(h) + (b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m
+ a*d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x
)*log(d*x + c))*log((d*x + c)^n))/(a*b*c^2*f - a^2*c*d*f + (b^2*c*d*g - a*b
*d^2*g)*x^3 - (a*b*d^2*f + a^2*d^2*g - (c*d*f + c^2*g)*b^2)*x^2 + (b^2*c^2*
f + a*b*c^2*g - (d^2*f + c*d*g)*a^2)*x), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x, al
gorithm="fricas")
```

```
[Out] integral(log((g*x + f)^m*h)*log(((b*x + a)/(d*x + c))^n*e)/(b*d*x^2 + a*c +
 (b*c + a*d)*x), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)*ln(h*(g*x+f)**m)/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x, al
gorithm="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)*log(((b*x + a)/(d*x + c))^n*e)/((b*x + a)*(d*x
 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(h\left(f+g\,x\right)^m\right)\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{\left(a+b\,x\right)\,\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(h*(f + g*x)^m)*log(e*((a + b*x)/(c + d*x))^n))/((a + b*x)*(c + d*x
)),x)
```

```
[Out] int((log(h*(f + g*x)^m)*log(e*((a + b*x)/(c + d*x))^n))/((a + b*x)*(c + d*x
)), x)
```

3.70 $\int \frac{\log(h(f+gx)^m)}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal. Leaf size=108

$$\frac{b\mathrm{Int}\left(\frac{\log(h(f+gx)^m)}{(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)}{bc-ad}-\frac{d\mathrm{Int}\left(\frac{\log(h(f+gx)^m)}{(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)}{bc-ad}$$

```
[Out] b*Unintegrable(ln(h*(g*x+f)^m)/(b*x+a)/ln(e*((b*x+a)/(d*x+c))^n),x)/(-a*d+b
*c)-d*Unintegrable(ln(h*(g*x+f)^m)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))^n),x)/(-a
*d+b*c)
```

Rubi [A]
time = 0.38, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[h*(f + g*x)^m]/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x))^n])
,x]
```

```
[Out] (b*Defer[Int][Log[h*(f + g*x)^m]/((a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])
, x])/(b*c - a*d) - (d*Defer[Int][Log[h*(f + g*x)^m]/((c + d*x)*Log[e*((a +
 b*x)/(c + d*x))^n]), x])/(b*c - a*d)
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx &= \int \left(\frac{b\log\left(h(f+gx)^m\right)}{(bc-ad)(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}-\frac{d\log\left(h(f+gx)^m\right)}{(bc-ad)(c+dx)\log\left(e\left(\right.\right.}\right.\\ &= \frac{b\int \frac{\log(h(f+gx)^m)}{(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx}{bc-ad}-\frac{d\int \frac{\log(h(f+gx)^m)}{(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx}{bc-ad}\end{aligned}$$

Mathematica [A]
time = 1.25, size = 0, normalized size = 0.00

$$\int \frac{\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[h*(f + g*x)^m]/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x
))^n]),x]
```

```
[Out] Integrate[Log[h*(f + g*x)^m]/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x
))^n]), x]
```

Maple [A]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(h(gx+f)^{m}\right)}{\left(bx+a\right)\left(dx+c\right)\ln \left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))^n),x)
```

```
[Out] int(ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))^n),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n),x, al
gorithm="maxima")
```

```
[Out] integrate(log((g*x + f)^m*h)/((b*x + a)*(d*x + c)*log(((b*x + a)/(d*x + c))
^n*e)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n),x, al
gorithm="fricas")
```

```
[Out] integral(log((g*x + f)^m*h)/((b*d*x^2 + a*c + (b*c + a*d)*x)*log(((b*x + a)
/(d*x + c))^n*e)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(h*(g*x+f)**m)/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))**n),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n),x, al
gorithm="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)/((b*x + a)*(d*x + c)*log(((b*x + a)/(d*x + c))
^n*e)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(h\,{\left(f+g\,x\right)}^m\right)}{\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\,\left(a+b\,x\right)\,\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(h*(f + g*x)^m)/(log(e*((a + b*x)/(c + d*x))^n)*(a + b*x)*(c + d*x))
,x)
```

```
[Out] int(log(h*(f + g*x)^m)/(log(e*((a + b*x)/(c + d*x))^n)*(a + b*x)*(c + d*x))
, x)
```

3.71 $\int \frac{\log(h(f+gx)^m)}{(a+bx)(c+dx)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$

Optimal. Leaf size=93

$$-\frac{\log\left(h(f+gx)^m\right)}{(bc-ad)n\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}+\frac{gm\text{Int}\left(\frac{1}{(f+gx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)}{(bc-ad)n}$$

```
[Out] -ln(h*(g*x+f)^m)/(-a*d+b*c)/n/ln(e*((b*x+a)/(d*x+c))^n)+g*m*Unintegrable(1/
(g*x+f)/ln(e*((b*x+a)/(d*x+c))^n),x)/(-a*d+b*c)/n
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[h*(f + g*x)^m]/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x))^n]^
2),x]
```

```
[Out] -(Log[h*(f + g*x)^m]/((b*c - a*d)*n*Log[e*((a + b*x)/(c + d*x))^n])) + (g*m
*Defer[Int][1/((f + g*x)*Log[e*((a + b*x)/(c + d*x))^n]), x])/((b*c - a*d)*
n)
```

Rubi steps

$$\int \frac{\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=-\frac{\log\left(h(f+gx)^m\right)}{(bc-ad)n\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}+\frac{(gm)\int \frac{1}{(f+gx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx}{(bc-ad)n}$$

Mathematica [A]
time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[h*(f + g*x)^m]/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x
))^n]^2),x]
```

```
[Out] Integrate[Log[h*(f + g*x)^m]/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x
))^n]^2), x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(h(gx+f)^{m}\right)}{(bx+a)\,(dx+c)\ln \left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))^n)^2,x)
```

```
[Out] int(ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))^n)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n)^2,x,
algorithm="maxima")
```

```
[Out] g*m*integrate(1/(b*c*f*n - a*d*f*n + (b*c*g*n - a*d*g*n)*x + (b*c*f*n - a*d
*f*n + (b*c*g*n - a*d*g*n)*x)*log((b*x + a)^n) - (b*c*f*n - a*d*f*n + (b*c*
g*n - a*d*g*n)*x)*log((d*x + c)^n)), x) - (log((g*x + f)^m) + log(h))/(b*c*
n - a*d*n + (b*c*n - a*d*n)*log((b*x + a)^n) - (b*c*n - a*d*n)*log((d*x + c
)^n))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n)^2,x,
algorithm="fricas")
```

```
[Out] integral(log((g*x + f)^m*h)/((b*d*x^2 + a*c + (b*c + a*d)*x)*log(((b*x + a)
/(d*x + c))^n*e)^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(h*(g*x+f)**m)/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))**n)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n)^2,x,
algorithm="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)/((b*x + a)*(d*x + c)*log(((b*x + a)/(d*x + c))
^n*e)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(h\,{\left(f+g\,x\right)}^m\right)}{{\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}^2\,\left(a+b\,x\right)\,\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(h*(f + g*x)^m)/(log(e*((a + b*x)/(c + d*x))^n)^2*(a + b*x)*(c + d*x
)),x)
```

```
[Out] int(log(h*(f + g*x)^m)/(log(e*((a + b*x)/(c + d*x))^n)^2*(a + b*x)*(c + d*x
)), x)
```

3.72 $\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx$

Optimal. Leaf size=114

$$\frac{b\text{Int}\left(\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)},x\right)}{bc-ad}-\frac{d\text{Int}\left(\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)},x\right)}{bc-ad}$$

```
[Out] b*CannotIntegrate(ln(1+(-b*x-a)/(d*x+c))/(b*x+a)/ln((b*x+a)/(d*x+c))^2,x)/(
-a*d+b*c)-d*CannotIntegrate(ln(1+(-b*x-a)/(d*x+c))/(d*x+c)/ln((b*x+a)/(d*x+
c))^2,x)/(-a*d+b*c)
```

Rubi [A]
time = 0.37, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[1 - (a + b*x)/(c + d*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*
x)]^2),x]
```

```
[Out] (b*Defer[Int][Log[1 - (a + b*x)/(c + d*x)]/((a + b*x)*Log[(a + b*x)/(c + d*
x)]^2), x])/(b*c - a*d) - (d*Defer[Int][Log[1 - (a + b*x)/(c + d*x)]/((c +
d*x)*Log[(a + b*x)/(c + d*x)]^2), x])/(b*c - a*d)
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx &= \int \left(\frac{b\log\left(1-\frac{a+bx}{c+dx}\right)}{(bc-ad)(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)}-\frac{d\log\left(1-\frac{a+bx}{c+dx}\right)}{(bc-ad)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)\,dx\\ &= \frac{b\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx}{bc-ad}-\frac{d\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx}{bc-ad}\end{aligned}$$

Mathematica [A]
time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[1 - (a + b*x)/(c + d*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(
c + d*x)]^2),x]
```

```
[Out] Integrate[Log[1 - (a + b*x)/(c + d*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(
c + d*x)]^2), x]
```

Maple [A]
time = 2.70, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(1+\frac{-bx-a}{dx+c}\right)}{(bx+a)\,(dx+c)\ln\left(\frac{bx+a}{dx+c}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))^2,x)
```

```
[Out] int(ln(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x,
 algorithm="maxima")
```

```
[Out] -(log(-(b - d)*x - a + c) - log(b*x + a))/((b*c - a*d)*log(b*x + a) - (b*c
- a*d)*log(d*x + c)) - integrate(-1/(((b*d - d^2)*x^2 + a*c - c^2 + (b*c +
a*d - 2*c*d)*x)*log(b*x + a) - ((b*d - d^2)*x^2 + a*c - c^2 + (b*c + a*d -
2*c*d)*x)*log(d*x + c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x,
 algorithm="fricas")
```

```
[Out] integral(log(-((b - d)*x + a - c)/(d*x + c))/((b*d*x^2 + a*c + (b*c + a*d)*
x)*log((b*x + a)/(d*x + c))^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))**2,x)
```

```
[Out] Exception raised: TypeError >> '>' not supported between instances of 'Poly
' and 'int'
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x,
 algorithm="giac")
```

```
[Out] integrate(log(-(b*x + a)/(d*x + c) + 1)/((b*x + a)*(d*x + c)*log((b*x + a)/
(d*x + c))^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(1-\frac{a+b\,x}{c+d\,x}\right)}{\ln\left(\frac{a+b\,x}{c+d\,x}\right)^2\,\left(a+b\,x\right)\,\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(1 - (a + b*x)/(c + d*x))/(log((a + b*x)/(c + d*x))^2*(a + b*x)*(c +
 d*x)),x)
```

```
[Out] int(log(1 - (a + b*x)/(c + d*x))/(log((a + b*x)/(c + d*x))^2*(a + b*x)*(c +
 d*x)), x)
```

3.73 $\displaystyle\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx$

Optimal. Leaf size=114

$$\frac{b\text{Int}\left(\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)},x\right)}{bc-ad}-\frac{d\text{Int}\left(\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)},x\right)}{bc-ad}$$

```
[Out] b*CannotIntegrate(ln(1+(-d*x-c)/(b*x+a))/(b*x+a)/ln((b*x+a)/(d*x+c))^2,x)/(
-a*d+b*c)-d*CannotIntegrate(ln(1+(-d*x-c)/(b*x+a))/(d*x+c)/ln((b*x+a)/(d*x+
c))^2,x)/(-a*d+b*c)
```

Rubi [A]
time = 0.35, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[1 - (c + d*x)/(a + b*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*
x)]^2),x]
```

```
[Out] (b*Defer[Int][Log[1 - (c + d*x)/(a + b*x)]/((a + b*x)*Log[(a + b*x)/(c + d*
x)]^2), x])/(b*c - a*d) - (d*Defer[Int][Log[1 - (c + d*x)/(a + b*x)]/((c +
d*x)*Log[(a + b*x)/(c + d*x)]^2), x])/(b*c - a*d)
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx &= \int \left(\frac{b\log\left(1-\frac{c+dx}{a+bx}\right)}{(bc-ad)(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)}-\frac{d\log\left(1-\frac{c+dx}{a+bx}\right)}{(bc-ad)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx\\ &= \frac{b\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx}{bc-ad}-\frac{d\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx}{bc-ad}\end{aligned}$$

Mathematica [A]
time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[1 - (c + d*x)/(a + b*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(
c + d*x)]^2),x]
```

```
[Out] Integrate[Log[1 - (c + d*x)/(a + b*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(
c + d*x)]^2), x]
```

Maple [A]
time = 1.50, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(1+\frac{-dx-c}{bx+a}\right)}{(bx+a)\,(dx+c)\ln\left(\frac{bx+a}{dx+c}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1+(-d*x-c)/(b*x+a))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))^2,x)
```

```
[Out] int(ln(1+(-d*x-c)/(b*x+a))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+(-d*x-c)/(b*x+a))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x,
 algorithm="maxima")
```

```
[Out] -(log((b - d)*x + a - c) - log(b*x + a))/((b*c - a*d)*log(b*x + a) - (b*c -
 a*d)*log(d*x + c)) - integrate(-1/((((b^2 - b*d)*x^2 + a^2 - a*c + (a*(2*b
- d) - b*c)*x)*log(b*x + a) - ((b^2 - b*d)*x^2 + a^2 - a*c + (a*(2*b - d) -
 b*c)*x)*log(d*x + c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+(-d*x-c)/(b*x+a))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x,
 algorithm="fricas")
```

```
[Out] integral(log(((b - d)*x + a - c)/(b*x + a))/((b*d*x^2 + a*c + (b*c + a*d)*x
)*log((b*x + a)/(d*x + c))^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+(-d*x-c)/(b*x+a))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))**2,x)
```

```
[Out] Exception raised: TypeError >> '>' not supported between instances of 'Poly
' and 'int'
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+(-d*x-c)/(b*x+a))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x,
 algorithm="giac")
```

```
[Out] integrate(log(-(d*x + c)/(b*x + a) + 1)/((b*x + a)*(d*x + c)*log((b*x + a)/
(d*x + c))^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(1-\frac{c+d\,x}{a+b\,x}\right)}{{\ln\left(\frac{a+b\,x}{c+d\,x}\right)}^2\,\left(a+b\,x\right)\,\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(1 - (c + d*x)/(a + b*x))/(log((a + b*x)/(c + d*x))^2*(a + b*x)*(c +
 d*x)),x)
```

```
[Out] int(log(1 - (c + d*x)/(a + b*x))/(log((a + b*x)/(c + d*x))^2*(a + b*x)*(c +
 d*x)), x)
```

3.74 $\int \left(\frac{1}{(c+dx)(-a+c+(-b+d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right.$

Optimal. Leaf size=45

$$-\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(bc-ad)\log\left(\frac{a+bx}{c+dx}\right)}$$

```
[Out] -ln(1+(-b*x-a)/(d*x+c))/(-a*d+b*c)/ln((b*x+a)/(d*x+c))
```

Rubi [F]
time = 0.38, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(\frac{1}{(c+dx)(-a+c+(-b+d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)*(-a + c + (-b + d)*x)*Log[(a + b*x)/(c + d*x)]) + Log[1 -
(a + b*x)/(c + d*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*x)]^2),x]
```

```
[Out] Defer[Int][1/((c + d*x)*(-a + c + (-b + d)*x)*Log[(a + b*x)/(c + d*x)]), x]
 + (b*Defer[Int][Log[1 - (a + b*x)/(c + d*x)]/((a + b*x)*Log[(a + b*x)/(c +
 d*x)]^2), x])/(b*c - a*d) - (d*Defer[Int][Log[1 - (a + b*x)/(c + d*x)]/((c
 + d*x)*Log[(a + b*x)/(c + d*x)]^2), x])/(b*c - a*d)
```

Rubi steps

$$\begin{aligned}\int \left(\frac{1}{(c+dx)(-a+c+(-b+d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx &= \int \frac{}{(c+dx)(-a+c+} \\ &= \int \frac{}{(c+dx)(-a+c+} \\ &= \frac{b\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx}{bc-ad}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 44, normalized size = 0.98

$$\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(-bc+ad)\log\left(\frac{a+bx}{c+dx}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)*(-a + c + (-b + d)*x)*Log[(a + b*x)/(c + d*x)]) + Lo
g[1 - (a + b*x)/(c + d*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*x)]^2)
,x]
```

```
[Out] Log[1 - (a + b*x)/(c + d*x)]/((-(b*c) + a*d)*Log[(a + b*x)/(c + d*x)])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\,(-a+c+(-b+d)\,x)\ln\left(\frac{bx+a}{dx+c}\right)}+\frac{\ln\left(1+\frac{-bx-a}{dx+c}\right)}{(bx+a)\,(dx+c)\ln\left(\frac{bx+a}{dx+c}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(-a+c+(-b+d)*x)/ln((b*x+a)/(d*x+c))+ln(1+(-b*x-a)/(d*x+c))/(b
*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))^2,x)
```

```
[Out] int(1/(d*x+c)/(-a+c+(-b+d)*x)/ln((b*x+a)/(d*x+c))+ln(1+(-b*x-a)/(d*x+c))/(b
*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))^2,x)
```

Maxima [A]
time = 0.32, size = 59, normalized size = 1.31

$$-\frac{\log\left(-(b-d)x-a+c\right)-\log\left(bx+a\right)}{(bc-ad)\log\left(bx+a\right)-(bc-ad)\log\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(-a+c+(-b+d)*x)/log((b*x+a)/(d*x+c))+log(1+(-b*x-a)/(d*
x+c))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(log(-(b - d)*x - a + c) - log(b*x + a))/((b*c - a*d)*log(b*x + a) - (b*c
 - a*d)*log(d*x + c))
```

Fricas [A]
time = 0.37, size = 50, normalized size = 1.11

$$-\frac{\log\left(-\frac{(b-d)x+a-c}{dx+c}\right)}{(bc-ad)\log\left(\frac{bx+a}{dx+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(-a+c+(-b+d)*x)/log((b*x+a)/(d*x+c))+log(1+(-b*x-a)/(d*
x+c))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -log(-((b - d)*x + a - c)/(d*x + c))/((b*c - a*d)*log((b*x + a)/(d*x + c)))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(-a+c+(-b+d)*x)/ln((b*x+a)/(d*x+c))+ln(1+(-b*x-a)/(d*x+
c))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))**2,x)
```

```
[Out] Exception raised: TypeError >> '>' not supported between instances of 'Poly
' and 'int'
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(-a+c+(-b+d)*x)/log((b*x+a)/(d*x+c))+log(1+(-b*x-a)/(d*
x+c))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(-1/(((b - d)*x + a - c)*(d*x + c)*log((b*x + a)/(d*x + c))) + log
(-(b*x + a)/(d*x + c) + 1)/((b*x + a)*(d*x + c)*log((b*x + a)/(d*x + c))^2)
, x)
```

Mupad [B]
time = 0.70, size = 44, normalized size = 0.98

$$\frac{\ln\left(1-\frac{a+b\,x}{c+d\,x}\right)}{\ln\left(\frac{a+b\,x}{c+d\,x}\right)\,\left(a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(1 - (a + b*x)/(c + d*x))/(log((a + b*x)/(c + d*x))^2*(a + b*x)*(c +
 d*x)) - 1/(log((a + b*x)/(c + d*x))*(c + d*x)*(a - c + x*(b - d))),x)
```

```
[Out] log(1 - (a + b*x)/(c + d*x))/(log((a + b*x)/(c + d*x))*(a*d - b*c))
```

3.75 $\int \left(-\frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)$

Optimal. Leaf size=45

$$-\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(bc-ad)\log\left(\frac{a+bx}{c+dx}\right)}$$

```
[Out] -ln(1+(-d*x-c)/(b*x+a))/(-a*d+b*c)/ln((b*x+a)/(d*x+c))
```

Rubi [F]
time = 0.36, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(-\frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx$$

Verification is not applicable to the result.

```
[In] Int[-(1/((a + b*x)*(a - c + (b - d)*x)*Log[(a + b*x)/(c + d*x)])) + Log[1 -
 (c + d*x)/(a + b*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*x)]^2),x]
[Out] -Defer[Int][1/((a + b*x)*(a - c + (b - d)*x)*Log[(a + b*x)/(c + d*x)]), x]
+ (b*Defer[Int][Log[1 - (c + d*x)/(a + b*x)]/((a + b*x)*Log[(a + b*x)/(c +
d*x)]^2), x])/(b*c - a*d) - (d*Defer[Int][Log[1 - (c + d*x)/(a + b*x)]/((c
+ d*x)*Log[(a + b*x)/(c + d*x)]^2), x])/(b*c - a*d)
```

Rubi steps

$$\begin{aligned}\int \left(-\frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx &= -\int \frac{1}{(a+bx)(a-c+(b\\ &= -\int \frac{1}{(a+bx)(a-c+(b\\ &= \frac{b\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)}dx}{bc-ad} -\end{aligned}$$

Mathematica [A]
time = 0.06, size = 45, normalized size = 1.00

$$-\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(bc-ad)\log\left(\frac{a+bx}{c+dx}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[-(1/((a + b*x)*(a - c + (b - d)*x)*Log[(a + b*x)/(c + d*x)])) + L
og[1 - (c + d*x)/(a + b*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*x)]^2
),x]
```

```
[Out] -(Log[1 - (c + d*x)/(a + b*x)]/((b*c - a*d)*Log[(a + b*x)/(c + d*x)]))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 11.26, size = 503, normalized size = 11.18

method	result
risch	$\frac{2i\ln(bx-dx+a-c)}{(ad-cb)\left(\pi\operatorname{csgn}\left(\frac{i(bx+a)}{dx+c}\right)\operatorname{csgn}(i(bx+a))\operatorname{csgn}\left(\frac{i}{dx+c}\right)-\pi\operatorname{csgn}\left(\frac{i(bx+a)}{dx+c}\right)^2\operatorname{csgn}(i(bx+a))-\pi\operatorname{csgn}\left(\frac{i(bx+a)}{dx+c}\right)^2\operatorname{csgn}\left(\frac{i}{dx+c}\right)+\pi\operatorname{csgn}\left(\frac{i(b}{d}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(b*x+a)/(a-c+(b-d)*x)/ln((b*x+a)/(d*x+c))+ln(1+(-d*x-c)/(b*x+a))/(b*
x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/(a*d-b*c)/(Pi*csgn(I*(b*x+a)/(d*x+c))*csgn(I*(b*x+a))*csgn(I/(d*x+c))-P
i*csgn(I*(b*x+a)/(d*x+c))^2*csgn(I*(b*x+a))-Pi*csgn(I*(b*x+a)/(d*x+c))^2*cs
gn(I/(d*x+c))+Pi*csgn(I*(b*x+a)/(d*x+c))^3+2*I*ln(b*x+a)-2*I*ln(d*x+c))*ln(
b*x-d*x+a-c)-(I*Pi*csgn(I*(b*x-d*x+a-c))*csgn(I/(b*x+a))*csgn(I/(b*x+a)*(b*
x-d*x+a-c))-I*Pi*csgn(I*(b*x-d*x+a-c))*csgn(I/(b*x+a)*(b*x-d*x+a-c))^2-I*Pi
*csgn(I/(b*x+a))*csgn(I/(b*x+a)*(b*x-d*x+a-c))^2+I*Pi*csgn(I/(b*x+a)*(b*x-d
*x+a-c))^3+2*ln(b*x+a))/(a*d-b*c)/(-I*Pi*csgn(I*(b*x+a)/(d*x+c))^3+I*Pi*csg
n(I*(b*x+a)/(d*x+c))^2*csgn(I*(b*x+a))+I*Pi*csgn(I*(b*x+a)/(d*x+c))^2*csgn(
I/(d*x+c))-I*Pi*csgn(I*(b*x+a)/(d*x+c))*csgn(I*(b*x+a))*csgn(I/(d*x+c))+2*l
n(b*x+a)-2*ln(d*x+c))
```

Maxima [A]
time = 0.38, size = 58, normalized size = 1.29

$$-\frac{\log\left((b-d)x+a-c\right)-\log\left(bx+a\right)}{(bc-ad)\log\left(bx+a\right)-(bc-ad)\log\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(b*x+a)/(a-c+(b-d)*x)/log((b*x+a)/(d*x+c))+log(1+(-d*x-c)/(b*x
+a))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(log((b - d)*x + a - c) - log(b*x + a))/((b*c - a*d)*log(b*x + a) - (b*c -
 a*d)*log(d*x + c))
```

Fricas [A]
time = 0.34, size = 49, normalized size = 1.09

$$-\frac{\log\left(\frac{(b-d)x+a-c}{bx+a}\right)}{(bc-ad)\log\left(\frac{bx+a}{dx+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(b*x+a)/(a-c+(b-d)*x)/log((b*x+a)/(d*x+c))+log(1+(-d*x-c)/(b*x
+a))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -log(((b - d)*x + a - c)/(b*x + a))/((b*c - a*d)*log((b*x + a)/(d*x + c)))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(b*x+a)/(a-c+(b-d)*x)/ln((b*x+a)/(d*x+c))+ln(1+(-d*x-c)/(b*x+a
))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))**2,x)
```

```
[Out] Exception raised: TypeError >> '>' not supported between instances of 'Poly
' and 'int'
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(b*x+a)/(a-c+(b-d)*x)/log((b*x+a)/(d*x+c))+log(1+(-d*x-c)/(b*x
+a))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(-1/(((b - d)*x + a - c)*(b*x + a)*log((b*x + a)/(d*x + c))) + log
(-(d*x + c)/(b*x + a) + 1)/((b*x + a)*(d*x + c)*log((b*x + a)/(d*x + c))^2)
, x)
```

Mupad [B]
time = 0.54, size = 44, normalized size = 0.98

$$\frac{\ln\left(1-\frac{c+d\,x}{a+b\,x}\right)}{\ln\left(\frac{a+b\,x}{c+d\,x}\right)\,\left(a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(1 - (c + d*x)/(a + b*x))/(log((a + b*x)/(c + d*x))^2*(a + b*x)*(c +
 d*x)) - 1/(log((a + b*x)/(c + d*x))*(a + b*x)*(a - c + x*(b - d))),x)
```

```
[Out] log(1 - (c + d*x)/(a + b*x))/(log((a + b*x)/(c + d*x))*(a*d - b*c))
```

3.76 $\int \frac{x^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx$

Optimal. Leaf size=560

$$-\frac{anx}{2bg}+\frac{cnx}{2dg}+\frac{a^2n\log(a+bx)}{2b^2g}-\frac{nx^2\log(a+bx)}{2g}-\frac{c^2n\log(c+dx)}{2d^2g}+\frac{nx^2\log(c+dx)}{2g}+\frac{x^2\left(n\log(a+bx)-\log\right.}{}$$

```
[Out] -1/2*a*n*x/b/g+1/2*c*n*x/d/g+1/2*a^2*n*ln(b*x+a)/b^2/g-1/2*n*x^2*ln(b*x+a)/
g-1/2*c^2*n*ln(d*x+c)/d^2/g+1/2*n*x^2*ln(d*x+c)/g+1/2*x^2*(n*ln(b*x+a)-ln(e
*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/g+1/2*f*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+
c))^n)-n*ln(d*x+c))*ln(-g*x^2+f)/g^2-1/2*f*n*ln(b*x+a)*ln(b*(f^(1/2)-x*g^(1
/2))/(b*f^(1/2)+a*g^(1/2)))/g^2+1/2*f*n*ln(d*x+c)*ln(d*(f^(1/2)-x*g^(1/2))/
(d*f^(1/2)+c*g^(1/2)))/g^2-1/2*f*n*ln(b*x+a)*ln(b*(f^(1/2)+x*g^(1/2))/(b*f^
(1/2)-a*g^(1/2)))/g^2+1/2*f*n*ln(d*x+c)*ln(d*(f^(1/2)+x*g^(1/2))/(d*f^(1/2)
-c*g^(1/2)))/g^2-1/2*f*n*polylog(2,-(b*x+a)*g^(1/2)/(b*f^(1/2)-a*g^(1/2)))/
g^2-1/2*f*n*polylog(2,(b*x+a)*g^(1/2)/(b*f^(1/2)+a*g^(1/2)))/g^2+1/2*f*n*po
lylog(2,-(d*x+c)*g^(1/2)/(d*f^(1/2)-c*g^(1/2)))/g^2+1/2*f*n*polylog(2,(d*x+
c)*g^(1/2)/(d*f^(1/2)+c*g^(1/2)))/g^2
```

Rubi [A]
time = 0.52, antiderivative size = 560, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2593, 272, 45, 2463, 2442, 266, 2441, 2440, 2438}

$$-\frac{fn\text{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^2}-\frac{fn\text{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)}{2g^2}+\frac{fn\text{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^2}+\frac{fn\text{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right)}{2g^2}+\frac{a^2n\log(a+bx)}{2b^2g}+\frac{f\log\left(f-gx^2\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)}{2g^2}+\frac{x^2\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)}{2g}-\frac{fn\log(a+bx)\log\left(\frac{b\left(\sqrt{f}-\sqrt{g}x\right)}{a\sqrt{g}+b\sqrt{f}}\right)}{2g^2}-\frac{fn\log(a+bx)\log\left(\frac{b\left(\sqrt{f}+\sqrt{g}x\right)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^2}-\frac{nx^2\log(a+bx)}{2g}-\frac{anx}{2bg}-\frac{c^2n\log(c+dx)}{2d^2g}+\frac{fn\log(c+dx)\log\left(\frac{d\left(\sqrt{f}-\sqrt{g}x\right)}{c\sqrt{g}+d\sqrt{f}}\right)}{2g^2}+\frac{fn\log(c+dx)\log\left(\frac{d\left(\sqrt{f}+\sqrt{g}x\right)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^2}+\frac{nx^2\log(c+dx)}{2g}+\frac{cnx}{2dg}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Log[e*((a + b*x)/(c + d*x))^n])/(f - g*x^2),x]
```

```
[Out] -1/2*(a*n*x)/(b*g) + (c*n*x)/(2*d*g) + (a^2*n*Log[a + b*x])/(2*b^2*g) - (n*
x^2*Log[a + b*x])/(2*g) - (c^2*n*Log[c + d*x])/(2*d^2*g) + (n*x^2*Log[c + d
*x])/(2*g) + (x^2*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[
c + d*x]))/(2*g) - (f*n*Log[a + b*x]*Log[(b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[
f] + a*Sqrt[g])])/(2*g^2) + (f*n*Log[c + d*x]*Log[(d*(Sqrt[f] - Sqrt[g]*x))
/(d*Sqrt[f] + c*Sqrt[g])])/(2*g^2) - (f*n*Log[a + b*x]*Log[(b*(Sqrt[f] + Sq
rt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])])/(2*g^2) + (f*n*Log[c + d*x]*Log[(d*(Sqr
t[f] + Sqrt[g]*x))/(d*Sqrt[f] - c*Sqrt[g])])/(2*g^2) + (f*(n*Log[a + b*x] -
 Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f - g*x^2])/(2*g^2) -
 (f*n*PolyLog[2, -((Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))])/(2*g^2) -
 (f*n*PolyLog[2, (Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*g^2) + (f
*n*PolyLog[2, -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))])/(2*g^2) + (f
*n*PolyLog[2, (Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*g^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3 \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx &= n\int \frac{x^3\log(a+bx)}{f-gx^2}\,dx - n\int \frac{x^3\log(c+dx)}{f-gx^2}\,dx - \Bigg(n\log(a+bx) - \log\Bigg(e\Bigg(\frac{a+bx}{c+dx} \\
&= n\int \left(-\frac{x\log(a+bx)}{g} + \frac{fx\log(a+bx)}{g\left(f-gx^2\right)}\right)dx - n\int \Bigg(-\frac{x\log(c+dx)}{g} + \frac{fx\log(c - }{g\,(f-g} \\
&= -\frac{n\int x\log(a+bx)\,dx}{g} + \frac{n\int x\log(c+dx)\,dx}{g} + \frac{(fn)\int \frac{x\log(a+bx)}{f-gx^2}\,dx}{g} - \frac{(fn)\int \frac{x\log}{f-}}{g} \\
&= -\frac{nx^2\log(a+bx)}{2g} + \frac{nx^2\log(c+dx)}{2g} + \frac{x^2\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(\right.}{2g} \\
&= -\frac{nx^2\log(a+bx)}{2g} + \frac{nx^2\log(c+dx)}{2g} + \frac{x^2\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(\right.}{2g} \\
&= -\frac{anx}{2bg} + \frac{cnx}{2dg} + \frac{a^2n\log(a+bx)}{2b^2g} - \frac{nx^2\log(a+bx)}{2g} - \frac{c^2n\log(c+dx)}{2d^2g} + \frac{nx^2\log(c}{2g} \\
&= -\frac{anx}{2bg} + \frac{cnx}{2dg} + \frac{a^2n\log(a+bx)}{2b^2g} - \frac{nx^2\log(a+bx)}{2g} - \frac{c^2n\log(c+dx)}{2d^2g} + \frac{nx^2\log(c}{2g} \\
&= -\frac{anx}{2bg} + \frac{cnx}{2dg} + \frac{a^2n\log(a+bx)}{2b^2g} - \frac{nx^2\log(a+bx)}{2g} - \frac{c^2n\log(c+dx)}{2d^2g} + \frac{nx^2\log(c}{2g}
\end{aligned}
$$

<u>Mathematica</u> [A]
time = 0.26, size = 461, normalized size = 0.82

$$
\frac{-gx^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + \frac{gn\left(a^2d^2\log(a+bx) - b\left(d(-bc+ad)x + bc^2\log(c+dx)\right)\right)}{b^2d^2} - f\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\sqrt{f}-\sqrt{g}\,x\right) - f\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\sqrt{f}+\sqrt{g}\,x\right) + fn\left(\left(\log\left(\frac{\sqrt{g}\,(a+bx)}{b\sqrt{f}+a\sqrt{g}}\right) - \log\left(\frac{\sqrt{g}\,(c+dx)}{d\sqrt{f}+c\sqrt{g}}\right)\right)\log\left(\sqrt{f}-\sqrt{g}\,x\right) + \text{Li}_2\left(\frac{b\left(\sqrt{f}-\sqrt{g}\,x\right)}{b\sqrt{f}+a\sqrt{g}}\right) - \text{Li}_2\left(\frac{d\left(\sqrt{f}-\sqrt{g}\,x\right)}{d\sqrt{f}+c\sqrt{g}}\right)\right) + fn\left(\left(\log\left(-\frac{\sqrt{g}\,(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right) - \log\left(-\frac{\sqrt{g}\,(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)\right)\log\left(\sqrt{f}+\sqrt{g}\,x\right) + \text{Li}_2\left(\frac{b\left(\sqrt{f}+\sqrt{g}\,x\right)}{b\sqrt{f}-a\sqrt{g}}\right) - \text{Li}_2\left(\frac{d\left(\sqrt{f}+\sqrt{g}\,x\right)}{d\sqrt{f}-c\sqrt{g}}\right)\right)}{2g^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Log[e*((a + b*x)/(c + d*x))^n])/(f - g*x^2),x]
```

```
[Out] (-(g*x^2*Log[e*((a + b*x)/(c + d*x))^n]) + (g*n*(a^2*d^2*Log[a + b*x] - b*(
d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])))/(b^2*d^2) - f*Log[e*((a + b*x)/(
c + d*x))^n]*Log[Sqrt[f] - Sqrt[g]*x] - f*Log[e*((a + b*x)/(c + d*x))^n]*Lo
g[Sqrt[f] + Sqrt[g]*x] + f*n*((Log[(Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[
```

```
g])] - Log[(Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])*Log[Sqrt[f] - Sqrt
[g]*x] + PolyLog[2, (b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])] - Po
lyLog[2, (d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])]) + f*n*((Log[-(
(Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))] - Log[-((Sqrt[g]*(c + d*x))/(
d*Sqrt[f] - c*Sqrt[g]))])*Log[Sqrt[f] + Sqrt[g]*x] + PolyLog[2, (b*(Sqrt[f]
 + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])] - PolyLog[2, (d*(Sqrt[f] + Sqrt[g]*
x))/(d*Sqrt[f] - c*Sqrt[g])]))/(2*g^2)
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \ln \left(e \left(\frac{b x +a}{d x +c}\right)^{n}\right)}{-g \,x^{2}+f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)
[Out] int(x^3*ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="maxima")
[Out] -integrate(x^3*log(((b*x + a)/(d*x + c))^n*e)/(g*x^2 - f), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="fricas")
[Out] integral(-x^3*log(((b*x + a)/(d*x + c))^n*e)/(g*x^2 - f), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(e*((b*x+a)/(d*x+c))**n)/(-g*x**2+f),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="giac")
[Out] integrate(-x^3*log(((b*x + a)/(d*x + c))^n*e)/(g*x^2 - f), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{f-g\,x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*log(e*((a + b*x)/(c + d*x))^n))/(f - g*x^2),x)
[Out] int((x^3*log(e*((a + b*x)/(c + d*x))^n))/(f - g*x^2), x)
```

3.77 $\int \frac{x^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx$

Optimal. Leaf size=550

$$-\frac{n(a+bx)\log(a+bx)}{bg}+\frac{n(c+dx)\log(c+dx)}{dg}+\frac{x\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{g}-\frac{\sqrt{f}}{\cdots}$$

```
[Out] -n*(b*x+a)*ln(b*x+a)/b/g+n*(d*x+c)*ln(d*x+c)/d/g+x*(n*ln(b*x+a)-ln(e*((b*x+
a)/(d*x+c))^n)-n*ln(d*x+c))/g-arctanh(x*g^(1/2)/f^(1/2))*(n*ln(b*x+a)-ln(e*
((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))*f^(1/2)/g^(3/2)-1/2*n*ln(b*x+a)*ln(b*(f^(
1/2)-x*g^(1/2))/(b*f^(1/2)+a*g^(1/2)))*f^(1/2)/g^(3/2)+1/2*n*ln(d*x+c)*ln(d
*(f^(1/2)-x*g^(1/2))/(d*f^(1/2)+c*g^(1/2)))*f^(1/2)/g^(3/2)+1/2*n*ln(b*x+a)
*ln(b*(f^(1/2)+x*g^(1/2))/(b*f^(1/2)-a*g^(1/2)))*f^(1/2)/g^(3/2)-1/2*n*ln(d
*x+c)*ln(d*(f^(1/2)+x*g^(1/2))/(d*f^(1/2)-c*g^(1/2)))*f^(1/2)/g^(3/2)+1/2*n
*polylog(2,-(b*x+a)*g^(1/2)/(b*f^(1/2)-a*g^(1/2)))*f^(1/2)/g^(3/2)-1/2*n*po
lylog(2,(b*x+a)*g^(1/2)/(b*f^(1/2)+a*g^(1/2)))*f^(1/2)/g^(3/2)-1/2*n*polylo
g(2,-(d*x+c)*g^(1/2)/(d*f^(1/2)-c*g^(1/2)))*f^(1/2)/g^(3/2)+1/2*n*polylog(2
,(d*x+c)*g^(1/2)/(d*f^(1/2)+c*g^(1/2)))*f^(1/2)/g^(3/2)
```

Rubi [A]
time = 0.41, antiderivative size = 550, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2593, 327, 214, 2463, 2436, 2332, 2456, 2441, 2440, 2438}

$$\frac{\sqrt{f}\, n\text{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^{3/2}}-\frac{\sqrt{f}\, n\text{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)}{2g^{3/2}}-\frac{\sqrt{f}\, n\text{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^{3/2}}+\frac{\sqrt{f}\, n\text{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right)}{2g^{3/2}}-\frac{\sqrt{f}\tanh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)}{g^{3/2}}+\frac{x\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)}{g}-\frac{\sqrt{f}\, n\log(a+bx)\log\left(\frac{b\left(\sqrt{f}-\sqrt{g}x\right)}{a\sqrt{g}+b\sqrt{f}}\right)}{2g^{3/2}}+\frac{\sqrt{f}\, n\log(a+bx)\log\left(\frac{b\left(\sqrt{f}+\sqrt{g}x\right)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^{3/2}}-\frac{n(a+bx)\log(a+bx)}{bg}+\frac{\sqrt{f}\, n\log(c+dx)\log\left(\frac{d\left(\sqrt{f}-\sqrt{g}x\right)}{c\sqrt{g}+d\sqrt{f}}\right)}{2g^{3/2}}-\frac{\sqrt{f}\, n\log(c+dx)\log\left(\frac{d\left(\sqrt{f}+\sqrt{g}x\right)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^{3/2}}+\frac{n(c+dx)\log(c+dx)}{dg}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Log[e*((a + b*x)/(c + d*x))^n])/(f - g*x^2),x]
```

```
[Out] -((n*(a + b*x)*Log[a + b*x])/(b*g)) + (n*(c + d*x)*Log[c + d*x])/(d*g) + (x
*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/g - (S
qrt[f]*ArcTanh[(Sqrt[g]*x)/Sqrt[f]]*(n*Log[a + b*x] - Log[e*((a + b*x)/(c +
 d*x))^n] - n*Log[c + d*x]))/g^(3/2) - (Sqrt[f]*n*Log[a + b*x]*Log[(b*(Sqrt
[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*g^(3/2)) + (Sqrt[f]*n*Log[c
+ d*x]*Log[(d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*g^(3/2))
+ (Sqrt[f]*n*Log[a + b*x]*Log[(b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt
[g])])/(2*g^(3/2)) - (Sqrt[f]*n*Log[c + d*x]*Log[(d*(Sqrt[f] + Sqrt[g]*x))/
(d*Sqrt[f] - c*Sqrt[g])])/(2*g^(3/2)) + (Sqrt[f]*n*PolyLog[2, -((Sqrt[g]*(a
 + b*x))/(b*Sqrt[f] - a*Sqrt[g]))])/(2*g^(3/2)) - (Sqrt[f]*n*PolyLog[2, (Sq
rt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*g^(3/2)) - (Sqrt[f]*n*PolyLog
[2, -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))])/(2*g^(3/2)) + (Sqrt[f]
*n*PolyLog[2, (Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*g^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
```

```
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\int \frac{x^2 \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = n\int \frac{x^2\log(a+bx)}{f-gx^2}\,dx - n\int \frac{x^2\log(c+dx)}{f-gx^2}\,dx - \Big(n\log(a+bx) - \log\Big(e\Big(\frac{a+bx}{c+dx}$$

$$= \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{g} + n\int\Big(-\frac{\log(a+bx)}{g} + \frac{f\log(}{g\,(f}$$

$$= \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{g} - \frac{\sqrt{f}\,\tanh^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)(n\log}{}$$

$$= \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{g} - \frac{\sqrt{f}\,\tanh^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)(n\log}{}$$

$$= -\frac{n(a+bx)\log(a+bx)}{bg} + \frac{n(c+dx)\log(c+dx)}{dg} + \frac{x\Big(n\log(a+bx) - \log\Big(e\Big(\frac{a+bx}{c+dx}}{g}$$

$$= -\frac{n(a+bx)\log(a+bx)}{bg} + \frac{n(c+dx)\log(c+dx)}{dg} + \frac{x\Big(n\log(a+bx) - \log\Big(e\Big(\frac{a+bx}{c+dx}}{g}$$

$$= -\frac{n(a+bx)\log(a+bx)}{bg} + \frac{n(c+dx)\log(c+dx)}{dg} + \frac{x\Big(n\log(a+bx) - \log\Big(e\Big(\frac{a+bx}{c+dx}}{g}$$

$$= -\frac{n(a+bx)\log(a+bx)}{bg} + \frac{n(c+dx)\log(c+dx)}{dg} + \frac{x\Big(n\log(a+bx) - \log\Big(e\Big(\frac{a+bx}{c+dx}}{g}$$

<u>Mathematica</u> **[A]**
time = 0.18, size = 467, normalized size = 0.85

$$\frac{-\frac{2\sqrt{g}(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} + \frac{2(bc-ad)\sqrt{g}\,n\log(c+dx)}{bd} - \sqrt{f}\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\sqrt{f}-\sqrt{g}\,x\right) + \sqrt{f}\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\sqrt{f}+\sqrt{g}\,x\right) + \sqrt{f}\,n\left(\left(\log\left(\frac{\sqrt{g}(a+bx)}{b\sqrt{f}+a\sqrt{g}}\right) - \log\left(\frac{\sqrt{g}(c+dx)}{d\sqrt{f}+c\sqrt{g}}\right)\right)\log\left(\sqrt{f}-\sqrt{g}\,x\right) + \mathrm{Li}_2\left(\frac{b\left(\sqrt{f}-\sqrt{g}\,x\right)}{b\sqrt{f}+a\sqrt{g}}\right) - \mathrm{Li}_2\left(\frac{d\left(\sqrt{f}-\sqrt{g}\,x\right)}{d\sqrt{f}+c\sqrt{g}}\right)\right) - \sqrt{f}\,n\left(\left(\log\left(-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right) - \log\left(-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)\right)\log\left(\sqrt{f}+\sqrt{g}\,x\right) + \mathrm{Li}_2\left(\frac{b\left(\sqrt{f}+\sqrt{g}\,x\right)}{b\sqrt{f}-a\sqrt{g}}\right) - \mathrm{Li}_2\left(\frac{d\left(\sqrt{f}+\sqrt{g}\,x\right)}{d\sqrt{f}-c\sqrt{g}}\right)\right)}{2g^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Log[e*((a + b*x)/(c + d*x))^n])/(f - g*x^2),x]
```

```
[Out] ((-2*Sqrt[g]*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b + (2*(b*c - a*d)*S
qrt[g]*n*Log[c + d*x])/(b*d) - Sqrt[f]*Log[e*((a + b*x)/(c + d*x))^n]*Log[S
```

```
qrt[f] - Sqrt[g]*x] + Sqrt[f]*Log[e*((a + b*x)/(c + d*x))^n]*Log[Sqrt[f] +
Sqrt[g]*x] + Sqrt[f]*n*((Log[(Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])] -
 Log[(Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])*Log[Sqrt[f] - Sqrt[g]*x]
 + PolyLog[2, (b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])] - PolyLog[
2, (d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])]) - Sqrt[f]*n*((Log[-(
(Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))] - Log[-((Sqrt[g]*(c + d*x))/(
d*Sqrt[f] - c*Sqrt[g]))])*Log[Sqrt[f] + Sqrt[g]*x] + PolyLog[2, (b*(Sqrt[f]
 + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])] - PolyLog[2, (d*(Sqrt[f] + Sqrt[g]*
x))/(d*Sqrt[f] - c*Sqrt[g])]))/(2*g^(3/2))
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{x^2 \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{-g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)
```

```
[Out] int(x^2*ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1048 vs. 2(440) = 880.
time = 0.61, size = 1048, normalized size = 1.91

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*c*(c^2/((b*c*d^3 - a*d^4)*g*x + (b*c^2*d^2 - a*c*d^3)*g) + a^2*lo
g(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g) + (b*c^2 - 2*a*c*d)*log(
d*x + c)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*g))*d - 2*(c^3/((b*c*d^4 -
a*d^5)*g*x + (b*c^2*d^3 - a*c*d^4)*g) + a^3*log(b*x + a)/((b^4*c^2 - 2*a*b^
3*c*d + a^2*b^2*d^2)*g) + (2*b*c^3 - 3*a*c^2*d)*log(d*x + c)/((b^2*c^2*d^3
- 2*a*b*c*d^4 + a^2*d^5)*g) - x/(b*d^2*g))*b*d^2 + 2*a*(c^2/((b*c*d^3 - a*d
^4)*g*x + (b*c^2*d^2 - a*c*d^3)*g) + a^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c
*d + a^2*b*d^2)*g) + (b*c^2 - 2*a*c*d)*log(d*x + c)/((b^2*c^2*d^2 - 2*a*b*c
*d^3 + a^2*d^4)*g))*d^2 - 2*a*c*d*(c/((b*c*d^2 - a*d^3)*g*x + (b*c^2*d - a*
c*d^2)*g) + a*log(b*x + a)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g) - a*log(d*x
+ c)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g)) - 2*b*d*(a^2*log(b*x + a)/((b^3*c
 - a*b^2*d)*g) - c^2*log(d*x + c)/((b*c*d^2 - a*d^3)*g) + x/(b*d*g)) + 2*b*
c*(a*log(b*x + a)/((b^2*c - a*b*d)*g) - c*log(d*x + c)/((b*c*d - a*d^2)*g))
 - (log(sqrt(g)*x - sqrt(f))*log((b*sqrt(g)*x - b*sqrt(f))/(b*sqrt(f) + a*s
qrt(g)) + 1) + dilog(-(b*sqrt(g)*x - b*sqrt(f))/(b*sqrt(f) + a*sqrt(g))))*s
qrt(f)/g^(3/2) + (log(sqrt(g)*x + sqrt(f))*log(-(b*sqrt(g)*x + b*sqrt(f))/(
```

```
b*sqrt(f) - a*sqrt(g)) + 1) + dilog((b*sqrt(g)*x + b*sqrt(f))/(b*sqrt(f) -
a*sqrt(g))))*sqrt(f)/g^(3/2) + (log(sqrt(g)*x - sqrt(f))*log((d*sqrt(g)*x -
 d*sqrt(f))/(d*sqrt(f) + c*sqrt(g)) + 1) + dilog(-(d*sqrt(g)*x - d*sqrt(f))
/(d*sqrt(f) + c*sqrt(g))))*sqrt(f)/g^(3/2) - (log(sqrt(g)*x + sqrt(f))*log(
-(d*sqrt(g)*x + d*sqrt(f))/(d*sqrt(f) - c*sqrt(g)) + 1) + dilog((d*sqrt(g)*
x + d*sqrt(f))/(d*sqrt(f) - c*sqrt(g))))*sqrt(f)/g^(3/2))*n - 1/2*(f*log((g
*x - sqrt(f*g))/(g*x + sqrt(f*g)))/(sqrt(f*g)*g) + 2*x/g)*log(((b*x + a)/(d
*x + c))^n*e)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="fricas")
[Out] integral(-x^2*log(((b*x + a)/(d*x + c))^n*e)/(g*x^2 - f), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(e*((b*x+a)/(d*x+c))**n)/(-g*x**2+f),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="giac")
[Out] integrate(-x^2*log(((b*x + a)/(d*x + c))^n*e)/(g*x^2 - f), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{f-g\,x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*log(e*((a + b*x)/(c + d*x))^n))/(f - g*x^2),x)
[Out] int((x^2*log(e*((a + b*x)/(c + d*x))^n))/(f - g*x^2), x)
```

3.78 $\int \frac{x \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\, dx$

Optimal. Leaf size=403

$$-\frac{n\log(a+bx)\log\left(\frac{b\left(\sqrt{f}-\sqrt{g}\,x\right)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g}+\frac{n\log(c+dx)\log\left(\frac{d\left(\sqrt{f}-\sqrt{g}\,x\right)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g}-\frac{n\log(a+bx)\log\left(\frac{b\left(\sqrt{f}+\sqrt{g}\,x\right)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g}$$

```
[Out] 1/2*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))*ln(-g*x^2+f)/g-1/2*
n*ln(b*x+a)*ln(b*(f^(1/2)-x*g^(1/2))/(b*f^(1/2)+a*g^(1/2)))/g+1/2*n*ln(d*x+
c)*ln(d*(f^(1/2)-x*g^(1/2))/(d*f^(1/2)+c*g^(1/2)))/g-1/2*n*ln(b*x+a)*ln(b*(
f^(1/2)+x*g^(1/2))/(b*f^(1/2)-a*g^(1/2)))/g+1/2*n*ln(d*x+c)*ln(d*(f^(1/2)+x
*g^(1/2))/(d*f^(1/2)-c*g^(1/2)))/g-1/2*n*polylog(2,-(b*x+a)*g^(1/2)/(b*f^(1
/2)-a*g^(1/2)))/g-1/2*n*polylog(2,(b*x+a)*g^(1/2)/(b*f^(1/2)+a*g^(1/2)))/g+
1/2*n*polylog(2,-(d*x+c)*g^(1/2)/(d*f^(1/2)-c*g^(1/2)))/g+1/2*n*polylog(2,(
d*x+c)*g^(1/2)/(d*f^(1/2)+c*g^(1/2)))/g
```

Rubi [A]
time = 0.26, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2593, 266, 2463, 2441, 2440, 2438}

$$-\frac{n\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g}-\frac{n\,\text{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)}{2g}+\frac{n\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g}+\frac{n\,\text{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right)}{2g}+\frac{\log\left(f-gx^2\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)}{2g}-\frac{n\log(a+bx)\log\left(\frac{b\left(\sqrt{f}-\sqrt{g}\,x\right)}{a\sqrt{g}+b\sqrt{f}}\right)}{2g}-\frac{n\log(a+bx)\log\left(\frac{b\left(\sqrt{f}+\sqrt{g}\,x\right)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g}+\frac{n\log(c+dx)\log\left(\frac{d\left(\sqrt{f}-\sqrt{g}\,x\right)}{c\sqrt{g}+d\sqrt{f}}\right)}{2g}+\frac{n\log(c+dx)\log\left(\frac{d\left(\sqrt{f}+\sqrt{g}\,x\right)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[e*((a + b*x)/(c + d*x))^n])/(f - g*x^2),x]
```

```
[Out] -1/2*(n*Log[a + b*x]*Log[(b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])]
)/g + (n*Log[c + d*x]*Log[(d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])
])/(2*g) - (n*Log[a + b*x]*Log[(b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqr
t[g])])/(2*g) + (n*Log[c + d*x]*Log[(d*(Sqrt[f] + Sqrt[g]*x))/(d*Sqrt[f] -
c*Sqrt[g])])/(2*g) + ((n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*
Log[c + d*x])*Log[f - g*x^2])/(2*g) - (n*PolyLog[2, -((Sqrt[g]*(a + b*x))/(
b*Sqrt[f] - a*Sqrt[g]))])/(2*g) - (n*PolyLog[2, (Sqrt[g]*(a + b*x))/(b*Sqrt
[f] + a*Sqrt[g])])/(2*g) + (n*PolyLog[2, -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] -
 c*Sqrt[g]))])/(2*g) + (n*PolyLog[2, (Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqr
t[g])])/(2*g)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\int \frac{x \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = n\int \frac{x\log(a+bx)}{f-gx^2}\,dx - n\int \frac{x\log(c+dx)}{f-gx^2}\,dx - \left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.$$

$$= \frac{\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\log\left(f-gx^2\right)}{2g} + n\int \left(\frac{\log(}{2\sqrt{g}\left(\sqrt{\right.}}\right.$$

$$= \frac{\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\log\left(f-gx^2\right)}{2g} + \frac{n\int \frac{\log(a+bx)}{\sqrt{f}-\sqrt{g}\,x}}{2\sqrt{g}}$$

$$= -\frac{n\log(a+bx)\log\left(\frac{b\left(\sqrt{f}-\sqrt{g}\,x\right)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g} + \frac{n\log(c+dx)\log\left(\frac{d\left(\sqrt{f}-\sqrt{g}\,x\right)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g} - $$

$$= -\frac{n\log(a+bx)\log\left(\frac{b\left(\sqrt{f}-\sqrt{g}\,x\right)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g} + \frac{n\log(c+dx)\log\left(\frac{d\left(\sqrt{f}-\sqrt{g}\,x\right)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g} - $$

$$= -\frac{n\log(a+bx)\log\left(\frac{b\left(\sqrt{f}-\sqrt{g}\,x\right)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g} + \frac{n\log(c+dx)\log\left(\frac{d\left(\sqrt{f}-\sqrt{g}\,x\right)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g} - $$

Mathematica [A]
time = 0.09, size = 413, normalized size = 1.02

$$-\frac{-n\log\left(\frac{\sqrt{g}(a+bx)}{b\sqrt{f}+a\sqrt{g}}\right)\log\left(\sqrt{f}-\sqrt{g}\,x\right)+\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\sqrt{f}-\sqrt{g}\,x\right)+n\log\left(\frac{\sqrt{g}(c+dx)}{d\sqrt{f}+c\sqrt{g}}\right)\log\left(\sqrt{f}-\sqrt{g}\,x\right)-n\log\left(-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)\log\left(\sqrt{f}+\sqrt{g}\,x\right)+\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\sqrt{f}+\sqrt{g}\,x\right)+n\log\left(-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)\log\left(\sqrt{f}+\sqrt{g}\,x\right)-n\mathrm{Li}_2\left(\frac{b\left(\sqrt{f}-\sqrt{g}\,x\right)}{b\sqrt{f}+a\sqrt{g}}\right)+n\mathrm{Li}_2\left(\frac{d\left(\sqrt{f}-\sqrt{g}\,x\right)}{d\sqrt{f}+c\sqrt{g}}\right)-n\mathrm{Li}_2\left(\frac{b\left(\sqrt{f}+\sqrt{g}\,x\right)}{b\sqrt{f}-a\sqrt{g}}\right)+n\mathrm{Li}_2\left(\frac{d\left(\sqrt{f}+\sqrt{g}\,x\right)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[e*((a + b*x)/(c + d*x))^n])/(f - g*x^2),x]
```

```
[Out] -1/2*(-(n*Log[(Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])]*Log[Sqrt[f] - Sq
rt[g]*x]) + Log[e*((a + b*x)/(c + d*x))^n]*Log[Sqrt[f] - Sqrt[g]*x] + n*Log
[(Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])]*Log[Sqrt[f] - Sqrt[g]*x] - n*
Log[-((Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))]*Log[Sqrt[f] + Sqrt[g]*x
] + Log[e*((a + b*x)/(c + d*x))^n]*Log[Sqrt[f] + Sqrt[g]*x] + n*Log[-((Sqrt
[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))]*Log[Sqrt[f] + Sqrt[g]*x] - n*PolyL
og[2, (b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])] + n*PolyLog[2, (d*
(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])] - n*PolyLog[2, (b*(Sqrt[f]
+ Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])] + n*PolyLog[2, (d*(Sqrt[f] + Sqrt[g]
*x))/(d*Sqrt[f] - c*Sqrt[g])])/g
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{x \ln \left(e \left(\frac{bx+a}{dx+c}\right)^{n}\right)}{-g\,x^{2}+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)
```

```
[Out] int(x*ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="maxima")
```

```
[Out] -integrate(x*log(((b*x + a)/(d*x + c))^n*e)/(g*x^2 - f), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(-x*log(((b*x + a)/(d*x + c))^n*e)/(g*x^2 - f), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(e*((b*x+a)/(d*x+c))**n)/(-g*x**2+f),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="giac")
[Out] integrate(-x*log(((b*x + a)/(d*x + c))^n*e)/(g*x^2 - f), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{f-g\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(e*((a + b*x)/(c + d*x))^n))/(f - g*x^2),x)
[Out] int((x*log(e*((a + b*x)/(c + d*x))^n))/(f - g*x^2), x)
```

3.79 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx$

Optimal. Leaf size=291

$$\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{\left(d\sqrt{f}-c\sqrt{g}\right)(a+bx)}{\left(b\sqrt{f}-a\sqrt{g}\right)(c+dx)}\right)}{2\sqrt{f}\sqrt{g}}-\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{\left(d\sqrt{f}+c\sqrt{g}\right)(a+bx)}{\left(b\sqrt{f}+a\sqrt{g}\right)(c+dx)}\right)}{2\sqrt{f}\sqrt{g}}+\frac{n\mathrm{Li}_2\left(\left(d\sqrt{\right.\right.}{\left(b\sqrt{\right.}}{2}$$

```
[Out] 1/2*ln(e*((b*x+a)/(d*x+c))^n)*ln(1-(b*x+a)*(d*f^(1/2)-c*g^(1/2))/(d*x+c)/(b
*f^(1/2)-a*g^(1/2)))/f^(1/2)/g^(1/2)-1/2*ln(e*((b*x+a)/(d*x+c))^n)*ln(1-(b*
x+a)*(d*f^(1/2)+c*g^(1/2))/(d*x+c)/(b*f^(1/2)+a*g^(1/2)))/f^(1/2)/g^(1/2)+1
/2*n*polylog(2,(b*x+a)*(d*f^(1/2)-c*g^(1/2))/(d*x+c)/(b*f^(1/2)-a*g^(1/2)))
/f^(1/2)/g^(1/2)-1/2*n*polylog(2,(b*x+a)*(d*f^(1/2)+c*g^(1/2))/(d*x+c)/(b*f
^(1/2)+a*g^(1/2)))/f^(1/2)/g^(1/2)
```

Rubi [A]

time = 0.24, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2576, 2404, 2354, 2438}

$$\frac{n\text{PolyLog}\left(2,\frac{(a+bx)\left(d\sqrt{f}-c\sqrt{g}\right)}{(c+dx)\left(b\sqrt{f}-a\sqrt{g}\right)}\right)}{2\sqrt{f}\sqrt{g}}-\frac{n\text{PolyLog}\left(2,\frac{(a+bx)\left(c\sqrt{g}+d\sqrt{f}\right)}{(c+dx)\left(a\sqrt{g}+b\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}+\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(a+bx)\left(d\sqrt{f}-c\sqrt{g}\right)}{(c+dx)\left(b\sqrt{f}-a\sqrt{g}\right)}\right)}{2\sqrt{f}\sqrt{g}}-\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(a+bx)\left(c\sqrt{g}+d\sqrt{f}\right)}{(c+dx)\left(a\sqrt{g}+b\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*((a + b*x)/(c + d*x))^n]/(f - g*x^2),x]
```

```
[Out] (Log[e*((a + b*x)/(c + d*x))^n]*Log[1 - ((d*Sqrt[f] - c*Sqrt[g])*(a + b*x))
/((b*Sqrt[f] - a*Sqrt[g])*(c + d*x))])/(2*Sqrt[f]*Sqrt[g]) - (Log[e*((a + b
*x)/(c + d*x))^n]*Log[1 - ((d*Sqrt[f] + c*Sqrt[g])*(a + b*x))/((b*Sqrt[f] +
 a*Sqrt[g])*(c + d*x))])/(2*Sqrt[f]*Sqrt[g]) + (n*PolyLog[2, ((d*Sqrt[f] -
c*Sqrt[g])*(a + b*x))/((b*Sqrt[f] - a*Sqrt[g])*(c + d*x))])/(2*Sqrt[f]*Sqrt
[g]) - (n*PolyLog[2, ((d*Sqrt[f] + c*Sqrt[g])*(a + b*x))/((b*Sqrt[f] + a*Sq
rt[g])*(c + d*x))])/(2*Sqrt[f]*Sqrt[g])
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
```

```
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2576

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g = Coef
f[P2x, x, 1], h = Coeff[P2x, x, 2]}, Dist[b*c - a*d, Subst[Int[(b^2*f - a*b
*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g + c^2*h
)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b*x)/(c
+ d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x, 2] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = n\int \frac{\log(a+bx)}{f-gx^2}\,dx - n\int \frac{\log(c+dx)}{f-gx^2}\,dx - \left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) -$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{\sqrt{f}\,\sqrt{g}} + n\int \left(\frac{\text{lo}}{2\sqrt{f}}\right.$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{\sqrt{f}\,\sqrt{g}} + \frac{n\int \frac{\log(a+bx)}{\sqrt{f}-\sqrt{g}}}{2\sqrt{f}}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{\sqrt{f}\,\sqrt{g}} - \frac{n\log(a+bx)}{2}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{\sqrt{f}\,\sqrt{g}} - \frac{n\log(a+bx)}{2}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{\sqrt{f}\,\sqrt{g}} - \frac{n\log(a+bx)}{2}$$

<u>Mathematica</u> **[A]**
time = 0.08, size = 421, normalized size = 1.45

$$\frac{n\log\left(\frac{\sqrt{g}(a+bx)}{b\sqrt{f}+a\sqrt{g}}\right)\log\left(\sqrt{f}-\sqrt{g}\,x\right) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\sqrt{f}-\sqrt{g}\,x\right) - n\log\left(\frac{\sqrt{g}(c+dx)}{d\sqrt{f}+c\sqrt{g}}\right)\log\left(\sqrt{f}-\sqrt{g}\,x\right) - n\log\left(-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)\log\left(\sqrt{f}+\sqrt{g}\,x\right) + \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\sqrt{f}+\sqrt{g}\,x\right) + n\log\left(-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)\log\left(\sqrt{f}+\sqrt{g}\,x\right) + n\text{Li}_2\left(\frac{b\left(\sqrt{f}-\sqrt{g}\,x\right)}{b\sqrt{f}+a\sqrt{g}}\right) - n\text{Li}_2\left(\frac{d\left(\sqrt{f}-\sqrt{g}\,x\right)}{d\sqrt{f}+c\sqrt{g}}\right) - n\text{Li}_2\left(\frac{b\left(\sqrt{f}+\sqrt{g}\,x\right)}{b\sqrt{f}-a\sqrt{g}}\right) + n\text{Li}_2\left(\frac{d\left(\sqrt{f}+\sqrt{g}\,x\right)}{d\sqrt{f}-c\sqrt{g}}\right)}{2\sqrt{f}\,\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*((a + b*x)/(c + d*x))^n]/(f - g*x^2),x]
```

```
[Out] (n*Log[(Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])]*Log[Sqrt[f] - Sqrt[g]*x
] - Log[e*((a + b*x)/(c + d*x))^n]*Log[Sqrt[f] - Sqrt[g]*x] - n*Log[(Sqrt[g
]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])]*Log[Sqrt[f] - Sqrt[g]*x] - n*Log[-((S
qrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))]*Log[Sqrt[f] + Sqrt[g]*x] + Log[
e*((a + b*x)/(c + d*x))^n]*Log[Sqrt[f] + Sqrt[g]*x] + n*Log[-((Sqrt[g]*(c +
 d*x))/(d*Sqrt[f] - c*Sqrt[g]))]*Log[Sqrt[f] + Sqrt[g]*x] + n*PolyLog[2, (b
*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])] - n*PolyLog[2, (d*(Sqrt[f]
```

```
 - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])] - n*PolyLog[2, (b*(Sqrt[f] + Sqrt[g
]*x))/(b*Sqrt[f] - a*Sqrt[g])] + n*PolyLog[2, (d*(Sqrt[f] + Sqrt[g]*x))/(d*
Sqrt[f] - c*Sqrt[g])])/(2*Sqrt[f]*Sqrt[g])
```

Maple [**F**]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{-g\,x^{2}+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)
```

Maxima [**A**]
time = 0.55, size = 350, normalized size = 1.20

$$\frac{\left(\log\left(\sqrt{g}\,x-\sqrt{f}\right)\log\left(\frac{b\sqrt{g}\,x-b\sqrt{f}}{b\sqrt{f}+a\sqrt{g}}+1\right)-\log\left(\sqrt{g}\,x+\sqrt{f}\right)\log\left(-\frac{b\sqrt{g}\,x+b\sqrt{f}}{b\sqrt{f}-a\sqrt{g}}+1\right)-\log\left(\sqrt{g}\,x-\sqrt{f}\right)\log\left(\frac{d\sqrt{g}\,x-d\sqrt{f}}{d\sqrt{f}+c\sqrt{g}}+1\right)+\log\left(\sqrt{g}\,x+\sqrt{f}\right)\log\left(-\frac{d\sqrt{g}\,x+d\sqrt{f}}{d\sqrt{f}-c\sqrt{g}}+1\right)+\mathrm{Li}_2\left(-\frac{b\sqrt{g}\,x-b\sqrt{f}}{b\sqrt{f}+a\sqrt{g}}\right)-\mathrm{Li}_2\left(\frac{b\sqrt{g}\,x+b\sqrt{f}}{b\sqrt{f}-a\sqrt{g}}\right)-\mathrm{Li}_2\left(-\frac{d\sqrt{g}\,x-d\sqrt{f}}{d\sqrt{f}+c\sqrt{g}}\right)+\mathrm{Li}_2\left(\frac{d\sqrt{g}\,x+d\sqrt{f}}{d\sqrt{f}-c\sqrt{g}}\right)\right)n}{2\sqrt{fg}}-\frac{\log\left(\left(\frac{bx+a}{dx+c}\right)^{n}e\right)\log\left(\frac{gx-\sqrt{fg}}{gx+\sqrt{fg}}\right)}{2\sqrt{fg}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="maxima")
```

```
[Out] 1/2*(log(sqrt(g)*x - sqrt(f))*log((b*sqrt(g)*x - b*sqrt(f))/(b*sqrt(f) + a*
sqrt(g)) + 1) - log(sqrt(g)*x + sqrt(f))*log(-(b*sqrt(g)*x + b*sqrt(f))/(b*
sqrt(f) - a*sqrt(g)) + 1) - log(sqrt(g)*x - sqrt(f))*log((d*sqrt(g)*x - d*s
qrt(f))/(d*sqrt(f) + c*sqrt(g)) + 1) + log(sqrt(g)*x + sqrt(f))*log(-(d*sqr
t(g)*x + d*sqrt(f))/(d*sqrt(f) - c*sqrt(g)) + 1) + dilog(-(b*sqrt(g)*x - b*
sqrt(f))/(b*sqrt(f) + a*sqrt(g))) - dilog((b*sqrt(g)*x + b*sqrt(f))/(b*sqrt
(f) - a*sqrt(g))) - dilog(-(d*sqrt(g)*x - d*sqrt(f))/(d*sqrt(f) + c*sqrt(g)
)) + dilog((d*sqrt(g)*x + d*sqrt(f))/(d*sqrt(f) - c*sqrt(g))))*n/sqrt(f*g)
- 1/2*log(((b*x + a)/(d*x + c))^n*e)*log((g*x - sqrt(f*g))/(g*x + sqrt(f*g)
))/sqrt(f*g)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(-log(((b*x + a)/(d*x + c))^n*e)/(g*x^2 - f), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)/(-g*x**2+f),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(-log(((b*x + a)/(d*x + c))^n*e)/(g*x^2 - f), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{f-g\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*((a + b*x)/(c + d*x))^n)/(f - g*x^2),x)
```

```
[Out] int(log(e*((a + b*x)/(c + d*x))^n)/(f - g*x^2), x)
```

3.80 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x(f-gx^2)}\,dx$

Optimal. Leaf size=518

$$\frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f}-\frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f}-\frac{\log(x)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{f}$$

```
[Out] n*ln(-b*x/a)*ln(b*x+a)/f-n*ln(-d*x/c)*ln(d*x+c)/f-ln(x)*(n*ln(b*x+a)-ln(e*(
(b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/f+1/2*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^
n)-n*ln(d*x+c))*ln(-g*x^2+f)/f-1/2*n*ln(b*x+a)*ln(b*(f^(1/2)-x*g^(1/2))/(b*
f^(1/2)+a*g^(1/2)))/f+1/2*n*ln(d*x+c)*ln(d*(f^(1/2)-x*g^(1/2))/(d*f^(1/2)+c
*g^(1/2)))/f-1/2*n*ln(b*x+a)*ln(b*(f^(1/2)+x*g^(1/2))/(b*f^(1/2)-a*g^(1/2))
)/f+1/2*n*ln(d*x+c)*ln(d*(f^(1/2)+x*g^(1/2))/(d*f^(1/2)-c*g^(1/2)))/f+n*pol
ylog(2,1+b*x/a)/f-n*polylog(2,1+d*x/c)/f-1/2*n*polylog(2,-(b*x+a)*g^(1/2)/(
b*f^(1/2)-a*g^(1/2)))/f-1/2*n*polylog(2,(b*x+a)*g^(1/2)/(b*f^(1/2)+a*g^(1/2
)))/f+1/2*n*polylog(2,-(d*x+c)*g^(1/2)/(d*f^(1/2)-c*g^(1/2)))/f+1/2*n*polyl
og(2,(d*x+c)*g^(1/2)/(d*f^(1/2)+c*g^(1/2)))/f
```

Rubi [A]
time = 0.42, antiderivative size = 518, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 11, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2593, 272, 36, 29, 31, 2463, 2441, 2352, 266, 2440, 2438}

$$-\frac{n\text{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f}-\frac{n\text{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)}{2f}+\frac{n\text{PolyLog}\left(2,\frac{bx}{a}+1\right)}{f}+\frac{n\text{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f}+\frac{n\text{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right)}{2f}-\frac{n\text{PolyLog}\left(2,\frac{dx}{c}+1\right)}{f}+\frac{\log\left(f-gx^2\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)}{2f}-\frac{\log(x)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)}{f}-\frac{n\log(a+bx)\log\left(\frac{b\left(\sqrt{f}-\sqrt{g}x\right)}{a\sqrt{g}+b\sqrt{f}}\right)}{2f}-\frac{n\log(a+bx)\log\left(\frac{b\left(\sqrt{f}+\sqrt{g}x\right)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f}+\frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f}+\frac{n\log(c+dx)\log\left(\frac{d\left(\sqrt{f}-\sqrt{g}x\right)}{c\sqrt{g}+d\sqrt{f}}\right)}{2f}+\frac{n\log(c+dx)\log\left(\frac{d\left(\sqrt{f}+\sqrt{g}x\right)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f}-\frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*((a + b*x)/(c + d*x))^n]/(x*(f - g*x^2)),x]
```

```
[Out] (n*Log[-((b*x)/a)]*Log[a + b*x])/f - (n*Log[-((d*x)/c)]*Log[c + d*x])/f - (
Log[x]*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/
f - (n*Log[a + b*x]*Log[(b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])])
/(2*f) + (n*Log[c + d*x]*Log[(d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[
g])])/(2*f) - (n*Log[a + b*x]*Log[(b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*
Sqrt[g])])/(2*f) + (n*Log[c + d*x]*Log[(d*(Sqrt[f] + Sqrt[g]*x))/(d*Sqrt[f]
 - c*Sqrt[g])])/(2*f) + ((n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] -
 n*Log[c + d*x])*Log[f - g*x^2])/(2*f) - (n*PolyLog[2, -((Sqrt[g]*(a + b*x)
)/(b*Sqrt[f] - a*Sqrt[g]))])/(2*f) - (n*PolyLog[2, (Sqrt[g]*(a + b*x))/(b*S
qrt[f] + a*Sqrt[g])])/(2*f) + (n*PolyLog[2, 1 + (b*x)/a])/f + (n*PolyLog[2,
 -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))])/(2*f) + (n*PolyLog[2, (Sq
rt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*f) - (n*PolyLog[2, 1 + (d*x)/
c])/f
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
```

```
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\,(f-gx^2)}\,dx = n\int \frac{\log(a+bx)}{x\,(f-gx^2)}\,dx - n\int \frac{\log(c+dx)}{x\,(f-gx^2)}\,dx - \left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - \right.$$

$$= n\int \left(\frac{\log(a+bx)}{fx} - \frac{gx\log(a+bx)}{f\,(-f+gx^2)}\right)\,dx - n\int \left(\frac{\log(c+dx)}{fx} - \frac{gx\log(c+dx)}{f\,(-f+gx^2)}\right)\,dx$$

$$= \frac{n\int \frac{\log(a+bx)}{x}\,dx}{f} - \frac{n\int \frac{\log(c+dx)}{x}\,dx}{f} - \frac{(gn)\int \frac{x\log(a+bx)}{-f+gx^2}\,dx}{f} + \frac{(gn)\int \frac{x\log(c+dx)}{-f+gx^2}\,dx}{f} - \frac{(n\,l}{}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{\log(x)\left(n\log(a+bx) - \log\left(e\right.\right.}{f}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{\log(x)\left(n\log(a+bx) - \log\left(e\right.\right.}{f}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{\log(x)\left(n\log(a+bx) - \log\left(e\right.\right.}{f}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{\log(x)\left(n\log(a+bx) - \log\left(e\right.\right.}{f}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{\log(x)\left(n\log(a+bx) - \log\left(e\right.\right.}{f}$$

<u>Mathematica</u> [A]
time = 0.14, size = 487, normalized size = 0.94

$$-\frac{2n\log(x)\log\left(1+\frac{bx}{a}\right) - 2\log(x)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - 2n\log(x)\log\left(1+\frac{dx}{c}\right) - n\log\left(\frac{\sqrt{g}\,(a+bx)}{b\sqrt{f}+a\sqrt{g}}\right)\log\left(\sqrt{f}-\sqrt{g}\,x\right) + \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\sqrt{f}-\sqrt{g}\,x\right) + n\log\left(\frac{\sqrt{g}\,(c+dx)}{d\sqrt{f}+c\sqrt{g}}\right)\log\left(\sqrt{f}-\sqrt{g}\,x\right) - n\log\left(-\frac{\sqrt{g}\,(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)\log\left(\sqrt{f}+\sqrt{g}\,x\right) + \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\sqrt{f}+\sqrt{g}\,x\right) + n\log\left(-\frac{\sqrt{g}\,(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)\log\left(\sqrt{f}+\sqrt{g}\,x\right) + 2n\text{Li}_2\left(-\frac{bx}{a}\right) - 2n\text{Li}_2\left(-\frac{dx}{c}\right) - n\text{Li}_2\left(\frac{b\left(\sqrt{f}-\sqrt{g}\,x\right)}{b\sqrt{f}+a\sqrt{g}}\right) + n\text{Li}_2\left(\frac{d\left(\sqrt{f}-\sqrt{g}\,x\right)}{d\sqrt{f}+c\sqrt{g}}\right) - n\text{Li}_2\left(\frac{b\left(\sqrt{f}+\sqrt{g}\,x\right)}{b\sqrt{f}-a\sqrt{g}}\right) + n\text{Li}_2\left(\frac{d\left(\sqrt{f}+\sqrt{g}\,x\right)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*((a + b*x)/(c + d*x))^n]/(x*(f - g*x^2)),x]
```

```
[Out] -1/2*(2*n*Log[x]*Log[1 + (b*x)/a] - 2*Log[x]*Log[e*((a + b*x)/(c + d*x))^n]
 - 2*n*Log[x]*Log[1 + (d*x)/c] - n*Log[(Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*S
qrt[g])]*Log[Sqrt[f] - Sqrt[g]*x] + Log[e*((a + b*x)/(c + d*x))^n]*Log[Sqrt
```

```
[f] - Sqrt[g]*x] + n*Log[(Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])]*Log[S
qrt[f] - Sqrt[g]*x] - n*Log[-((Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))]
*Log[Sqrt[f] + Sqrt[g]*x] + Log[e*((a + b*x)/(c + d*x))^n]*Log[Sqrt[f] + Sq
rt[g]*x] + n*Log[-((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))]*Log[Sqrt[f
] + Sqrt[g]*x] + 2*n*PolyLog[2, -((b*x)/a)] - 2*n*PolyLog[2, -((d*x)/c)] -
n*PolyLog[2, (b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])] + n*PolyLog
[2, (d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])] - n*PolyLog[2, (b*(S
qrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])] + n*PolyLog[2, (d*(Sqrt[f] +
Sqrt[g]*x))/(d*Sqrt[f] - c*Sqrt[g])])/f
```

Maple [**F**]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(e \left(\frac{b x +a}{d x +c}\right)^{n}\right)}{x \left(-g \,x^{2}+f \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)/x/(-g*x^2+f),x)
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)/x/(-g*x^2+f),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(-g*x^2+f),x, algorithm="maxima")
[Out] -integrate(log(((b*x + a)/(d*x + c))^n*e)/((g*x^2 - f)*x), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(-g*x^2+f),x, algorithm="fricas")
[Out] integral(-log(((b*x + a)/(d*x + c))^n*e)/(g*x^3 - f*x), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)/x/(-g*x**2+f),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(-g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(-log(((b*x + a)/(d*x + c))^n*e)/((g*x^2 - f)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{x\,\left(f-g\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*((a + b*x)/(c + d*x))^n)/(x*(f - g*x^2)),x)
```

```
[Out] int(log(e*((a + b*x)/(c + d*x))^n)/(x*(f - g*x^2)), x)
```

3.81 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f-gx^2)}\,dx$

Optimal. Leaf size=596

$$\frac{bn\log(x)}{af}-\frac{dn\log(x)}{cf}-\frac{bn\log(a+bx)}{af}-\frac{n\log(a+bx)}{fx}+\frac{dn\log(c+dx)}{cf}+\frac{n\log(c+dx)}{fx}+\frac{n\log(a+bx)-\log}{}$$

```
[Out] b*n*ln(x)/a/f-d*n*ln(x)/c/f-b*n*ln(b*x+a)/a/f-n*ln(b*x+a)/f/x+d*n*ln(d*x+c)
/c/f+n*ln(d*x+c)/f/x+(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/f/
x-arctanh(x*g^(1/2)/f^(1/2))*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*
x+c))*g^(1/2)/f^(3/2)-1/2*n*ln(b*x+a)*ln(b*(f^(1/2)-x*g^(1/2))/(b*f^(1/2)+a
*g^(1/2)))*g^(1/2)/f^(3/2)+1/2*n*ln(d*x+c)*ln(d*(f^(1/2)-x*g^(1/2))/(d*f^(1
/2)+c*g^(1/2)))*g^(1/2)/f^(3/2)+1/2*n*ln(b*x+a)*ln(b*(f^(1/2)+x*g^(1/2))/(b
*f^(1/2)-a*g^(1/2)))*g^(1/2)/f^(3/2)-1/2*n*ln(d*x+c)*ln(d*(f^(1/2)+x*g^(1/2
))/(d*f^(1/2)-c*g^(1/2)))*g^(1/2)/f^(3/2)+1/2*n*polylog(2,-(b*x+a)*g^(1/2)/
(b*f^(1/2)-a*g^(1/2)))*g^(1/2)/f^(3/2)-1/2*n*polylog(2,(b*x+a)*g^(1/2)/(b*f
^(1/2)+a*g^(1/2)))*g^(1/2)/f^(3/2)-1/2*n*polylog(2,-(d*x+c)*g^(1/2)/(d*f^(1
/2)-c*g^(1/2)))*g^(1/2)/f^(3/2)+1/2*n*polylog(2,(d*x+c)*g^(1/2)/(d*f^(1/2)+
c*g^(1/2)))*g^(1/2)/f^(3/2)
```

Rubi [A]
time = 0.41, antiderivative size = 596, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 12, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2593, 331, 214, 2463, 2442, 36, 29, 31, 2456, 2441, 2440, 2438}

$$\frac{\sqrt{g}\,n\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f^{3/2}}-\frac{\sqrt{g}\,n\,\text{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)}{2f^{3/2}}-\frac{\sqrt{g}\,n\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f^{3/2}}+\frac{\sqrt{g}\,n\,\text{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right)}{2f^{3/2}}-\frac{\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)\right)}{f^{3/2}}+\frac{-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log(a+bx)-n\log(c+dx)}{fx}-\frac{\sqrt{g}\,n\log(a+bx)\log\left(\frac{b\left(\sqrt{f}-\sqrt{g}x\right)}{a\sqrt{g}+b\sqrt{f}}\right)}{2f^{3/2}}+\frac{\sqrt{g}\,n\log(a+bx)\log\left(\frac{b\left(\sqrt{f}+\sqrt{g}x\right)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f^{3/2}}+\frac{bn\log(x)}{af}-\frac{bn\log(a+bx)}{af}-\frac{n\log(a+bx)}{fx}+\frac{\sqrt{g}\,n\log(c+dx)\log\left(\frac{d\left(\sqrt{f}-\sqrt{g}x\right)}{c\sqrt{g}+d\sqrt{f}}\right)}{2f^{3/2}}-\frac{\sqrt{g}\,n\log(c+dx)\log\left(\frac{d\left(\sqrt{f}+\sqrt{g}x\right)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f^{3/2}}-\frac{dn\log(x)}{cf}+\frac{dn\log(c+dx)}{cf}+\frac{n\log(c+dx)}{fx}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*((a + b*x)/(c + d*x))^n]/(x^2*(f - g*x^2)),x]
```

```
[Out] (b*n*Log[x])/(a*f) - (d*n*Log[x])/(c*f) - (b*n*Log[a + b*x])/(a*f) - (n*Log
[a + b*x])/(f*x) + (d*n*Log[c + d*x])/(c*f) + (n*Log[c + d*x])/(f*x) + (n*L
og[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])/(f*x) - (Sqr
t[g]*ArcTanh[(Sqrt[g]*x)/Sqrt[f]]*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d
*x))^n] - n*Log[c + d*x]))/f^(3/2) - (Sqrt[g]*n*Log[a + b*x]*Log[(b*(Sqrt[f
] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*f^(3/2)) + (Sqrt[g]*n*Log[c +
d*x]*Log[(d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*f^(3/2)) +
(Sqrt[g]*n*Log[a + b*x]*Log[(b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g
])])/(2*f^(3/2)) - (Sqrt[g]*n*Log[c + d*x]*Log[(d*(Sqrt[f] + Sqrt[g]*x))/(d
*Sqrt[f] - c*Sqrt[g])])/(2*f^(3/2)) + (Sqrt[g]*n*PolyLog[2, -((Sqrt[g]*(a +
 b*x))/(b*Sqrt[f] - a*Sqrt[g]))])/(2*f^(3/2)) - (Sqrt[g]*n*PolyLog[2, (Sqrt
[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*f^(3/2)) - (Sqrt[g]*n*PolyLog[2
```

```
, -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))])/(2*f^(3/2)) + (Sqrt[g]*n
*PolyLog[2, (Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*f^(3/2))
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
```

```
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\int \frac{\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2\left(f-gx^2\right)}\,dx = n\int \frac{\log(a+bx)}{x^2\left(f-gx^2\right)}\,dx - n\int \frac{\log(c+dx)}{x^2\left(f-gx^2\right)}\,dx - \left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)$$

$$= \frac{n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)}{fx} + n\int \left(\frac{\log(a+bx)}{fx^2} + \frac{g\log(a+bx)}{f\left(f-gx^2\right)}\right.$$

$$= \frac{n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)}{fx} - \frac{\sqrt{g}\,\tanh^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\left(n\log(a+bx\right.}{f}$$

$$= -\frac{n\log(a+bx)}{fx} + \frac{n\log(c+dx)}{fx} + \frac{n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)}{fx} -$$

$$= -\frac{n\log(a+bx)}{fx} + \frac{n\log(c+dx)}{fx} + \frac{n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)}{fx} -$$

$$= \frac{bn\log(x)}{af} - \frac{dn\log(x)}{cf} - \frac{bn\log(a+bx)}{af} - \frac{n\log(a+bx)}{fx} + \frac{dn\log(c+dx)}{cf} + \frac{n\log(c}{fx}$$

$$= \frac{bn\log(x)}{af} - \frac{dn\log(x)}{cf} - \frac{bn\log(a+bx)}{af} - \frac{n\log(a+bx)}{fx} + \frac{dn\log(c+dx)}{cf} + \frac{n\log(c}{fx}$$

$$= \frac{bn\log(x)}{af} - \frac{dn\log(x)}{cf} - \frac{bn\log(a+bx)}{af} - \frac{n\log(a+bx)}{fx} + \frac{dn\log(c+dx)}{cf} + \frac{n\log(c}{fx}$$

<u>**Mathematica**</u> [A]
time = 0.22, size = 479, normalized size = 0.80

$$\frac{-\frac{2\sqrt{f}\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x} + \frac{2\sqrt{f}\,n((bc-ad)\log(x)-bc\log(a+bx)+ad\log(c+dx))}{ac} - \sqrt{g}\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\sqrt{f}-\sqrt{g}\,x\right) + \sqrt{g}\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\sqrt{f}+\sqrt{g}\,x\right) + \sqrt{g}\,n\left(\left(\log\left(\frac{\sqrt{g}\,(a+bx)}{b\sqrt{f}+a\sqrt{g}}\right) - \log\left(\frac{\sqrt{g}\,(c+dx)}{d\sqrt{f}+c\sqrt{g}}\right)\right)\log\left(\sqrt{f}-\sqrt{g}\,x\right) + \operatorname{Li}_2\left(\frac{b\left(\sqrt{f}-\sqrt{g}\,x\right)}{b\sqrt{f}+a\sqrt{g}}\right) - \operatorname{Li}_2\left(\frac{d\left(\sqrt{f}-\sqrt{g}\,x\right)}{d\sqrt{f}+c\sqrt{g}}\right)\right) - \sqrt{g}\,n\left(\left(\log\left(-\frac{\sqrt{g}\,(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right) - \log\left(-\frac{\sqrt{g}\,(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)\right)\log\left(\sqrt{f}+\sqrt{g}\,x\right) + \operatorname{Li}_2\left(\frac{b\left(\sqrt{f}+\sqrt{g}\,x\right)}{b\sqrt{f}-a\sqrt{g}}\right) - \operatorname{Li}_2\left(\frac{d\left(\sqrt{f}+\sqrt{g}\,x\right)}{d\sqrt{f}-c\sqrt{g}}\right)\right)}{2f^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*((a + b*x)/(c + d*x))^n]/(x^2*(f - g*x^2)),x]
```

```
[Out] ((-2*Sqrt[f]*Log[e*((a + b*x)/(c + d*x))^n])/x + (2*Sqrt[f]*n*((b*c - a*d)*
Log[x] - b*c*Log[a + b*x] + a*d*Log[c + d*x]))/(a*c) - Sqrt[g]*Log[e*((a +
```

```
b*x)/(c + d*x))^n]*Log[Sqrt[f] - Sqrt[g]*x] + Sqrt[g]*Log[e*((a + b*x)/(c +
 d*x))^n]*Log[Sqrt[f] + Sqrt[g]*x] + Sqrt[g]*n*((Log[(Sqrt[g]*(a + b*x))/(b
*Sqrt[f] + a*Sqrt[g])] - Log[(Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])*
Log[Sqrt[f] - Sqrt[g]*x] + PolyLog[2, (b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f]
+ a*Sqrt[g])] - PolyLog[2, (d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g]
)]) - Sqrt[g]*n*((Log[-((Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))] - Log
[-((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))])*Log[Sqrt[f] + Sqrt[g]*x]
+ PolyLog[2, (b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])] - PolyLog[2
, (d*(Sqrt[f] + Sqrt[g]*x))/(d*Sqrt[f] - c*Sqrt[g])]))/(2*f^(3/2))
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(e \left(\frac{b x +a}{d x +c}\right)^{n}\right)}{x^{2} \left(-g \,x^{2}+f \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)/x^2/(-g*x^2+f),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)/x^2/(-g*x^2+f),x)
```

Maxima [A]
time = 0.60, size = 970, normalized size = 1.63

$$\frac{1}{2}\left(2acd\left(\frac{b^2\log(bx+a)}{(ab^2c^2-2a^2bcd+a^3d^2)f}+\frac{d}{(bc^2d-acd^2)fx+(bc^3-ac^2d)f}-\frac{(2bcd-ad^2)\log(dx+c)}{(b^2c^4-2abc^3d+a^2c^2d^2)f}-\frac{\log(x)}{ac^2f}\right)+2bd^2\left(\frac{c}{(bcd^2-ad^3)fx+(bc^2d-acd^2)f}+\frac{a\log(bx+a)}{(b^2c^2-2abcd+a^2d^2)f}-\frac{a\log(dx+c)}{(b^2c^2-2abcd+a^2d^2)f}\right)-2bcd\left(\frac{b\log(bx+a)}{(b^2c^2-2abcd+a^2d^2)f}-\frac{b\log(dx+c)}{(b^2c^2-2abcd+a^2d^2)f}+\frac{1}{(bcd-ad^2)fx+(bc^2-acd)f}\right)-2ad^2\left(\frac{b\log(bx+a)}{(b^2c^2-2abcd+a^2d^2)f}-\frac{b\log(dx+c)}{(b^2c^2-2abcd+a^2d^2)f}+\frac{1}{(bcd-ad^2)fx+(bc^2-acd)f}\right)-2bc\left(\frac{b\log(bx+a)}{(abc-a^2d)f}-\frac{d\log(dx+c)}{(bc^2-acd)f}-\frac{\log(x)}{acf}\right)+2bd\left(\frac{\log(bx+a)}{(bc-ad)f}-\frac{\log(dx+c)}{(bc-ad)f}\right)+\frac{\left(\log\left(\sqrt{g}x-\sqrt{f}\right)\log\left(\frac{b\sqrt{g}x-b\sqrt{f}}{b\sqrt{f}+a\sqrt{g}}+1\right)+\mathrm{Li}_2\left(-\frac{b\sqrt{g}x-b\sqrt{f}}{b\sqrt{f}+a\sqrt{g}}\right)\right)\sqrt{g}}{f^{\frac{3}{2}}}-\frac{\left(\log\left(\sqrt{g}x+\sqrt{f}\right)\log\left(-\frac{b\sqrt{g}x+b\sqrt{f}}{b\sqrt{f}-a\sqrt{g}}+1\right)+\mathrm{Li}_2\left(\frac{b\sqrt{g}x+b\sqrt{f}}{b\sqrt{f}-a\sqrt{g}}\right)\right)\sqrt{g}}{f^{\frac{3}{2}}}-\frac{\left(\log\left(\sqrt{g}x-\sqrt{f}\right)\log\left(\frac{d\sqrt{g}x-d\sqrt{f}}{d\sqrt{f}+c\sqrt{g}}+1\right)+\mathrm{Li}_2\left(-\frac{d\sqrt{g}x-d\sqrt{f}}{d\sqrt{f}+c\sqrt{g}}\right)\right)\sqrt{g}}{f^{\frac{3}{2}}}+\frac{\left(\log\left(\sqrt{g}x+\sqrt{f}\right)\log\left(-\frac{d\sqrt{g}x+d\sqrt{f}}{d\sqrt{f}-c\sqrt{g}}+1\right)+\mathrm{Li}_2\left(\frac{d\sqrt{g}x+d\sqrt{f}}{d\sqrt{f}-c\sqrt{g}}\right)\right)\sqrt{g}}{f^{\frac{3}{2}}}\right)n-\frac{1}{2}\left(\frac{g\log\left(\frac{gx-\sqrt{fg}}{gx+\sqrt{fg}}\right)}{\sqrt{fg}\,f}+\frac{2}{fx}\right)\log\left(\left(\frac{bx+a}{dx+c}\right)^{n}e\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(-g*x^2+f),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*c*d*(b^2*log(b*x + a)/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*f) + d/
((b*c^2*d - a*c*d^2)*f*x + (b*c^3 - a*c^2*d)*f) - (2*b*c*d - a*d^2)*log(d*x
 + c)/((b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*f) - log(x)/(a*c^2*f)) + 2*b*d
^2*(c/((b*c*d^2 - a*d^3)*f*x + (b*c^2*d - a*c*d^2)*f) + a*log(b*x + a)/((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*f) - a*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*f)) - 2*b*c*d*(b*log(b*x + a)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*f) -
b*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*f) + 1/((b*c*d - a*d^2)*f*x
 + (b*c^2 - a*c*d)*f)) - 2*a*d^2*(b*log(b*x + a)/((b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*f) - b*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*f) + 1/((b*c*d
- a*d^2)*f*x + (b*c^2 - a*c*d)*f)) - 2*b*c*(b*log(b*x + a)/((a*b*c - a^2*d)
*f) - d*log(d*x + c)/((b*c^2 - a*c*d)*f) - log(x)/(a*c*f)) + 2*b*d*(log(b*x
 + a)/((b*c - a*d)*f) - log(d*x + c)/((b*c - a*d)*f)) + (log(sqrt(g)*x - sq
rt(f))*log((b*sqrt(g)*x - b*sqrt(f))/(b*sqrt(f) + a*sqrt(g)) + 1) + dilog(-
(b*sqrt(g)*x - b*sqrt(f))/(b*sqrt(f) + a*sqrt(g))))*sqrt(g)/f^(3/2) - (log(
sqrt(g)*x + sqrt(f))*log(-(b*sqrt(g)*x + b*sqrt(f))/(b*sqrt(f) - a*sqrt(g))
 + 1) + dilog((b*sqrt(g)*x + b*sqrt(f))/(b*sqrt(f) - a*sqrt(g))))*sqrt(g)/f
^(3/2) - (log(sqrt(g)*x - sqrt(f))*log((d*sqrt(g)*x - d*sqrt(f))/(d*sqrt(f)
```

```
 + c*sqrt(g)) + 1) + dilog(-(d*sqrt(g)*x - d*sqrt(f))/(d*sqrt(f) + c*sqrt(g
))))*sqrt(g)/f^(3/2) + (log(sqrt(g)*x + sqrt(f))*log(-(d*sqrt(g)*x + d*sqrt
(f))/(d*sqrt(f) - c*sqrt(g)) + 1) + dilog((d*sqrt(g)*x + d*sqrt(f))/(d*sqrt
(f) - c*sqrt(g))))*sqrt(g)/f^(3/2))*n - 1/2*(g*log((g*x - sqrt(f*g))/(g*x +
 sqrt(f*g)))/(sqrt(f*g)*f) + 2/(f*x))*log(((b*x + a)/(d*x + c))^n*e)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(-g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(-log(((b*x + a)/(d*x + c))^n*e)/(g*x^4 - f*x^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)/x**2/(-g*x**2+f),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(-g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(-log(((b*x + a)/(d*x + c))^n*e)/((g*x^2 - f)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{x^2\,\left(f-g\,x^2\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*((a + b*x)/(c + d*x))^n)/(x^2*(f - g*x^2)),x)
```

```
[Out] int(log(e*((a + b*x)/(c + d*x))^n)/(x^2*(f - g*x^2)), x)
```

3.82 $\int \frac{x^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx$

Optimal. Leaf size=1046

$$\frac{anx}{2bh}-\frac{cnx}{2dh}-\frac{a^2n\log(a+bx)}{2b^2h}+\frac{nx^2\log(a+bx)}{2h}-\frac{gn(a+bx)\log(a+bx)}{bh^2}+\frac{c^2n\log(c+dx)}{2d^2h}-\frac{nx^2\log(c+dx)}{2h}+$$

```
[Out] 1/2*a*n*x/b/h-1/2*c*n*x/d/h-1/2*a^2*n*ln(b*x+a)/b^2/h+1/2*n*x^2*ln(b*x+a)/h
-g*n*(b*x+a)*ln(b*x+a)/b/h^2+1/2*c^2*n*ln(d*x+c)/d^2/h-1/2*n*x^2*ln(d*x+c)/
h+g*n*(d*x+c)*ln(d*x+c)/d/h^2+g*x*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*
ln(d*x+c))/h^2-1/2*x^2*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/
h-1/2*(-f*h+g^2)*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))*ln(h*x
^2+g*x+f)/h^3+1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x-(-4*f*h+g^2)^(1/2))/(2*a*h-b*(
g-(-4*f*h+g^2)^(1/2))))*(g^2-f*h-g*(-3*f*h+g^2)/(-4*f*h+g^2)^(1/2))/h^3-1/2
*n*ln(d*x+c)*ln(-d*(g+2*h*x-(-4*f*h+g^2)^(1/2))/(2*c*h-d*(g-(-4*f*h+g^2)^(1
/2))))*(g^2-f*h-g*(-3*f*h+g^2)/(-4*f*h+g^2)^(1/2))/h^3+1/2*n*polylog(2,2*h*
(b*x+a)/(2*a*h-b*(g-(-4*f*h+g^2)^(1/2))))*(g^2-f*h-g*(-3*f*h+g^2)/(-4*f*h+g
^2)^(1/2))/h^3-1/2*n*polylog(2,2*h*(d*x+c)/(2*c*h-d*(g-(-4*f*h+g^2)^(1/2)))
)*(g^2-f*h-g*(-3*f*h+g^2)/(-4*f*h+g^2)^(1/2))/h^3+1/2*n*ln(b*x+a)*ln(-b*(g+
2*h*x+(-4*f*h+g^2)^(1/2))/(2*a*h-b*(g+(-4*f*h+g^2)^(1/2))))*(g^2-f*h+g*(-3*
f*h+g^2)/(-4*f*h+g^2)^(1/2))/h^3-1/2*n*ln(d*x+c)*ln(-d*(g+2*h*x+(-4*f*h+g^2
)^(1/2))/(2*c*h-d*(g+(-4*f*h+g^2)^(1/2))))*(g^2-f*h+g*(-3*f*h+g^2)/(-4*f*h+
g^2)^(1/2))/h^3+1/2*n*polylog(2,2*h*(b*x+a)/(2*a*h-b*(g+(-4*f*h+g^2)^(1/2))
))*(g^2-f*h+g*(-3*f*h+g^2)/(-4*f*h+g^2)^(1/2))/h^3-1/2*n*polylog(2,2*h*(d*x
+c)/(2*c*h-d*(g+(-4*f*h+g^2)^(1/2))))*(g^2-f*h+g*(-3*f*h+g^2)/(-4*f*h+g^2)^
(1/2))/h^3-g*(-3*f*h+g^2)*arctanh((2*h*x+g)/(-4*f*h+g^2)^(1/2))*(n*ln(b*x+a
)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/h^3/(-4*f*h+g^2)^(1/2)
```

Rubi [A]

time = 1.23, antiderivative size = 1046, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 14, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2593, 2465, 2436, 2332, 2442, 45, 2441, 2440, 2438, 715, 648, 632, 212, 642}

Antiderivative was successfully verified.

```
[In] Int[(x^3*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x + h*x^2),x]
```

```
[Out] (a*n*x)/(2*b*h) - (c*n*x)/(2*d*h) - (a^2*n*Log[a + b*x])/(2*b^2*h) + (n*x^2
*Log[a + b*x])/(2*h) - (g*n*(a + b*x)*Log[a + b*x])/(b*h^2) + (c^2*n*Log[c
+ d*x])/(2*d^2*h) - (n*x^2*Log[c + d*x])/(2*h) + (g*n*(c + d*x)*Log[c + d*x
])/(d*h^2) + (g*x*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[
```

```
c + d*x]))/h^2 - (x^2*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*
Log[c + d*x]))/(2*h) - (g*(g^2 - 3*f*h)*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*
h]]*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/(h^
3*Sqrt[g^2 - 4*f*h]) + ((g^2 - f*h - (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*n
*Log[a + b*x]*Log[-((b*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g - Sqr
t[g^2 - 4*f*h])))])/(2*h^3) - ((g^2 - f*h - (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*
f*h])*n*Log[c + d*x]*Log[-((d*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d*(
g - Sqrt[g^2 - 4*f*h])))])/(2*h^3) + ((g^2 - f*h + (g*(g^2 - 3*f*h))/Sqrt[g
^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*
h - b*(g + Sqrt[g^2 - 4*f*h])))])/(2*h^3) - ((g^2 - f*h + (g*(g^2 - 3*f*h))
/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x)
)/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h])))])/(2*h^3) - ((g^2 - f*h)*(n*Log[a +
b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f + g*x + h*x^2
])/(2*h^3) + ((g^2 - f*h - (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*n*PolyLog[2
, (2*h*(a + b*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h]))])/(2*h^3) + ((g^2 - f
*h + (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a
*h - b*(g + Sqrt[g^2 - 4*f*h]))])/(2*h^3) - ((g^2 - f*h - (g*(g^2 - 3*f*h))
/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g - Sqrt[g^2 -
 4*f*h]))])/(2*h^3) - ((g^2 - f*h + (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*n*
PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])/(2*h^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
```

```
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\int \frac{x^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = n\int \frac{x^3\log(a+bx)}{f+gx+hx^2}\,dx - n\int \frac{x^3\log(c+dx)}{f+gx+hx^2}\,dx - \left(n\log(a+bx) - \log\left(e\left(\frac{a+ \cdots}{c+ \cdots}\right.\right.\right.$$

$$= n\int\left(-\frac{g\log(a+bx)}{h^2} + \frac{x\log(a+bx)}{h} + \frac{(fg+(g^2-fh)\,x)\log(a+bx)}{h^2\,(f+gx+hx^2)}\right)dx - \cdots$$

$$= \frac{gx\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h^2} - \frac{x^2\left(n\log(a+bx) - \log\left(e \cdots\right.\right.}{2 \cdots}$$

$$= \frac{nx^2\log(a+bx)}{2h} - \frac{nx^2\log(c+dx)}{2h} + \frac{gx\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(\cdots\right.}{h^2}$$

$$= \frac{nx^2\log(a+bx)}{2h} - \frac{gn(a+bx)\log(a+bx)}{bh^2} - \frac{nx^2\log(c+dx)}{2h} + \frac{gn(c+dx)\log(c \cdots}{dh^2}$$

$$= \frac{anx}{2bh} - \frac{cnx}{2dh} - \frac{a^2n\log(a+bx)}{2b^2h} + \frac{nx^2\log(a+bx)}{2h} - \frac{gn(a+bx)\log(a+bx)}{bh^2} + \frac{c^2 \cdots}{\cdots}$$

$$= \frac{anx}{2bh} - \frac{cnx}{2dh} - \frac{a^2n\log(a+bx)}{2b^2h} + \frac{nx^2\log(a+bx)}{2h} - \frac{gn(a+bx)\log(a+bx)}{bh^2} + \frac{c^2 \cdots}{\cdots}$$

$$= \frac{anx}{2bh} - \frac{cnx}{2dh} - \frac{a^2n\log(a+bx)}{2b^2h} + \frac{nx^2\log(a+bx)}{2h} - \frac{gn(a+bx)\log(a+bx)}{bh^2} + \frac{c^2 \cdots}{\cdots}$$

__Mathematica__ [A]
time = 0.98, size = 1240, normalized size = 1.19

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x + h*x^2),x]
```

```
[Out] (h^2*x^2*Log[e*((a + b*x)/(c + d*x))^n] - (2*g*h*(a + b*x)*Log[e*((a + b*x)
/(c + d*x))^n])/b + (2*(b*c - a*d)*g*h*n*Log[c + d*x])/(b*d) + (h^2*n*(-(a^
2*d^2*Log[a + b*x]) + b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])))/(b^2*d^
2) + (2*f*g*h*Log[e*((a + b*x)/(c + d*x))^n]*Log[g - Sqrt[g^2 - 4*f*h] + 2*
h*x])/Sqrt[g^2 - 4*f*h] + (g^2 - f*h)*(1 - g/Sqrt[g^2 - 4*f*h])*Log[e*((a +
 b*x)/(c + d*x))^n]*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] - (2*f*g*h*Log[e*((a
 + b*x)/(c + d*x))^n]*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x])/Sqrt[g^2 - 4*f*h]
 + (g^2 - f*h)*(1 + g/Sqrt[g^2 - 4*f*h])*Log[e*((a + b*x)/(c + d*x))^n]*Log
[g + Sqrt[g^2 - 4*f*h] + 2*h*x] - (2*f*g*h*n*((Log[(2*h*(a + b*x))/(-(b*g)
+ 2*a*h + b*Sqrt[g^2 - 4*f*h])] - Log[(2*h*(c + d*x))/(-(d*g) + 2*c*h + d*S
qrt[g^2 - 4*f*h])])*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + PolyLog[2, (b*(-g
+ Sqrt[g^2 - 4*f*h] - 2*h*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - Pol
yLog[2, (d*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*c*h + d*(-g + Sqrt[g^2 - 4*
f*h]))]))/Sqrt[g^2 - 4*f*h] - ((g^2 - f*h)*(-g + Sqrt[g^2 - 4*f*h])*n*((Log
[(2*h*(a + b*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - Log[(2*h*(c + d*
x))/(-(d*g) + 2*c*h + d*Sqrt[g^2 - 4*f*h])])*Log[g - Sqrt[g^2 - 4*f*h] + 2*
h*x] + PolyLog[2, (b*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(-(b*g) + 2*a*h + b*
Sqrt[g^2 - 4*f*h])] - PolyLog[2, (d*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*c*
h + d*(-g + Sqrt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h] + (2*f*g*h*n*((Log[(2*
h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))] - Log[(2*h*(c + d*x))/(2*
c*h - d*(g + Sqrt[g^2 - 4*f*h]))])*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x] + Pol
yLog[2, (b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*a*h + b*(g + Sqrt[g^2 - 4*f
*h]))] - PolyLog[2, (d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*c*h + d*(g + Sq
rt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h] - ((g^2 - f*h)*(g + Sqrt[g^2 - 4*f*h
])*n*((Log[(2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))] - Log[(2*h*
(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])*Log[g + Sqrt[g^2 - 4*f*h]
+ 2*h*x] + PolyLog[2, (b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*a*h + b*(g +
Sqrt[g^2 - 4*f*h]))] - PolyLog[2, (d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*c
*h + d*(g + Sqrt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h])/(2*h^3)
```

Maple [F]
time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{x^3 \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{h\,x^2+gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

```
[Out] int(x^3*ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*f*h-g^2>0)', see `assume?` for mo
re deta
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="fricas
")
```

```
[Out] integral(x^3*log(((b*x + a)/(d*x + c))^n*e)/(h*x^2 + g*x + f), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(e*((b*x+a)/(d*x+c))**n)/(h*x**2+g*x+f),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="giac")
```

```
[Out] integrate(x^3*log(((b*x + a)/(d*x + c))^n*e)/(h*x^2 + g*x + f), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{h\,x^2+g\,x+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x + h*x^2),x)
```

```
[Out] int((x^3*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x + h*x^2), x)
```

3.83 $\int \frac{x^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\, dx$

Optimal. Leaf size=831

$$\frac{n(a+bx)\log(a+bx)}{bh} - \frac{n(c+dx)\log(c+dx)}{dh} - \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h} + \frac{(g^2 - 2f}{}$$

```
[Out] n*(b*x+a)*ln(b*x+a)/b/h-n*(d*x+c)*ln(d*x+c)/d/h-x*(n*ln(b*x+a)-ln(e*((b*x+a
)/(d*x+c))^n)-n*ln(d*x+c))/h+1/2*g*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n
*ln(d*x+c))*ln(h*x^2+g*x+f)/h^2-1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x-(-4*f*h+g^2)
^(1/2))/(2*a*h-b*(g-(-4*f*h+g^2)^(1/2))))*(g+(2*f*h-g^2)/(-4*f*h+g^2)^(1/2)
)/h^2+1/2*n*ln(d*x+c)*ln(-d*(g+2*h*x-(-4*f*h+g^2)^(1/2))/(2*c*h-d*(g-(-4*f*
h+g^2)^(1/2))))*(g+(2*f*h-g^2)/(-4*f*h+g^2)^(1/2))/h^2-1/2*n*polylog(2,2*h*
(b*x+a)/(2*a*h-b*(g-(-4*f*h+g^2)^(1/2))))*(g+(2*f*h-g^2)/(-4*f*h+g^2)^(1/2)
)/h^2+1/2*n*polylog(2,2*h*(d*x+c)/(2*c*h-d*(g-(-4*f*h+g^2)^(1/2))))*(g+(2*f
*h-g^2)/(-4*f*h+g^2)^(1/2))/h^2-1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x+(-4*f*h+g^2)
^(1/2))/(2*a*h-b*(g+(-4*f*h+g^2)^(1/2))))*(g+(-2*f*h+g^2)/(-4*f*h+g^2)^(1/2
))/h^2+1/2*n*ln(d*x+c)*ln(-d*(g+2*h*x+(-4*f*h+g^2)^(1/2))/(2*c*h-d*(g+(-4*f
*h+g^2)^(1/2))))*(g+(-2*f*h+g^2)/(-4*f*h+g^2)^(1/2))/h^2-1/2*n*polylog(2,2*
h*(b*x+a)/(2*a*h-b*(g+(-4*f*h+g^2)^(1/2))))*(g+(-2*f*h+g^2)/(-4*f*h+g^2)^(1
/2))/h^2+1/2*n*polylog(2,2*h*(d*x+c)/(2*c*h-d*(g+(-4*f*h+g^2)^(1/2))))*(g+(
-2*f*h+g^2)/(-4*f*h+g^2)^(1/2))/h^2+(-2*f*h+g^2)*arctanh((2*h*x+g)/(-4*f*h+
g^2)^(1/2))*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/h^2/(-4*f*h
+g^2)^(1/2)
```

Rubi [A]

time = 0.81, antiderivative size = 831, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 12, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2593, 2465, 2436, 2332, 2441, 2440, 2438, 717, 648, 632, 212, 642}

Antiderivative was successfully verified.

```
[In] Int[(x^2*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x + h*x^2),x]
```

```
[Out] (n*(a + b*x)*Log[a + b*x])/(b*h) - (n*(c + d*x)*Log[c + d*x])/(d*h) - (x*(n
*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/h + ((g^2
 - 2*f*h)*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*h]]*(n*Log[a + b*x] - Log[e*((
a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/(h^2*Sqrt[g^2 - 4*f*h]) - ((g - (
g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g - Sqrt[g^2 - 4*f
*h] + 2*h*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h])))])/(2*h^2) + ((g - (g^2 -
 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g - Sqrt[g^2 - 4*f*h] +
```

```
 2*h*x))/(2*c*h - d*(g - Sqrt[g^2 - 4*f*h]))))])/(2*h^2) - ((g + (g^2 - 2*f*
h)/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g + Sqrt[g^2 - 4*f*h] + 2*h*
x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))))])/(2*h^2) + ((g + (g^2 - 2*f*h)/Sq
rt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(
2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))))])/(2*h^2) + (g*(n*Log[a + b*x] - Log[e*
((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f + g*x + h*x^2])/(2*h^2) -
((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h
- b*(g - Sqrt[g^2 - 4*f*h]))])/(2*h^2) - ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f
*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))])/(2*
h^2) + ((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/
(2*c*h - d*(g - Sqrt[g^2 - 4*f*h]))])/(2*h^2) + ((g + (g^2 - 2*f*h)/Sqrt[g^
2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h])
)])/(2*h^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\int \frac{x^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = n\int \frac{x^2\log(a+bx)}{f+gx+hx^2}\,dx - n\int \frac{x^2\log(c+dx)}{f+gx+hx^2}\,dx - \left(n\log(a+bx) - \log\left(e\left(\frac{a+\cdots}{c+\cdots}\right.\right.\right.$$

$$= -\frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h} + n\int\left(\frac{\log(a+bx)}{h} - \frac{(f+\cdots}{h\,\cdots}\right.$$

$$= -\frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h} + \frac{n\int\log(a+bx)\,dx}{h} - \frac{n\int\cdots}{\cdots}$$

$$= -\frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h} + \frac{g\left(n\log(a+bx) - \log\left(e\left(\cdots\right.\right.\right.}{\cdots}$$

$$= \frac{n(a+bx)\log(a+bx)}{bh} - \frac{n(c+dx)\log(c+dx)}{dh} - \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)\cdots\right.\right.}{h}$$

$$= \frac{n(a+bx)\log(a+bx)}{bh} - \frac{n(c+dx)\log(c+dx)}{dh} - \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)\cdots\right.\right.}{h}$$

$$= \frac{n(a+bx)\log(a+bx)}{bh} - \frac{n(c+dx)\log(c+dx)}{dh} - \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)\cdots\right.\right.}{h}$$

$$= \frac{n(a+bx)\log(a+bx)}{bh} - \frac{n(c+dx)\log(c+dx)}{dh} - \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)\cdots\right.\right.}{h}$$

Mathematica [A]
time = 2.33, size = 1105, normalized size = 1.33

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x + h*x^2),x]
```

```
[Out] (2*d*h*Sqrt[g^2 - 4*f*h]*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] - 2*(b*c
- a*d)*h*Sqrt[g^2 - 4*f*h]*n*Log[c + d*x] - 2*b*d*f*h*Log[e*((a + b*x)/(c +
 d*x))^n]*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + b*d*g*(g - Sqrt[g^2 - 4*f*h]
)*Log[e*((a + b*x)/(c + d*x))^n]*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + 2*b*d
*f*h*Log[e*((a + b*x)/(c + d*x))^n]*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x] - b*
d*g*(g + Sqrt[g^2 - 4*f*h])*Log[e*((a + b*x)/(c + d*x))^n]*Log[g + Sqrt[g^2
 - 4*f*h] + 2*h*x] + 2*b*d*f*h*n*((Log[(2*h*(a + b*x))/(-(b*g) + 2*a*h + b*
Sqrt[g^2 - 4*f*h])] - Log[(2*h*(c + d*x))/(-(d*g) + 2*c*h + d*Sqrt[g^2 - 4*
f*h])])*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + PolyLog[2, (b*(-g + Sqrt[g^2 -
 4*f*h] - 2*h*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - PolyLog[2, (d*(
-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*c*h + d*(-g + Sqrt[g^2 - 4*f*h]))]) - b
*d*g*(g - Sqrt[g^2 - 4*f*h])*n*((Log[(2*h*(a + b*x))/(-(b*g) + 2*a*h + b*Sq
rt[g^2 - 4*f*h])] - Log[(2*h*(c + d*x))/(-(d*g) + 2*c*h + d*Sqrt[g^2 - 4*f*
h])])*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + PolyLog[2, (b*(-g + Sqrt[g^2 - 4
*f*h] - 2*h*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - PolyLog[2, (d*(-g
 + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*c*h + d*(-g + Sqrt[g^2 - 4*f*h]))]) - 2*b
*d*f*h*n*((Log[(2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))] - Log[(
2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])*Log[g + Sqrt[g^2 - 4*f
*h] + 2*h*x] + PolyLog[2, (b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*a*h + b*(
g + Sqrt[g^2 - 4*f*h]))] - PolyLog[2, (d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(
-2*c*h + d*(g + Sqrt[g^2 - 4*f*h]))]) + b*d*g*(g + Sqrt[g^2 - 4*f*h])*n*((L
og[(2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))] - Log[(2*h*(c + d*x
))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x]
 + PolyLog[2, (b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*a*h + b*(g + Sqrt[g^2
 - 4*f*h]))] - PolyLog[2, (d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*c*h + d*(
g + Sqrt[g^2 - 4*f*h]))]))/(2*b*d*h^2*Sqrt[g^2 - 4*f*h])
```

Maple [F]
time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \ln \left(e \left(\frac{b x +a}{d x +c}\right)^{n}\right)}{h \,x^{2}+g x +f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

```
[Out] int(x^2*ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*f*h-g^2>0)', see `assume?` for mo
re deta
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="fricas
")
```

```
[Out] integral(x^2*log(((b*x + a)/(d*x + c))^n*e)/(h*x^2 + g*x + f), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(e*((b*x+a)/(d*x+c))**n)/(h*x**2+g*x+f),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="giac")
```

```
[Out] integrate(x^2*log(((b*x + a)/(d*x + c))^n*e)/(h*x^2 + g*x + f), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{h\,x^2+g\,x+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x + h*x^2),x)
```

```
[Out] int((x^2*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x + h*x^2), x)
```

3.84 $\int \frac{x \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx$

Optimal. Leaf size=685

$$-\frac{g\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)}{h\sqrt{g^2-4fh}}+\frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\log(a+}{}$$

```
[Out] -1/2*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))*ln(h*x^2+g*x+f)/h+
1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x-(-4*f*h+g^2)^(1/2))/(2*a*h-b*(g-(-4*f*h+g^2)
^(1/2))))*(1-g/(-4*f*h+g^2)^(1/2))/h-1/2*n*ln(d*x+c)*ln(-d*(g+2*h*x-(-4*f*h
+g^2)^(1/2))/(2*c*h-d*(g-(-4*f*h+g^2)^(1/2))))*(1-g/(-4*f*h+g^2)^(1/2))/h+1
/2*n*polylog(2,2*h*(b*x+a)/(2*a*h-b*(g-(-4*f*h+g^2)^(1/2))))*(1-g/(-4*f*h+g
^2)^(1/2))/h-1/2*n*polylog(2,2*h*(d*x+c)/(2*c*h-d*(g-(-4*f*h+g^2)^(1/2))))*
(1-g/(-4*f*h+g^2)^(1/2))/h+1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x+(-4*f*h+g^2)^(1/2
))/(2*a*h-b*(g+(-4*f*h+g^2)^(1/2))))*(1+g/(-4*f*h+g^2)^(1/2))/h-1/2*n*ln(d*
x+c)*ln(-d*(g+2*h*x+(-4*f*h+g^2)^(1/2))/(2*c*h-d*(g+(-4*f*h+g^2)^(1/2))))*(
1+g/(-4*f*h+g^2)^(1/2))/h+1/2*n*polylog(2,2*h*(b*x+a)/(2*a*h-b*(g+(-4*f*h+g
^2)^(1/2))))*(1+g/(-4*f*h+g^2)^(1/2))/h-1/2*n*polylog(2,2*h*(d*x+c)/(2*c*h-
d*(g+(-4*f*h+g^2)^(1/2))))*(1+g/(-4*f*h+g^2)^(1/2))/h-g*arctanh((2*h*x+g)/(
-4*f*h+g^2)^(1/2))*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/h/(-
4*f*h+g^2)^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 685, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2593, 2465, 2441, 2440, 2438, 648, 632, 212, 642}

Antiderivative was successfully verified.

```
[In] Int[(x*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x + h*x^2),x]
```

```
[Out] -((g*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*h]]*(n*Log[a + b*x] - Log[e*((a + b
*x)/(c + d*x))^n] - n*Log[c + d*x]))/(h*Sqrt[g^2 - 4*f*h])) + ((1 - g/Sqrt[
g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a
*h - b*(g - Sqrt[g^2 - 4*f*h])))])/(2*h) - ((1 - g/Sqrt[g^2 - 4*f*h])*n*Log
[c + d*x]*Log[-((d*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d*(g - Sqrt[g^
2 - 4*f*h])))])/(2*h) + ((1 + g/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*
(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h])))])/(2*
h) - ((1 + g/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4*f
*h] + 2*h*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h])))])/(2*h) - ((n*Log[a + b*
x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f + g*x + h*x^2])
```

```
/(2*h) + ((1 - g/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h - b
*(g - Sqrt[g^2 - 4*f*h]))])/(2*h) + ((1 + g/Sqrt[g^2 - 4*f*h])*n*PolyLog[2,
 (2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))])/(2*h) - ((1 - g/Sqrt
[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g - Sqrt[g^2 - 4*f*
h]))])/(2*h) - ((1 + g/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/(2*c
*h - d*(g + Sqrt[g^2 - 4*f*h]))])/(2*h)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
```

```
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\int \frac{x \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = n\int \frac{x\log(a+bx)}{f+gx+hx^2}\,dx - n\int \frac{x\log(c+dx)}{f+gx+hx^2}\,dx - \left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.$$

$$= n\int \left(\frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)\log(a+bx)}{g-\sqrt{g^2-4fh}+2hx} + \frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)\log(a+bx)}{g+\sqrt{g^2-4fh}+2hx}\right) dx$$

$$= -\frac{\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2h} + \left(\left(\left(1-\right.\right.\right.$$

$$= -\frac{g\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h\sqrt{g^2-4fh}} + -$$

$$= -\frac{g\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h\sqrt{g^2-4fh}} + -$$

$$= -\frac{g\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h\sqrt{g^2-4fh}} + -$$

Mathematica [A]
time = 1.77, size = 669, normalized size = 0.98

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x + h*x^2),x]
```

```
[Out] ((-2*g*ArcTan[(g + 2*h*x)/Sqrt[-g^2 + 4*f*h]]*(Log[e*((a + b*x)/(c + d*x))^
n] - n*Log[(a + b*x)/(c + d*x)]))/Sqrt[-g^2 + 4*f*h] + (Log[e*((a + b*x)/(c
 + d*x))^n] - n*Log[(a + b*x)/(c + d*x)])*Log[f + x*(g + h*x)] + (n*(Log[a/
b + x] - Log[c/d + x] - Log[(a + b*x)/(c + d*x)])*(2*g*ArcTan[(g + 2*h*x)/S
qrt[-g^2 + 4*f*h]] - Sqrt[-g^2 + 4*f*h]*Log[f + x*(g + h*x)]))/Sqrt[-g^2 +
```

```
4*f*h] + ((-g + Sqrt[g^2 - 4*f*h])*n*(Log[a/b + x]*Log[1 - (2*h*(a + b*x))/
(2*a*h + b*(-g + Sqrt[g^2 - 4*f*h]))] + PolyLog[2, (2*h*(a + b*x))/(2*a*h +
 b*(-g + Sqrt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h] + ((g + Sqrt[g^2 - 4*f*h]
)*n*(Log[a/b + x]*Log[1 + (2*h*(a + b*x))/(-2*a*h + b*(g + Sqrt[g^2 - 4*f*h
]))] + PolyLog[2, (2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))]))/Sq
rt[g^2 - 4*f*h] - ((-g + Sqrt[g^2 - 4*f*h])*n*(Log[c/d + x]*Log[1 - (2*h*(c
 + d*x))/(2*c*h + d*(-g + Sqrt[g^2 - 4*f*h]))] + PolyLog[2, (2*h*(c + d*x))
/(2*c*h + d*(-g + Sqrt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h] - ((g + Sqrt[g^2
 - 4*f*h])*n*(Log[c/d + x]*Log[1 + (2*h*(c + d*x))/(-2*c*h + d*(g + Sqrt[g^
2 - 4*f*h]))] + PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h
]))]))/Sqrt[g^2 - 4*f*h])/(2*h)
```

Maple [F]
time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{x \ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{h\,x^{2}+gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

```
[Out] int(x*ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*f*h-g^2>0)', see `assume?` for mo
re deta
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="fricas")
```

```
[Out] integral(x*log(((b*x + a)/(d*x + c))^n*e)/(h*x^2 + g*x + f), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(e*((b*x+a)/(d*x+c))**n)/(h*x**2+g*x+f),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="giac")
```

```
[Out] integrate(x*log(((b*x + a)/(d*x + c))^n*e)/(h*x^2 + g*x + f), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{h\,x^2+g\,x+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x + h*x^2),x)
```

```
[Out] int((x*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x + h*x^2), x)
```

3.85 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx$

Optimal. Leaf size=401

$$-\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{2(d^2f-cdg+c^2h)(a+bx)}{\left(2bdf-bcg-adg+2ach-(bc-ad)\sqrt{g^2-4fh}\right)(c+dx)}\right)}{\sqrt{g^2-4fh}}+\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{\cdots}{(2bdf-bcg-\cdots}\right.}{\sqrt{g^2\cdots}}$$

```
[Out] -ln(e*((b*x+a)/(d*x+c))^n)*ln(1-2*(c^2*h-c*d*g+d^2*f)*(b*x+a)/(d*x+c)/(2*b*
d*f-b*c*g-a*d*g+2*a*c*h-(-a*d+b*c)*(-4*f*h+g^2)^(1/2)))/(-4*f*h+g^2)^(1/2)+
ln(e*((b*x+a)/(d*x+c))^n)*ln(1-2*(c^2*h-c*d*g+d^2*f)*(b*x+a)/(d*x+c)/(2*b*d
*f-b*c*g-a*d*g+2*a*c*h+(-a*d+b*c)*(-4*f*h+g^2)^(1/2)))/(-4*f*h+g^2)^(1/2)-n
*polylog(2,2*(c^2*h-c*d*g+d^2*f)*(b*x+a)/(d*x+c)/(2*b*d*f-b*c*g-a*d*g+2*a*c
*h-(-a*d+b*c)*(-4*f*h+g^2)^(1/2)))/(-4*f*h+g^2)^(1/2)+n*polylog(2,2*(c^2*h-
c*d*g+d^2*f)*(b*x+a)/(d*x+c)/(2*b*d*f-b*c*g-a*d*g+2*a*c*h+(-a*d+b*c)*(-4*f*
h+g^2)^(1/2)))/(-4*f*h+g^2)^(1/2)
```

Rubi [A]

time = 0.53, antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2576, 2404, 2354, 2438}

$$-\frac{n\,\mathrm{PolyLog}\left(2,\frac{2(a+bx)(c^2h-cdg+d^2f)}{(c+dx)\left(-\sqrt{g^2-4fh}\,(bc-ad)+2ach-adg-bcg+2bdf\right)}\right)}{\sqrt{g^2-4fh}}+\frac{n\,\mathrm{PolyLog}\left(2,\frac{2(a+bx)(c^2h-cdg+d^2f)}{(c+dx)\left(\sqrt{g^2-4fh}\,(bc-ad)+2ach-adg-bcg+2bdf\right)}\right)}{\sqrt{g^2-4fh}}-\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{2(a+bx)(c^2h-cdg+d^2f)}{(c+dx)\left(-\sqrt{g^2-4fh}\,(bc-ad)+2ach-adg-bcg+2bdf\right)}\right)}{\sqrt{g^2-4fh}}+\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{2(a+bx)(c^2h-cdg+d^2f)}{(c+dx)\left(\sqrt{g^2-4fh}\,(bc-ad)+2ach-adg-bcg+2bdf\right)}\right)}{\sqrt{g^2-4fh}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*((a + b*x)/(c + d*x))^n]/(f + g*x + h*x^2),x]
```

```
[Out] -((Log[e*((a + b*x)/(c + d*x))^n]*Log[1 - (2*(d^2*f - c*d*g + c^2*h)*(a + b
*x))/((2*b*d*f - b*c*g - a*d*g + 2*a*c*h - (b*c - a*d)*Sqrt[g^2 - 4*f*h])*(
c + d*x))]])/Sqrt[g^2 - 4*f*h]) + (Log[e*((a + b*x)/(c + d*x))^n]*Log[1 - (2
*(d^2*f - c*d*g + c^2*h)*(a + b*x))/((2*b*d*f - b*c*g - a*d*g + 2*a*c*h + (
b*c - a*d)*Sqrt[g^2 - 4*f*h])*(c + d*x))]])/Sqrt[g^2 - 4*f*h] - (n*PolyLog[2
, (2*(d^2*f - c*d*g + c^2*h)*(a + b*x))/((2*b*d*f - b*c*g - a*d*g + 2*a*c*h
 - (b*c - a*d)*Sqrt[g^2 - 4*f*h])*(c + d*x))])/Sqrt[g^2 - 4*f*h] + (n*PolyL
og[2, (2*(d^2*f - c*d*g + c^2*h)*(a + b*x))/((2*b*d*f - b*c*g - a*d*g + 2*a
*c*h + (b*c - a*d)*Sqrt[g^2 - 4*f*h])*(c + d*x))])/Sqrt[g^2 - 4*f*h]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2576

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g = Coef
f[P2x, x, 1], h = Coeff[P2x, x, 2]}, Dist[b*c - a*d, Subst[Int[(b^2*f - a*b
*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g + c^2*h
)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b*x)/(c
+ d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x, 2] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = n\int \frac{\log(a+bx)}{f+gx+hx^2}\,dx - n\int \frac{\log(c+dx)}{f+gx+hx^2}\,dx - \left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right.\right.$$

$$= n\int \left(\frac{2h\log(a+bx)}{\sqrt{g^2-4fh}\left(g-\sqrt{g^2-4fh}+2hx\right)} - \frac{2h\log(a+bx)}{\sqrt{g^2-4fh}\left(g+\sqrt{g^2-4fh}+2\right.}\right.$$

$$= \frac{2\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{\sqrt{g^2-4fh}} + \frac{(2hn)}{}$$

$$= \frac{2\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{\sqrt{g^2-4fh}} + \frac{n\log(}{}$$

$$= \frac{2\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{\sqrt{g^2-4fh}} + \frac{n\log(}{}$$

$$= \frac{2\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{\sqrt{g^2-4fh}} + \frac{n\log(}{}$$

Mathematica [A]
time = 0.29, size = 604, normalized size = 1.51

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*((a + b*x)/(c + d*x))^n]/(f + g*x + h*x^2),x]
```

```
[Out] (-2*Sqrt[g^2 - 4*f*h]*n*ArcTan[(g + 2*h*x)/Sqrt[-g^2 + 4*f*h]]*Log[a/b + x]
 + 2*Sqrt[g^2 - 4*f*h]*n*ArcTan[(g + 2*h*x)/Sqrt[-g^2 + 4*f*h]]*Log[c/d + x
] + 2*Sqrt[g^2 - 4*f*h]*ArcTan[(g + 2*h*x)/Sqrt[-g^2 + 4*f*h]]*Log[e*((a +
b*x)/(c + d*x))^n] + Sqrt[-g^2 + 4*f*h]*n*Log[a/b + x]*Log[1 - (2*h*(a + b*
x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - Sqrt[-g^2 + 4*f*h]*n*Log[a/b
+ x]*Log[1 + (2*h*(a + b*x))/(-2*a*h + b*(g + Sqrt[g^2 - 4*f*h]))] - Sqrt[-
```

```
g^2 + 4*f*h]*n*Log[c/d + x]*Log[1 - (2*h*(c + d*x))/(-(d*g) + 2*c*h + d*Sqr
t[g^2 - 4*f*h])] + Sqrt[-g^2 + 4*f*h]*n*Log[c/d + x]*Log[1 + (2*h*(c + d*x)
)/(-2*c*h + d*(g + Sqrt[g^2 - 4*f*h]))] + Sqrt[-g^2 + 4*f*h]*n*PolyLog[2, (
2*h*(a + b*x))/(2*a*h + b*(-g + Sqrt[g^2 - 4*f*h]))] - Sqrt[-g^2 + 4*f*h]*n
*PolyLog[2, (2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))] - Sqrt[-g^
2 + 4*f*h]*n*PolyLog[2, (2*h*(c + d*x))/(-(d*g) + 2*c*h + d*Sqrt[g^2 - 4*f*
h])] + Sqrt[-g^2 + 4*f*h]*n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g + Sqrt
[g^2 - 4*f*h]))])/Sqrt[-(g^2 - 4*f*h)^2]
```

Maple [F]
time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(e \left(\frac{bx+a}{dx+c}\right)^{n}\right)}{h\,x^{2}+gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*f*h-g^2>0)', see `assume?` for mo
re deta
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)/(d*x + c))^n*e)/(h*x^2 + g*x + f), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)/(h*x**2+g*x+f),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)/(d*x + c))^n*e)/(h*x^2 + g*x + f), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{h\,x^2+g\,x+f} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*((a + b*x)/(c + d*x))^n)/(f + g*x + h*x^2),x)
```

```
[Out] int(log(e*((a + b*x)/(c + d*x))^n)/(f + g*x + h*x^2), x)
```

3.86 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x(f+gx+hx^2)}\,dx$

Optimal. Leaf size=800

$$\frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f}-\frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f}-\frac{g\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right.\right.}{f\sqrt{g^2-4fh}}$$

```
[Out] n*ln(-b*x/a)*ln(b*x+a)/f-n*ln(-d*x/c)*ln(d*x+c)/f-ln(x)*(n*ln(b*x+a)-ln(e*(
(b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/f+1/2*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^
n)-n*ln(d*x+c))*ln(h*x^2+g*x+f)/f+n*polylog(2,1+b*x/a)/f-n*polylog(2,1+d*x/
c)/f-1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x+(-4*f*h+g^2)^(1/2))/(2*a*h-b*(g+(-4*f*h
+g^2)^(1/2))))*(1-g/(-4*f*h+g^2)^(1/2))/f+1/2*n*ln(d*x+c)*ln(-d*(g+2*h*x+(-
4*f*h+g^2)^(1/2))/(2*c*h-d*(g+(-4*f*h+g^2)^(1/2))))*(1-g/(-4*f*h+g^2)^(1/2)
)/f-1/2*n*polylog(2,2*h*(b*x+a)/(2*a*h-b*(g+(-4*f*h+g^2)^(1/2))))*(1-g/(-4*
f*h+g^2)^(1/2))/f+1/2*n*polylog(2,2*h*(d*x+c)/(2*c*h-d*(g+(-4*f*h+g^2)^(1/2
))))*(1-g/(-4*f*h+g^2)^(1/2))/f-1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x-(-4*f*h+g^2)
^(1/2))/(2*a*h-b*(g-(-4*f*h+g^2)^(1/2))))*(1+g/(-4*f*h+g^2)^(1/2))/f+1/2*n*
ln(d*x+c)*ln(-d*(g+2*h*x-(-4*f*h+g^2)^(1/2))/(2*c*h-d*(g-(-4*f*h+g^2)^(1/2)
)))*(1+g/(-4*f*h+g^2)^(1/2))/f-1/2*n*polylog(2,2*h*(b*x+a)/(2*a*h-b*(g-(-4*
f*h+g^2)^(1/2))))*(1+g/(-4*f*h+g^2)^(1/2))/f+1/2*n*polylog(2,2*h*(d*x+c)/(2
*c*h-d*(g-(-4*f*h+g^2)^(1/2))))*(1+g/(-4*f*h+g^2)^(1/2))/f-g*arctanh((2*h*x
+g)/(-4*f*h+g^2)^(1/2))*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))
/f/(-4*f*h+g^2)^(1/2)
```

Rubi [A]
time = 0.72, antiderivative size = 800, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 12, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2593, 2465, 2441, 2352, 2440, 2438, 719, 29, 648, 632, 212, 642}

Antiderivative was successfully verified.

```
[In] Int[Log[e*((a + b*x)/(c + d*x))^n]/(x*(f + g*x + h*x^2)),x]
```

```
[Out] (n*Log[-((b*x)/a)]*Log[a + b*x])/f - (n*Log[-((d*x)/c)]*Log[c + d*x])/f - (
g*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*h]]*(n*Log[a + b*x] - Log[e*((a + b*x)
/(c + d*x))^n] - n*Log[c + d*x]))/(f*Sqrt[g^2 - 4*f*h]) - (Log[x]*(n*Log[a
+ b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/f - ((1 + g/Sqrt
[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*
a*h - b*(g - Sqrt[g^2 - 4*f*h]))])/(2*f) + ((1 + g/Sqrt[g^2 - 4*f*h])*n*Lo
g[c + d*x]*Log[-((d*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d*(g - Sqrt[g
```

```
^2 - 4*f*h]))))])/(2*f) - ((1 - g/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b
*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))))])/(2
*f) + ((1 - g/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4*
f*h] + 2*h*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))))])/(2*f) + ((n*Log[a + b
*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f + g*x + h*x^2]
)/(2*f) - ((1 + g/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h -
b*(g - Sqrt[g^2 - 4*f*h]))])/(2*f) - ((1 - g/Sqrt[g^2 - 4*f*h])*n*PolyLog[2
, (2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))])/(2*f) + (n*PolyLog[
2, 1 + (b*x)/a])/f + ((1 + g/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x)
)/(2*c*h - d*(g - Sqrt[g^2 - 4*f*h]))])/(2*f) + ((1 - g/Sqrt[g^2 - 4*f*h])*
n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])/(2*f) -
(n*PolyLog[2, 1 + (d*x)/c])/f
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
```

```
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\int \frac{\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\left(f+gx+hx^2\right)}\,dx = n\int \frac{\log(a+bx)}{x\left(f+gx+hx^2\right)}\,dx - n\int \frac{\log(c+dx)}{x\left(f+gx+hx^2\right)}\,dx - \left(n\log(a+bx) - \log\left(e\left(\right.\right.\right.$$

$$= n\int \left(\frac{\log(a+bx)}{fx} + \frac{(-g-hx)\log(a+bx)}{f\left(f+gx+hx^2\right)}\right)dx - n\int\left(\frac{\log(c+dx)}{fx} + \frac{(-g-hx}{f\left(f+\right.}\right.$$

$$= -\frac{\log(x)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{f} + \frac{n\int \frac{\log(a+bx)}{x}\,dx}{f} + \frac{n\int}{}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{\log(x)\left(n\log(a+bx) - \log\left(\right.\right.}{f}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{g\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)(n}{}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{g\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)(n}{}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{g\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)(n}{}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{g\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)(n}{}$$

<u>Mathematica</u> [A]
time = 0.74, size = 625, normalized size = 0.78

$$\frac{2\log(x)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - \left(1+\frac{g}{\sqrt{g^2-4fh}}\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(g-\sqrt{g^2-4fh}+2hx\right) - \left(1-\frac{g}{\sqrt{g^2-4fh}}\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(g+\sqrt{g^2-4fh}+2hx\right) - 2n\left(\log(x)\left(\log\left(1+\frac{bx}{a}\right)-\log\left(1+\frac{dx}{c}\right)\right)+\text{Li}_2\left(-\frac{bx}{a}\right)-\text{Li}_2\left(-\frac{dx}{c}\right)\right) + \frac{\left(g+\sqrt{g^2-4fh}\right)n\left(\left(\log\left(\frac{2h(a+bx)}{-bg+2ah+b\sqrt{g^2-4fh}}\right)-\log\left(\frac{2h(c+dx)}{-dg+2ch+d\sqrt{g^2-4fh}}\right)\right)\log\left(g-\sqrt{g^2-4fh}+2hx\right)+\text{Li}_2\left(\frac{b\left(-g+\sqrt{g^2-4fh}-2hx\right)}{-bg+2ah+b\sqrt{g^2-4fh}}\right)-\text{Li}_2\left(\frac{d\left(-g+\sqrt{g^2-4fh}-2hx\right)}{2ch+d\left(-g+\sqrt{g^2-4fh}\right)}\right)\right)}{\sqrt{g^2-4fh}} + \frac{\left(-g+\sqrt{g^2-4fh}\right)n\left(\left(\log\left(\frac{2h(a+bx)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)-\log\left(\frac{2h(c+dx)}{2ch-d\left(g+\sqrt{g^2-4fh}\right)}\right)\right)\log\left(g+\sqrt{g^2-4fh}+2hx\right)+\text{Li}_2\left(\frac{b\left(g+\sqrt{g^2-4fh}+2hx\right)}{-2ah+b\left(g+\sqrt{g^2-4fh}\right)}\right)-\text{Li}_2\left(\frac{d\left(g+\sqrt{g^2-4fh}+2hx\right)}{-2ch+d\left(g+\sqrt{g^2-4fh}\right)}\right)\right)}{\sqrt{g^2-4fh}}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*((a + b*x)/(c + d*x))^n]/(x*(f + g*x + h*x^2)),x]
```

```
[Out] (2*Log[x]*Log[e*((a + b*x)/(c + d*x))^n] - (1 + g/Sqrt[g^2 - 4*f*h])*Log[e*
((a + b*x)/(c + d*x))^n]*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] - (1 - g/Sqrt[g
^2 - 4*f*h])*Log[e*((a + b*x)/(c + d*x))^n]*Log[g + Sqrt[g^2 - 4*f*h] + 2*h
*x] - 2*n*(Log[x]*(Log[1 + (b*x)/a] - Log[1 + (d*x)/c]) + PolyLog[2, -((b*x
)/a)] - PolyLog[2, -((d*x)/c)]) + ((g + Sqrt[g^2 - 4*f*h])*n*((Log[(2*h*(a
+ b*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - Log[(2*h*(c + d*x))/(-(d*
g) + 2*c*h + d*Sqrt[g^2 - 4*f*h])])*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + Po
lyLog[2, (b*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2
- 4*f*h])] - PolyLog[2, (d*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*c*h + d*(-g
 + Sqrt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h] + ((-g + Sqrt[g^2 - 4*f*h])*n*(
(Log[(2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))] - Log[(2*h*(c + d
*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*
x] + PolyLog[2, (b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*a*h + b*(g + Sqrt[g
^2 - 4*f*h]))] - PolyLog[2, (d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*c*h + d
*(g + Sqrt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h])/(2*f)
```

Maple [F]
time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{x\left(h\,x^{2}+gx+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)/x/(h*x^2+g*x+f),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)/x/(h*x^2+g*x+f),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(h*x^2+g*x+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*f*h-g^2>0)', see `assume?` for mo
re deta
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(h*x^2+g*x+f),x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)/(d*x + c))^n*e)/(h*x^3 + g*x^2 + f*x), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)/x/(h*x**2+g*x+f),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(h*x^2+g*x+f),x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)/(d*x + c))^n*e)/((h*x^2 + g*x + f)*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{x\,\left(h\,x^2+g\,x+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*((a + b*x)/(c + d*x))^n)/(x*(f + g*x + h*x^2)),x)
```

```
[Out] int(log(e*((a + b*x)/(c + d*x))^n)/(x*(f + g*x + h*x^2)), x)
```

3.87 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f+gx+hx^2)}\,dx$

Optimal. Leaf size=995

$$\frac{bn\log(x)}{af}-\frac{dn\log(x)}{cf}-\frac{bn\log(a+bx)}{af}-\frac{n\log(a+bx)}{fx}-\frac{gn\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f^2}+\frac{dn\log(c+dx)}{cf}+\frac{n\log(c+}{fx}$$

```
[Out] b*n*ln(x)/a/f-d*n*ln(x)/c/f-b*n*ln(b*x+a)/a/f-n*ln(b*x+a)/f/x-g*n*ln(-b*x/a
)*ln(b*x+a)/f^2+d*n*ln(d*x+c)/c/f+n*ln(d*x+c)/f/x+g*n*ln(-d*x/c)*ln(d*x+c)/
f^2+(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/f/x+g*ln(x)*(n*ln(b
*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/f^2-1/2*g*(n*ln(b*x+a)-ln(e*((
b*x+a)/(d*x+c))^n)-n*ln(d*x+c))*ln(h*x^2+g*x+f)/f^2-g*n*polylog(2,1+b*x/a)/
f^2+g*n*polylog(2,1+d*x/c)/f^2+1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x+(-4*f*h+g^2)^
(1/2))/(2*a*h-b*(g+(-4*f*h+g^2)^(1/2))))*(g+(2*f*h-g^2)/(-4*f*h+g^2)^(1/2))
/f^2-1/2*n*ln(d*x+c)*ln(-d*(g+2*h*x+(-4*f*h+g^2)^(1/2))/(2*c*h-d*(g+(-4*f*h
+g^2)^(1/2))))*(g+(2*f*h-g^2)/(-4*f*h+g^2)^(1/2))/f^2+1/2*n*polylog(2,2*h*(
b*x+a)/(2*a*h-b*(g+(-4*f*h+g^2)^(1/2))))*(g+(2*f*h-g^2)/(-4*f*h+g^2)^(1/2))
/f^2-1/2*n*polylog(2,2*h*(d*x+c)/(2*c*h-d*(g+(-4*f*h+g^2)^(1/2))))*(g+(2*f*
h-g^2)/(-4*f*h+g^2)^(1/2))/f^2+1/2*n*ln(b*x+a)*ln(-b*(g+2*h*x-(-4*f*h+g^2)^
(1/2))/(2*a*h-b*(g-(-4*f*h+g^2)^(1/2))))*(g+(-2*f*h+g^2)/(-4*f*h+g^2)^(1/2)
)/f^2-1/2*n*ln(d*x+c)*ln(-d*(g+2*h*x-(-4*f*h+g^2)^(1/2))/(2*c*h-d*(g-(-4*f*
h+g^2)^(1/2))))*(g+(-2*f*h+g^2)/(-4*f*h+g^2)^(1/2))/f^2+1/2*n*polylog(2,2*h
*(b*x+a)/(2*a*h-b*(g-(-4*f*h+g^2)^(1/2))))*(g+(-2*f*h+g^2)/(-4*f*h+g^2)^(1/
2))/f^2-1/2*n*polylog(2,2*h*(d*x+c)/(2*c*h-d*(g-(-4*f*h+g^2)^(1/2))))*(g+(-
2*f*h+g^2)/(-4*f*h+g^2)^(1/2))/f^2+(-2*f*h+g^2)*arctanh((2*h*x+g)/(-4*f*h+g
^2)^(1/2))*(n*ln(b*x+a)-ln(e*((b*x+a)/(d*x+c))^n)-n*ln(d*x+c))/f^2/(-4*f*h+
g^2)^(1/2)
```

Rubi [A]
time = 0.91, antiderivative size = 995, normalized size of antiderivative = 1.00, number of steps used = 40, number of rules used = 16, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {2593, 2465, 2442, 36, 29, 31, 2441, 2352, 2440, 2438, 723, 814, 648, 632, 212, 642}

Antiderivative was successfully verified.

```
[In] Int[Log[e*((a + b*x)/(c + d*x))^n]/(x^2*(f + g*x + h*x^2)),x]
```

```
[Out] (b*n*Log[x])/(a*f) - (d*n*Log[x])/(c*f) - (b*n*Log[a + b*x])/(a*f) - (n*Log
[a + b*x])/(f*x) - (g*n*Log[-((b*x)/a)]*Log[a + b*x])/f^2 + (d*n*Log[c + d*
```

```
x])/(c*f) + (n*Log[c + d*x])/(f*x) + (g*n*Log[-((d*x)/c)]*Log[c + d*x])/f^2
 + (n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])/(f*x)
 + ((g^2 - 2*f*h)*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*h]]*(n*Log[a + b*x] -
Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/(f^2*Sqrt[g^2 - 4*f*h]) +
 (g*Log[x]*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x
]))/f^2 + ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g
 - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h])))])/(2*f^
2) - ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g - Sq
rt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d*(g - Sqrt[g^2 - 4*f*h])))])/(2*f^2) +
((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g + Sqrt[g^
2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h])))])/(2*f^2) - ((g -
 (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4
*f*h] + 2*h*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h])))])/(2*f^2) - (g*(n*Log[
a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f + g*x + h
*x^2])/(2*f^2) + ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(
a + b*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h]))])/(2*f^2) + ((g - (g^2 - 2*f*
h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2
 - 4*f*h]))])/(2*f^2) - (g*n*PolyLog[2, 1 + (b*x)/a])/f^2 - ((g + (g^2 - 2*
f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g - Sqrt[g
^2 - 4*f*h]))])/(2*f^2) - ((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[
2, (2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])/(2*f^2) + (g*n*Pol
yLog[2, 1 + (d*x)/c])/f^2
```

<u>Rule 29</u>

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

<u>Rule 36</u>

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

<u>Rule 212</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

<u>Rule 632</u>

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rubi steps

$$\int \frac{\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2\left(f+gx+hx^2\right)}\,dx = n\int \frac{\log(a+bx)}{x^2\left(f+gx+hx^2\right)}\,dx - n\int \frac{\log(c+dx)}{x^2\left(f+gx+hx^2\right)}\,dx - \Big(n\log(a+bx) - \log \big($$

$$= \frac{n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)}{fx} + n\int \Big(\frac{\log(a+bx)}{fx^2} - \frac{g\log(a+}{f^2x}$$

$$= \frac{n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)}{fx} + \frac{n\int \frac{(g^2-fh+ghx)\log(a+bx)}{f+gx+hx^2}\,dx}{f^2} -$$

$$= -\frac{n\log(a+bx)}{fx} - \frac{gn\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f^2} + \frac{n\log(c+dx)}{fx} + \frac{gn\log\left(-\frac{dx}{c}\right)\log(}{f^2}$$

$$= -\frac{n\log(a+bx)}{fx} - \frac{gn\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f^2} + \frac{n\log(c+dx)}{fx} + \frac{gn\log\left(-\frac{dx}{c}\right)\log(}{f^2}$$

$$= \frac{bn\log(x)}{af} - \frac{dn\log(x)}{cf} - \frac{bn\log(a+bx)}{af} - \frac{n\log(a+bx)}{fx} - \frac{gn\log\left(-\frac{bx}{a}\right)\log(a+}{f^2}$$

$$= \frac{bn\log(x)}{af} - \frac{dn\log(x)}{cf} - \frac{bn\log(a+bx)}{af} - \frac{n\log(a+bx)}{fx} - \frac{gn\log\left(-\frac{bx}{a}\right)\log(a+}{f^2}$$

$$= \frac{bn\log(x)}{af} - \frac{dn\log(x)}{cf} - \frac{bn\log(a+bx)}{af} - \frac{n\log(a+bx)}{fx} - \frac{gn\log\left(-\frac{bx}{a}\right)\log(a+}{f^2}$$

Mathematica [A]
time = 0.62, size = 721, normalized size = 0.72

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*((a + b*x)/(c + d*x))^n]/(x^2*(f + g*x + h*x^2)),x]
```

```
[Out] ((-2*f*Log[e*((a + b*x)/(c + d*x))^n])/x - 2*g*Log[x]*Log[e*((a + b*x)/(c +
 d*x))^n] + (2*f*n*((b*c - a*d)*Log[x] - b*c*Log[a + b*x] + a*d*Log[c + d*x
]))/(a*c) + (g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*Log[e*((a + b*x)/(c + d*x
))^n]*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + (g + (-g^2 + 2*f*h)/Sqrt[g^2 - 4
*f*h])*Log[e*((a + b*x)/(c + d*x))^n]*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x] +
2*g*n*(Log[x]*(Log[1 + (b*x)/a] - Log[1 + (d*x)/c]) + PolyLog[2, -((b*x)/a)
] - PolyLog[2, -((d*x)/c)]) - ((g^2 - 2*f*h + g*Sqrt[g^2 - 4*f*h])*n*((Log[
(2*h*(a + b*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - Log[(2*h*(c + d*x
))/(-(d*g) + 2*c*h + d*Sqrt[g^2 - 4*f*h])])*Log[g - Sqrt[g^2 - 4*f*h] + 2*h
*x] + PolyLog[2, (b*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(-(b*g) + 2*a*h + b*S
qrt[g^2 - 4*f*h])] - PolyLog[2, (d*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*c*h
 + d*(-g + Sqrt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h] + ((g^2 - 2*f*h - g*Sqr
t[g^2 - 4*f*h])*n*((Log[(2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))
] - Log[(2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])*Log[g + Sqrt[
g^2 - 4*f*h] + 2*h*x] + PolyLog[2, (b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*
a*h + b*(g + Sqrt[g^2 - 4*f*h]))] - PolyLog[2, (d*(g + Sqrt[g^2 - 4*f*h] +
2*h*x))/(-2*c*h + d*(g + Sqrt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h])/(2*f^2)
```

Maple [F]
time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{x^{2}\left(h\,x^{2}+gx+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)/x^2/(h*x^2+g*x+f),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)/x^2/(h*x^2+g*x+f),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(h*x^2+g*x+f),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*f*h-g^2>0)', see `assume?` for mo
re deta
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(h*x^2+g*x+f),x, algorithm="fricas
")
```

```
[Out] integral(log(((b*x + a)/(d*x + c))^n*e)/(h*x^4 + g*x^3 + f*x^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)/x**2/(h*x**2+g*x+f),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(h*x^2+g*x+f),x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)/(d*x + c))^n*e)/((h*x^2 + g*x + f)*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{x^2\,\left(h\,x^2+g\,x+f\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*((a + b*x)/(c + d*x))^n)/(x^2*(f + g*x + h*x^2)),x)
```

```
[Out] int(log(e*((a + b*x)/(c + d*x))^n)/(x^2*(f + g*x + h*x^2)), x)
```

3.88 $\int \frac{\log\left(\frac{cx}{a+bx}\right)}{a+bx}\,dx$

Optimal. Leaf size=46

$$-\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b}-\frac{\text{Li}_2\left(1-\frac{a}{a+bx}\right)}{b}$$

```
[Out] -ln(a/(b*x+a))*ln(c*x/(b*x+a))/b-polylog(2,1-a/(b*x+a))/b
```

Rubi [A]
time = 0.10, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2544, 2458, 2378, 2370, 2352}

$$-\frac{\text{PolyLog}\left(2,1-\frac{a}{a+bx}\right)}{b}-\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*x)/(a + b*x)]/(a + b*x),x]
```

```
[Out] -((Log[a/(a + b*x)]*Log[(c*x)/(a + b*x)])/b) - PolyLog[2, 1 - a/(a + b*x)]/
b
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
```

```
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2544

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[(b*c - a*d)/(b*(c +
 d*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Dist[B*n*((b*c
- a*d)/g), Int[Log[(b*c - a*d)/(b*(c + d*x))]/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && EqQ[d*f - c*g, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(\frac{cx}{a+bx}\right)}{a+bx}\,dx &= -\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b}+\frac{a\int\frac{\log\left(\frac{a}{a+bx}\right)}{x(a+bx)}\,dx}{b}\\
&= -\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b}+\frac{a\text{Subst}\left(\int\frac{\log\left(\frac{a}{x}\right)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)}\,dx,x,a+bx\right)}{b^2}\\
&= -\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b}-\frac{a\text{Subst}\left(\int\frac{\log(ax)}{\left(-\frac{a}{b}+\frac{1}{bx}\right)x}\,dx,x,\frac{1}{a+bx}\right)}{b^2}\\
&= -\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b}-\frac{a\text{Subst}\left(\int\frac{\log(ax)}{\frac{1}{b}-\frac{ax}{b}}\,dx,x,\frac{1}{a+bx}\right)}{b^2}\\
&= -\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b}-\frac{\text{Li}_2\left(\frac{bx}{a+bx}\right)}{b}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 84, normalized size = 1.83

$$\frac{\log\left(-\frac{bx}{a}\right)\log\left(\frac{a}{a+bx}\right)}{b}+\frac{\log^2\left(\frac{a}{a+bx}\right)}{2b}-\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b}-\frac{\text{Li}_2\left(\frac{a+bx}{a}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*x)/(a + b*x)]/(a + b*x),x]
```

```
[Out] (Log[-((b*x)/a)]*Log[a/(a + b*x)])/b + Log[a/(a + b*x)]^2/(2*b) - (Log[a/(a
 + b*x)]*Log[(c*x)/(a + b*x)])/b - PolyLog[2, (a + b*x)/a]/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(46) = 92.
time = 1.66, size = 97, normalized size = 2.11

method	result	size
derivativedivides	$-\frac{\operatorname{dilog}\left(-\frac{\left(\frac{c}{b}-\frac{ac}{b(bx+a)}\right)b-c}{c}\right)}{b}-\frac{\ln\left(\frac{c}{b}-\frac{ac}{b(bx+a)}\right)\ln\left(-\frac{\left(\frac{c}{b}-\frac{ac}{b(bx+a)}\right)b-c}{c}\right)}{b}$	97
default	$-\frac{\operatorname{dilog}\left(-\frac{\left(\frac{c}{b}-\frac{ac}{b(bx+a)}\right)b-c}{c}\right)}{b}-\frac{\ln\left(\frac{c}{b}-\frac{ac}{b(bx+a)}\right)\ln\left(-\frac{\left(\frac{c}{b}-\frac{ac}{b(bx+a)}\right)b-c}{c}\right)}{b}$	97
risch	$-\frac{\operatorname{dilog}\left(-\frac{\left(\frac{c}{b}-\frac{ac}{b(bx+a)}\right)b-c}{c}\right)}{b}-\frac{\ln\left(\frac{c}{b}-\frac{ac}{b(bx+a)}\right)\ln\left(-\frac{\left(\frac{c}{b}-\frac{ac}{b(bx+a)}\right)b-c}{c}\right)}{b}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x/(b*x+a))/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -dilog(-((c/b-a*c/b/(b*x+a))*b-c)/c)/b-ln(c/b-a*c/b/(b*x+a))*ln(-((c/b-a*c/
b/(b*x+a))*b-c)/c)/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(45) = 90$.
time = 0.27, size = 95, normalized size = 2.07

$$\frac{\log\left(bx+a\right)\log\left(\frac{cx}{bx+a}\right)}{b}-\frac{\frac{c\log(bx+a)^2}{b}-\frac{2\left(\log\left(\frac{bx}{a}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{bx}{a}\right)\right)c}{b}}{2\,c}+\frac{\left(c\log\left(bx+a\right)-c\log\left(x\right)\right)\log\left(bx+a\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x/(b*x+a))/(b*x+a),x, algorithm="maxima")
```

```
[Out] log(b*x + a)*log(c*x/(b*x + a))/b - 1/2*(c*log(b*x + a)^2/b - 2*(log(b*x/a
+ 1)*log(x) + dilog(-b*x/a))*c/b)/c + (c*log(b*x + a) - c*log(x))*log(b*x +
 a)/(b*c)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x/(b*x+a))/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(log(c*x/(b*x + a))/(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left(\frac{cx}{a+bx}\right)}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x/(b*x+a))/(b*x+a),x)
```

```
[Out] Integral(log(c*x/(a + b*x))/(a + b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x/(b*x+a))/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(log(c*x/(b*x + a))/(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(\frac{c\,x}{a+b\,x}\right)}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((c*x)/(a + b*x))/(a + b*x),x)
```

```
[Out] int(log((c*x)/(a + b*x))/(a + b*x), x)
```

3.89 $\int \frac{\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx$

Optimal. Leaf size=20

$$\frac{\log^3\left(\frac{cx}{a+bx}\right)}{3a}$$

```
[Out] 1/3*ln(c*x/(b*x+a))^3/a
```

Rubi [A]
time = 0.05, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2562, 2339, 30}

$$\frac{\log^3\left(\frac{cx}{a+bx}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*x)/(a + b*x)]^2/(x*(a + b*x)),x]
```

```
[Out] Log[(c*x)/(a + b*x)]^3/(3*a)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\int \frac{\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx = \frac{\log^3\left(\frac{cx}{a+bx}\right)}{3a}$$

Mathematica [A]
time = 0.06, size = 20, normalized size = 1.00

$$\frac{\log^3\left(\frac{cx}{a+bx}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*x)/(a + b*x)]^2/(x*(a + b*x)),x]
```

```
[Out] Log[(c*x)/(a + b*x)]^3/(3*a)
```

Maple [A]
time = 0.56, size = 29, normalized size = 1.45

method	result	size
norman	$\frac{\ln\left(\frac{cx}{bx+a}\right)^3}{3a}$	19
derivativedivides	$\frac{\ln\left(\frac{c}{b}-\frac{ac}{b(bx+a)}\right)^3}{3a}$	29
default	$\frac{\ln\left(\frac{c}{b}-\frac{ac}{b(bx+a)}\right)^3}{3a}$	29
risch	$\frac{\ln\left(\frac{c}{b}-\frac{ac}{b(bx+a)}\right)^3}{3a}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x/(b*x+a))^2/x/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a*ln(c/b-a*c/b/(b*x+a))^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(18) = 36$.
time = 0.28, size = 141, normalized size = 7.05

$$-\left(\frac{\log(bx+a)}{a}-\frac{\log(x)}{a}\right)\log\left(\frac{cx}{bx+a}\right)^2-\frac{\left(c\log(bx+a)^2-2\,c\log(bx+a)\log(x)+c\log(x)^2\right)\log\left(\frac{cx}{bx+a}\right)}{ac}-\frac{c^2\log(bx+a)^3-3\,c^2\log(bx+a)^2\log(x)+3\,c^2\log(bx+a)\log(x)^2-c^2\log(x)^3}{3\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(log(b*x + a)/a - log(x)/a)*log(c*x/(b*x + a))^2 - (c*log(b*x + a)^2 - 2*c
*log(b*x + a)*log(x) + c*log(x)^2)*log(c*x/(b*x + a))/(a*c) - 1/3*(c^2*log(
```

```
b*x + a)^3 - 3*c^2*log(b*x + a)^2*log(x) + 3*c^2*log(b*x + a)*log(x)^2 - c^
2*log(x)^3)/(a*c^2)
```

Fricas [A]
time = 0.34, size = 18, normalized size = 0.90

$$\frac{\log\left(\frac{cx}{bx+a}\right)^3}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="fricas")
[Out] 1/3*log(c*x/(b*x + a))^3/a
```

Sympy [A]
time = 0.05, size = 14, normalized size = 0.70

$$\frac{\log\left(\frac{cx}{a+bx}\right)^3}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x/(b*x+a))**2/x/(b*x+a),x)
[Out] log(c*x/(a + b*x))**3/(3*a)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="giac")
[Out] integrate(log(c*x/(b*x + a))^2/((b*x + a)*x), x)
```

Mupad [B]
time = 0.26, size = 18, normalized size = 0.90

$$\frac{\ln\left(\frac{c\,x}{a+b\,x}\right)^3}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((c*x)/(a + b*x))^2/(x*(a + b*x)),x)
[Out] log((c*x)/(a + b*x))^3/(3*a)
```

3.90 $\int \frac{\log\left(\frac{a}{a+bx}\right)\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx$

Optimal. Leaf size=82

$$-\frac{\log^2\left(\frac{cx}{a+bx}\right)\mathrm{Li}_2\left(1-\frac{a}{a+bx}\right)}{a}+\frac{2\log\left(\frac{cx}{a+bx}\right)\mathrm{Li}_3\left(1-\frac{a}{a+bx}\right)}{a}-\frac{2\mathrm{Li}_4\left(1-\frac{a}{a+bx}\right)}{a}$$

```
[Out] -ln(c*x/(b*x+a))^2*polylog(2,1-a/(b*x+a))/a+2*ln(c*x/(b*x+a))*polylog(3,1-a
/(b*x+a))/a-2*polylog(4,1-a/(b*x+a))/a
```

Rubi [A]
time = 0.10, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {2588, 2590, 6745}

$$-\frac{\text{PolyLog}\left(2,1-\frac{a}{a+bx}\right)\log^2\left(\frac{cx}{a+bx}\right)}{a}+\frac{2\text{PolyLog}\left(3,1-\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{a}-\frac{2\text{PolyLog}\left(4,1-\frac{a}{a+bx}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[a/(a + b*x)]*Log[(c*x)/(a + b*x)]^2)/(x*(a + b*x)),x]
```

```
[Out] -((Log[(c*x)/(a + b*x)]^2*PolyLog[2, 1 - a/(a + b*x)])/a) + (2*Log[(c*x)/(a
 + b*x)]*PolyLog[3, 1 - a/(a + b*x)])/a - (2*PolyLog[4, 1 - a/(a + b*x)])/a
```

Rule 2588

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[(v - 1)*((c + d
*x)/(a + b*x))], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[(-h)*PolyLog[2,
 1 - v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(b*c - a*d)), x] + Dist[h*p
*r*s, Int[PolyLog[2, 1 - v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e,
 f, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 2590

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_)*PolyLog[n_, v_], x_Symbol] :> With[{g = Simplify[v*((c +
 d*x)/(a + b*x))], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[h*PolyLog[n +
 1, v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(b*c - a*d)), x] - Dist[h*p*
r*s, Int[PolyLog[n + 1, v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/((
a + b*x)*(c + d*x))), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e,
f, n, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(\frac{a}{a+bx}\right)\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx &= -\frac{\log^2\left(\frac{cx}{a+bx}\right)\text{Li}_2\left(1-\frac{a}{a+bx}\right)}{a}+2\int \frac{\log\left(\frac{cx}{a+bx}\right)\text{Li}_2\left(1-\frac{a}{a+bx}\right)}{x(a+bx)}\,dx\\ &= -\frac{\log^2\left(\frac{cx}{a+bx}\right)\text{Li}_2\left(1-\frac{a}{a+bx}\right)}{a}+\frac{2\log\left(\frac{cx}{a+bx}\right)\text{Li}_3\left(1-\frac{a}{a+bx}\right)}{a}-2\int \frac{\text{Li}_3\left(1-\frac{a}{a+bx}\right)}{x(a+bx)}\\ &= -\frac{\log^2\left(\frac{cx}{a+bx}\right)\text{Li}_2\left(1-\frac{a}{a+bx}\right)}{a}+\frac{2\log\left(\frac{cx}{a+bx}\right)\text{Li}_3\left(1-\frac{a}{a+bx}\right)}{a}-\frac{2\text{Li}_4\left(1-\frac{a}{a+bx}\right)}{a}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 76, normalized size = 0.93

$$-\frac{\log^2\left(\frac{cx}{a+bx}\right)\text{Li}_2\left(\frac{bx}{a+bx}\right)}{a}+\frac{2\log\left(\frac{cx}{a+bx}\right)\text{Li}_3\left(\frac{bx}{a+bx}\right)}{a}-\frac{2\text{Li}_4\left(\frac{bx}{a+bx}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[a/(a + b*x)]*Log[(c*x)/(a + b*x)]^2)/(x*(a + b*x)),x]
[Out] -((Log[(c*x)/(a + b*x)]^2*PolyLog[2, (b*x)/(a + b*x)])/a) + (2*Log[(c*x)/(a
 + b*x)]*PolyLog[3, (b*x)/(a + b*x)])/a - (2*PolyLog[4, (b*x)/(a + b*x)])/a
```

Maple [F]
time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(\frac{a}{bx+a}\right)\ln\left(\frac{cx}{bx+a}\right)^2}{x\left(bx+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a/(b*x+a))*ln(c*x/(b*x+a))^2/x/(b*x+a),x)
[Out] int(ln(a/(b*x+a))*ln(c*x/(b*x+a))^2/x/(b*x+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a/(b*x+a))*log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="maxima"
)
```

```
[Out] 1/4*(log(b*x + a)^4 - 4*log(b*x + a)^3*log(x))/a + integrate((a*log(a)*log(
c)^2 + 2*a*log(a)*log(c)*log(x) + a*log(a)*log(x)^2 + (a*(log(a) + 2*log(c)
) + (3*b*x + 2*a)*log(x))*log(b*x + a)^2 - (2*a*(log(a) + log(c))*log(x) +
a*log(x)^2 + (2*log(a)*log(c) + log(c)^2)*a)*log(b*x + a))/(a*b*x^2 + a^2*x
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a/(b*x+a))*log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="fricas"
)
```

```
[Out] integral(log(c*x/(b*x + a))^2*log(a/(b*x + a))/(b*x^2 + a*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)^{2}}{x\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a/(b*x+a))*ln(c*x/(b*x+a))**2/x/(b*x+a),x)
```

```
[Out] Integral(log(a/(a + b*x))*log(c*x/(a + b*x))**2/(x*(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a/(b*x+a))*log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(log(c*x/(b*x + a))^2*log(a/(b*x + a))/((b*x + a)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(\frac{c\,x}{a+b\,x}\right)^{2}\,\ln\left(\frac{a}{a+b\,x}\right)}{x\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log((c*x)/(a + b*x))^2*log(a/(a + b*x)))/(x*(a + b*x)),x)
```

```
[Out] int((log((c*x)/(a + b*x))^2*log(a/(a + b*x)))/(x*(a + b*x)), x)
```

3.91 $\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{(c+dx)(ag+bgx)}\,dx$

Optimal. Leaf size=150

$$-\frac{\log^2\left(\frac{e(a+bx)}{c+dx}\right)\mathrm{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}+\frac{2\log\left(\frac{e(a+bx)}{c+dx}\right)\mathrm{Li}_3\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}-\frac{2\mathrm{Li}_4\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}$$

```
[Out] -ln(e*(b*x+a)/(d*x+c))^2*polylog(2,1+(a*d-b*c)/b/(d*x+c))/(-a*d+b*c)/g+2*ln
(e*(b*x+a)/(d*x+c))*polylog(3,1+(a*d-b*c)/b/(d*x+c))/(-a*d+b*c)/g-2*polylog
(4,1+(a*d-b*c)/b/(d*x+c))/(-a*d+b*c)/g
```

Rubi [A]
time = 0.16, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}} = 0.055$, Rules used = {2588, 2590, 6745}

$$-\frac{\mathrm{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{g(bc-ad)}+\frac{2\mathrm{PolyLog}\left(3,1-\frac{bc-ad}{b(c+dx)}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)}{g(bc-ad)}-\frac{2\mathrm{PolyLog}\left(4,1-\frac{bc-ad}{b(c+dx)}\right)}{g(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x))/(c + d*x)]^2)/((c + d
*x)*(a*g + b*g*x)),x]
```

```
[Out] -((Log[(e*(a + b*x))/(c + d*x)]^2*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))]
)/((b*c - a*d)*g)) + (2*Log[(e*(a + b*x))/(c + d*x)]*PolyLog[3, 1 - (b*c -
a*d)/(b*(c + d*x))])/((b*c - a*d)*g) - (2*PolyLog[4, 1 - (b*c - a*d)/(b*(c
+ d*x))])/((b*c - a*d)*g)
```

Rule 2588

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[(v - 1)*((c + d
*x)/(a + b*x))], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[(-h)*PolyLog[2,
 1 - v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(b*c - a*d)), x] + Dist[h*p
*r*s, Int[PolyLog[2, 1 - v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e,
 f, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 2590

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_)*PolyLog[n_, v_], x_Symbol] :> With[{g = Simplify[v*((c +
 d*x)/(a + b*x))], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[h*PolyLog[n +
 1, v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(b*c - a*d)), x] - Dist[h*p*
```

```
r*s, Int[PolyLog[n + 1, v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/((
a + b*x)*(c + d*x))), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e,
f, n, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{(c+dx)(ag+bgx)}\,dx &= -\frac{\log^2\left(\frac{e(a+bx)}{c+dx}\right)\mathrm{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}+\frac{2\int\frac{\log\left(\frac{e(a+bx)}{c+dx}\right)\mathrm{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}\,dx}{g}\\ &= -\frac{\log^2\left(\frac{e(a+bx)}{c+dx}\right)\mathrm{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}+\frac{2\log\left(\frac{e(a+bx)}{c+dx}\right)\mathrm{Li}_3\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}-\\ &= -\frac{\log^2\left(\frac{e(a+bx)}{c+dx}\right)\mathrm{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}+\frac{2\log\left(\frac{e(a+bx)}{c+dx}\right)\mathrm{Li}_3\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}-\end{aligned}$$

Mathematica [A]
time = 0.03, size = 110, normalized size = 0.73

$$\frac{-\log^2\left(\frac{e(a+bx)}{c+dx}\right)\mathrm{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)+2\log\left(\frac{e(a+bx)}{c+dx}\right)\mathrm{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)-2\mathrm{Li}_4\left(\frac{d(a+bx)}{b(c+dx)}\right)}{(bc-ad)g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x))/(c + d*x)]^2)/(
(c + d*x)*(a*g + b*g*x)),x]
```

```
[Out] (-(Log[(e*(a + b*x))/(c + d*x)]^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])
+ 2*Log[(e*(a + b*x))/(c + d*x)]*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))] -
2*PolyLog[4, (d*(a + b*x))/(b*(c + d*x))])/((b*c - a*d)*g)
```

Maple [A]
time = 3.43, size = 238, normalized size = 1.59

method	result

default	$\frac{-\frac{\ln\left(\frac{e(bx+a)}{dx+c}\right)^{3}\ln\left(-\frac{\frac{e(bx+a)d}{dx+c}-be}{be}\right)}{3(ad-cb)}+\frac{\ln\left(\frac{e(bx+a)}{dx+c}\right)^{3}\ln\left(1-\frac{d(bx+a)}{b(dx+c)}\right)}{3ad-3cb}+\frac{\ln\left(\frac{e(bx+a)}{dx+c}\right)^{2}\mathrm{polylog}\left(2,\frac{d(bx+a)}{b(dx+c)}\right)}{ad-cb}-\frac{2\ln\left(\frac{e(bx+a)}{dx+c}\right)\mathrm{polylog}\left(3,\frac{d(bx}{b(dx}\right.}{ad-cb}}{g}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((-a*d+b*c)/b/(d*x+c))*ln(e*(b*x+a)/(d*x+c))^2/(d*x+c)/(b*g*x+a*g),x,
method=_RETURNVERBOSE)
```

```
[Out] 1/g*(-1/3/(a*d-b*c)*ln(e*(b*x+a)/(d*x+c))^3*ln(-(e*(b*x+a)/(d*x+c)*d-b*e)/b
/e)+1/3/(a*d-b*c)*ln(e*(b*x+a)/(d*x+c))^3*ln(1-d*(b*x+a)/b/(d*x+c))+1/(a*d-
b*c)*ln(e*(b*x+a)/(d*x+c))^2*polylog(2,d*(b*x+a)/b/(d*x+c))-2/(a*d-b*c)*ln(
e*(b*x+a)/(d*x+c))*polylog(3,d*(b*x+a)/b/(d*x+c))+2/(a*d-b*c)*polylog(4,d*(
b*x+a)/b/(d*x+c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*d+b*c)/b/(d*x+c))*log(e*(b*x+a)/(d*x+c))^2/(d*x+c)/(b*g*x
+a*g),x, algorithm="maxima")
```

```
[Out] -1/4*(4*log(b*x + a)*log(d*x + c)^3 - log(d*x + c)^4)/(b*c*g - a*d*g) - int
egrate((((d*log(b*c - a*d) - d*log(b))*a - (c*log(b*c - a*d) - c*log(b))*b)
*log(b*x + a)^2 + ((d*log(b*c - a*d) - d*log(b) + 2*d)*a - (c*(log(b*c - a*
d) + 2) - c*log(b))*b - (3*b*d*x + 2*b*c + a*d)*log(b*x + a))*log(d*x + c)^
2 + (d*log(b*c - a*d) - d*log(b))*a - (c*log(b*c - a*d) - c*log(b))*b + 2*(
(d*log(b*c - a*d) - d*log(b))*a - (c*log(b*c - a*d) - c*log(b))*b)*log(b*x
+ a) + ((b*c - a*d)*log(b*x + a)^2 - (2*d*log(b*c - a*d) - 2*d*log(b) + d)*
a + (c*(2*log(b*c - a*d) + 1) - 2*c*log(b))*b - 2*((d*log(b*c - a*d) - d*lo
g(b) + d)*a - (c*(log(b*c - a*d) + 1) - c*log(b))*b)*log(b*x + a))*log(d*x
+ c))/(a*b*c^2*g - a^2*c*d*g + (b^2*c*d*g - a*b*d^2*g)*x^2 + (b^2*c^2*g - a
^2*d^2*g)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*d+b*c)/b/(d*x+c))*log(e*(b*x+a)/(d*x+c))^2/(d*x+c)/(b*g*x
+a*g),x, algorithm="fricas")
```

```
[Out] integral(log((b*x + a)*e/(d*x + c))^2*log((b*c - a*d)/(b*d*x + b*c))/(b*d*g
*x^2 + a*c*g + (b*c + a*d)*g*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{d\int\frac{\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^3}{c+dx}\,dx}{3g\left(ad-bc\right)}-\frac{\log\left(\frac{-ad+bc}{b(c+dx)}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)^3}{3adg-3bcg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((-a*d+b*c)/b/(d*x+c))*ln(e*(b*x+a)/(d*x+c))**2/(d*x+c)/(b*g*x+
a*g),x)
```

```
[Out] -d*Integral(log(a*e/(c + d*x) + b*e*x/(c + d*x))**3/(c + d*x), x)/(3*g*(a*d
 - b*c)) - log((-a*d + b*c)/(b*(c + d*x)))*log(e*(a + b*x)/(c + d*x))**3/(3
*a*d*g - 3*b*c*g)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*d+b*c)/b/(d*x+c))*log(e*(b*x+a)/(d*x+c))^2/(d*x+c)/(b*g*x
+a*g),x, algorithm="giac")
```

```
[Out] integrate(log((b*x + a)*e/(d*x + c))^2*log((b*c - a*d)/((d*x + c)*b))/((b*g
*x + a*g)*(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)^2\,\ln\left(-\frac{a\,d-b\,c}{b\,\left(c+d\,x\right)}\right)}{\left(a\,g+b\,g\,x\right)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log((e*(a + b*x))/(c + d*x))^2*log(-(a*d - b*c)/(b*(c + d*x))))/((a*g
+ b*g*x)*(c + d*x)),x)
```

```
[Out] int((log((e*(a + b*x))/(c + d*x))^2*log(-(a*d - b*c)/(b*(c + d*x))))/((a*g
+ b*g*x)*(c + d*x)), x)
```

3.92 $\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(ag+bgx)}\,dx$

Optimal. Leaf size=160

$$-\frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\mathrm{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}+\frac{2n\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\mathrm{Li}_3\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}-\frac{2n^2\mathrm{Li}_4\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}$$

```
[Out] -ln(e*((b*x+a)/(d*x+c))^n)^2*polylog(2,1+(a*d-b*c)/b/(d*x+c))/(-a*d+b*c)/g+
2*n*ln(e*((b*x+a)/(d*x+c))^n)*polylog(3,1+(a*d-b*c)/b/(d*x+c))/(-a*d+b*c)/g
-2*n^2*polylog(4,1+(a*d-b*c)/b/(d*x+c))/(-a*d+b*c)/g
```

Rubi [A]
time = 0.16, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}} = 0.052$, Rules used = {2588, 2590, 6745}

$$-\frac{\mathrm{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(bc-ad)}+\frac{2n\mathrm{PolyLog}\left(3,1-\frac{bc-ad}{b(c+dx)}\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(bc-ad)}-\frac{2n^2\mathrm{PolyLog}\left(4,1-\frac{bc-ad}{b(c+dx)}\right)}{g(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[e*((a + b*x)/(c + d*x))^n]^2*Log[(b*c - a*d)/(b*(c + d*x))])/((c +
 d*x)*(a*g + b*g*x)),x]
```

```
[Out] -((Log[e*((a + b*x)/(c + d*x))^n]^2*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x)
)])/((b*c - a*d)*g)) + (2*n*Log[e*((a + b*x)/(c + d*x))^n]*PolyLog[3, 1 - (
b*c - a*d)/(b*(c + d*x))])/((b*c - a*d)*g) - (2*n^2*PolyLog[4, 1 - (b*c - a
*d)/(b*(c + d*x))])/((b*c - a*d)*g)
```

Rule 2588

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[(v - 1)*((c + d
*x)/(a + b*x))], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[(-h)*PolyLog[2,
 1 - v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(b*c - a*d)), x] + Dist[h*p
*r*s, Int[PolyLog[2, 1 - v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e,
 f, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 2590

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_)*PolyLog[n_, v_], x_Symbol] :> With[{g = Simplify[v*((c +
 d*x)/(a + b*x))], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[h*PolyLog[n +
 1, v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(b*c - a*d)), x] - Dist[h*p*
r*s, Int[PolyLog[n + 1, v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/((
```

```
a + b*x)*(c + d*x))), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e,
f, n, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(ag+bgx)}\,dx &= -\frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\mathrm{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}+\frac{(2n)\int\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\mathrm{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right.}{(a+bx)(c+dx)}}{g}\\ &= -\frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\mathrm{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}+\frac{2n\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\mathrm{Li}_3\left(1-\frac{bc-ad}{b(c+dx}\right.}{(bc-ad)g}\\ &= -\frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\mathrm{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}+\frac{2n\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\mathrm{Li}_3\left(1-\frac{bc-ad}{b(c+dx}\right.}{(bc-ad)g}\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 559 vs. 2(160) = 320.
time = 0.32, size = 559, normalized size = 3.49

$$\frac{\log\left(\frac{a+bx}{c+dx}\right)\left(3\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-3n\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{a+bx}{c+dx}\right)+n^2\log^2\left(\frac{a+bx}{c+dx}\right)\right)\log\left(\frac{bc-ad}{bc+bdx}\right)+\frac{3}{2}\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)^2\left(-\log^2\left(\frac{c}{d}+x\right)-2\log\left(\frac{a}{b}+x\right)\log(c+dx)+2\log\left(\frac{c}{d}+x\right)\log(c+dx)+2\log\left(\frac{a+bx}{c+dx}\right)\log(c+dx)+2\log\left(\frac{a}{b}+x\right)\log\left(\frac{b(c+dx)}{bc-ad}\right)+2\mathrm{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)\right)+3n\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+n\log\left(\frac{a+bx}{c+dx}\right)\right)\left(\log^2\left(\frac{a+bx}{c+dx}\right)\log\left(\frac{bc-ad}{bc+bdx}\right)+2\log\left(\frac{a+bx}{c+dx}\right)\mathrm{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)-2\mathrm{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)\right)-n^2\left(\log^3\left(\frac{a+bx}{c+dx}\right)\log\left(\frac{bc-ad}{bc+bdx}\right)+3\log^2\left(\frac{a+bx}{c+dx}\right)\mathrm{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)-6\log\left(\frac{a+bx}{c+dx}\right)\mathrm{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)+6\mathrm{Li}_4\left(\frac{d(a+bx)}{b(c+dx)}\right)\right)}{3(bc-ad)g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[e*((a + b*x)/(c + d*x))^n]^2*Log[(b*c - a*d)/(b*(c + d*x))])
/((c + d*x)*(a*g + b*g*x)),x]
```

```
[Out] (Log[(a + b*x)/(c + d*x)]*(3*Log[e*((a + b*x)/(c + d*x))^n]^2 - 3*n*Log[e*(
(a + b*x)/(c + d*x))^n]*Log[(a + b*x)/(c + d*x)] + n^2*Log[(a + b*x)/(c + d
*x)]^2)*Log[(b*c - a*d)/(b*c + b*d*x)] + (3*(Log[e*((a + b*x)/(c + d*x))^n]
 - n*Log[(a + b*x)/(c + d*x)])^2*(-Log[c/d + x]^2 - 2*Log[a/b + x]*Log[c +
d*x] + 2*Log[c/d + x]*Log[c + d*x] + 2*Log[(a + b*x)/(c + d*x)]*Log[c + d*x
] + 2*Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + 2*PolyLog[2, (d*(a + b*
x))/(-(b*c) + a*d)]))/2 + 3*n*(-Log[e*((a + b*x)/(c + d*x))^n] + n*Log[(a +
 b*x)/(c + d*x)])*(Log[(a + b*x)/(c + d*x)]^2*Log[(b*c - a*d)/(b*c + b*d*x)
] + 2*Log[(a + b*x)/(c + d*x)]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))] - 2*
PolyLog[3, (d*(a + b*x))/(b*(c + d*x))]) - n^2*(Log[(a + b*x)/(c + d*x)]^3*
Log[(b*c - a*d)/(b*c + b*d*x)] + 3*Log[(a + b*x)/(c + d*x)]^2*PolyLog[2, (d
*(a + b*x))/(b*(c + d*x))] - 6*Log[(a + b*x)/(c + d*x)]*PolyLog[3, (d*(a +
```

```
b*x))/(b*(c + d*x))] + 6*PolyLog[4, (d*(a + b*x))/(b*(c + d*x))]))/(3*(b*c
- a*d)*g)
```

Maple [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{2} \ln \left(\frac{-ad+cb}{b(dx+c)}\right)}{(dx+c)\,(bgx+ag)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)^2*ln((-a*d+b*c)/b/(d*x+c))/(d*x+c)/(b*g*x+a*g
),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)^2*ln((-a*d+b*c)/b/(d*x+c))/(d*x+c)/(b*g*x+a*g
),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log((-a*d+b*c)/b/(d*x+c))/(d*x+c)/(b
*g*x+a*g),x, algorithm="maxima")
```

```
[Out] integrate(log(((b*x + a)/(d*x + c))^n*e)^2*log((b*c - a*d)/((d*x + c)*b))/(
(b*g*x + a*g)*(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log((-a*d+b*c)/b/(d*x+c))/(d*x+c)/(b
*g*x+a*g),x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)/(d*x + c))^n*e)^2*log((b*c - a*d)/(b*d*x + b*c))/(b
*d*g*x^2 + a*c*g + (b*c + a*d)*g*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\log \left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)^{2} \log \left(-\frac{ad}{bc+bdx}+\frac{bc}{bc+bdx}\right)}{ac+adx+bcx+bdx^{2}}\,dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)**2*ln((-a*d+b*c)/b/(d*x+c))/(d*x+c)/(b
*g*x+a*g),x)
```

```
[Out] Integral(log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2*log(-a*d/(b*c + b*d*x)
+ b*c/(b*c + b*d*x))/(a*c + a*d*x + b*c*x + b*d*x**2), x)/g
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log((-a*d+b*c)/b/(d*x+c))/(d*x+c)/(b
*g*x+a*g),x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)/(d*x + c))^n*e)^2*log((b*c - a*d)/((d*x + c)*b))/(
(b*g*x + a*g)*(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(-\frac{a\,d-b\,c}{b\,\left(c+d\,x\right)}\right)\,{\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}^2}{\left(a\,g+b\,g\,x\right)\,\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(-(a*d - b*c)/(b*(c + d*x)))*log(e*((a + b*x)/(c + d*x))^n)^2)/((a*
g + b*g*x)*(c + d*x)),x)
```

```
[Out] int((log(-(a*d - b*c)/(b*(c + d*x)))*log(e*((a + b*x)/(c + d*x))^n)^2)/((a*
g + b*g*x)*(c + d*x)), x)
```

3.93 $\int \log\left(\frac{c(b+ax)}{x}\right)\,dx$

Optimal. Leaf size=25

$$x\log\left(ac+\frac{bc}{x}\right)+\frac{b\log(b+ax)}{a}$$

```
[Out] x*ln(a*c+b*c/x)+b*ln(a*x+b)/a
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2503, 2498, 269, 31}

$$x\log\left(ac+\frac{bc}{x}\right)+\frac{b\log(ax+b)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*(b + a*x))/x],x]
```

```
[Out] x*Log[a*c + (b*c)/x] + (b*Log[b + a*x])/a
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2503

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.), x_Symbol] :> Int[(a + b*Lo
g[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b, c, p, q}, x] && BinomialQ[v
, x] &&  !BinomialMatchQ[v, x]
```

Rubi steps

$$\int \log\left(\frac{c(b+ax)}{x}\right) dx = \int \log\left(ac+\frac{bc}{x}\right) dx$$
$$= x\log\left(ac+\frac{bc}{x}\right) + (bc)\int \frac{1}{\left(ac+\frac{bc}{x}\right)x} dx$$
$$= x\log\left(ac+\frac{bc}{x}\right) + (bc)\int \frac{1}{bc+acx} dx$$
$$= x\log\left(ac+\frac{bc}{x}\right) + \frac{b\log(b+ax)}{a}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.12

$$\frac{b\log(x)}{a} + \frac{(b+ax)\log\left(\frac{c(b+ax)}{x}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*(b + a*x))/x],x]
```

```
[Out] (b*Log[x])/a + ((b + a*x)*Log[(c*(b + a*x))/x])/a
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(25) = 50$.
time = 0.34, size = 54, normalized size = 2.16

method	result	size
risch	$x\ln\left(\frac{c(ax+b)}{x}\right)+\frac{b\ln(ax+b)}{a}$	26
derivativedivides	$-cb\left(\frac{\ln\left(-\frac{bc}{x}\right)}{ac}-\frac{\ln\left(ca+\frac{bc}{x}\right)\left(ca+\frac{bc}{x}\right)x}{a\,c^{2}b}\right)$	54
default	$-cb\left(\frac{\ln\left(-\frac{bc}{x}\right)}{ac}-\frac{\ln\left(ca+\frac{bc}{x}\right)\left(ca+\frac{bc}{x}\right)x}{a\,c^{2}b}\right)$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a*x+b)/x),x,method=_RETURNVERBOSE)
```

```
[Out] -c*b*(1/a/c*ln(-b*c/x)-ln(c*a+b*c/x)*(c*a+b*c/x)/a/c^2/b*x)
```

Maxima [A]
time = 0.26, size = 25, normalized size = 1.00

$$x\log\left(\frac{(ax+b)c}{x}\right) + \frac{b\log\left(ax+b\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)/x),x, algorithm="maxima")
```

```
[Out] x*log((a*x + b)*c/x) + b*log(a*x + b)/a
```

Fricas [A]
time = 0.36, size = 29, normalized size = 1.16

$$\frac{ax\log\left(\frac{acx+bc}{x}\right)+b\log\left(ax+b\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)/x),x, algorithm="fricas")
```

```
[Out] (a*x*log((a*c*x + b*c)/x) + b*log(a*x + b))/a
```

Sympy [A]
time = 0.04, size = 20, normalized size = 0.80

$$x\log\left(\frac{c(ax+b)}{x}\right)+\frac{b\log\left(ax+b\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a*x+b)/x),x)
```

```
[Out] x*log(c*(a*x + b)/x) + b*log(a*x + b)/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(25) = 50.
time = 3.16, size = 153, normalized size = 6.12

$$\frac{b^2c^2\left(\frac{\log\left(\frac{|acx+bc|}{|x|}\right)}{ac}-\frac{\log\left(\left|-ac+\frac{acx+bc}{x}\right|\right)}{ac}\right)-\frac{b^2c^2\log\left(-\left(b-\frac{a}{\frac{a}{b}-\frac{acx+bc}{bcx}}\right)c\left(\frac{a}{b}-\frac{acx+bc}{bcx}\right)\right)}{ac-\frac{acx+bc}{x}}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)/x),x, algorithm="giac")
```

```
[Out] (b^2*c^2*(log(abs(a*c*x + b*c)/abs(x))/(a*c) - log(abs(-a*c + (a*c*x + b*c)
/x))/(a*c)) - b^2*c^2*log(-(b - a/(a/b - (a*c*x + b*c)/(b*c*x)))*c*(a/b - (
a*c*x + b*c)/(b*c*x)))/(a*c - (a*c*x + b*c)/x))/(b*c)
```

Mupad [B]
time = 0.08, size = 25, normalized size = 1.00

$$x\,\ln\left(\frac{c\,\left(b+a\,x\right)}{x}\right)+\frac{b\,\ln\left(b+a\,x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((c*(b + a*x))/x),x)
```

```
[Out] x*log((c*(b + a*x))/x) + (b*log(b + a*x))/a
```

3.94 $\int \log^2\left(\frac{c(b+ax)}{x}\right)\,dx$

Optimal. Leaf size=67

$$\frac{(b+ax)\log^2\left(ac+\frac{bc}{x}\right)}{a}-\frac{2b\log\left(c\left(a+\frac{b}{x}\right)\right)\log\left(-\frac{b}{ax}\right)}{a}-\frac{2b\mathrm{Li}_2\left(1+\frac{b}{ax}\right)}{a}$$

```
[Out] (a*x+b)*ln(a*c+b*c/x)^2/a-2*b*ln(c*(a+b/x))*ln(-b/a/x)/a-2*b*polylog(2,1+b/
a/x)/a
```

Rubi [A]
time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2503, 2499, 2504, 2441, 2352}

$$-\frac{2b\mathrm{PolyLog}\left(2,\frac{b}{ax}+1\right)}{a}+\frac{(ax+b)\log^2\left(ac+\frac{bc}{x}\right)}{a}-\frac{2b\log\left(-\frac{b}{ax}\right)\log\left(c\left(a+\frac{b}{x}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*(b + a*x))/x]^2,x]
```

```
[Out] ((b + a*x)*Log[a*c + (b*c)/x]^2)/a - (2*b*Log[c*(a + b/x)]*Log[-(b/(a*x))])
/a - (2*b*PolyLog[2, 1 + b/(a*x)])/a
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2499

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)/(x_))^(p_.)]*(b_.))^(q_), x_Symbol] :>
 Simp[(e + d*x)*((a + b*Log[c*(d + e/x)^p])^q/d), x] + Dist[b*e*p*(q/d), In
t[(a + b*Log[c*(d + e/x)^p])^(q - 1)/x, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && IGtQ[q, 0]
```

Rule 2503

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.), x_Symbol] :> Int[(a + b*Lo
g[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b, c, p, q}, x] && BinomialQ[v
```

```
, x] &&  !BinomialMatchQ[v, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\int \log^2\left(\frac{c(b+ax)}{x}\right) dx &= \int \log^2\left(ac+\frac{bc}{x}\right) dx \\
&= \frac{(b+ax)\log^2\left(ac+\frac{bc}{x}\right)}{a} + \frac{(2b)\int \frac{\log\left(ac+\frac{bc}{x}\right)}{x}\,dx}{a} \\
&= \frac{(b+ax)\log^2\left(ac+\frac{bc}{x}\right)}{a} - \frac{(2b)\mathrm{Subst}\left(\int \frac{\log(ac+bcx)}{x}\,dx, x, \frac{1}{x}\right)}{a} \\
&= \frac{(b+ax)\log^2\left(ac+\frac{bc}{x}\right)}{a} - \frac{2b\log\left(c\left(a+\frac{b}{x}\right)\right)\log\left(-\frac{b}{ax}\right)}{a} + \frac{(2b^2c)\,\mathrm{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{ac+bcx}\,d\right.}{a} \\
&= \frac{(b+ax)\log^2\left(ac+\frac{bc}{x}\right)}{a} - \frac{2b\log\left(c\left(a+\frac{b}{x}\right)\right)\log\left(-\frac{b}{ax}\right)}{a} - \frac{2b\mathrm{Li}_2\left(1+\frac{b}{ax}\right)}{a}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 63, normalized size = 0.94

$$
\frac{\log\left(\frac{c(b+ax)}{x}\right)\left(-2b\log\left(-\frac{b}{ax}\right)+(b+ax)\log\left(\frac{c(b+ax)}{x}\right)\right)-2b\mathrm{Li}_2\left(1+\frac{b}{ax}\right)}{a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*(b + a*x))/x]^2,x]
```

```
[Out] (Log[(c*(b + a*x))/x]*(-2*b*Log[-(b/(a*x))] + (b + a*x)*Log[(c*(b + a*x))/x
]) - 2*b*PolyLog[2, 1 + b/(a*x)])/a
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$
\int \ln\left(\frac{c(ax+b)}{x}\right)^2 dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a*x+b)/x)^2,x)
```

```
[Out] int(ln(c*(a*x+b)/x)^2,x)
```

Maxima [A]
time = 0.27, size = 113, normalized size = 1.69

$$x\log\left(\frac{(ax+b)c}{x}\right)^2+\frac{2\,b\log\left(ax+b\right)\log\left(\frac{(ax+b)c}{x}\right)}{a}+\frac{\left(\frac{c\log(ax+b)^2}{a}-\frac{2\left(\log\left(\frac{ax}{b}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{ax}{b}\right)\right)c}{a}\right)b-\frac{2\left(c\log(ax+b)-c\log(x)\right)b\log(ax+b)}{a}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)/x)^2,x, algorithm="maxima")
```

```
[Out] x*log((a*x + b)*c/x)^2 + 2*b*log(a*x + b)*log((a*x + b)*c/x)/a + ((c*log(a*
x + b)^2/a - 2*(log(a*x/b + 1)*log(x) + dilog(-a*x/b))*c/a)*b - 2*(c*log(a*
x + b) - c*log(x))*b*log(a*x + b)/a)/c
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)/x)^2,x, algorithm="fricas")
```

```
[Out] integral(log((a*c*x + b*c)/x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$2b\int\frac{\log\left(ac+\frac{bc}{x}\right)}{ax+b}\,dx+x\log\left(\frac{c(ax+b)}{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a*x+b)/x)**2,x)
```

```
[Out] 2*b*Integral(log(a*c + b*c/x)/(a*x + b), x) + x*log(c*(a*x + b)/x)**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)/x)^2,x, algorithm="giac")
```

```
[Out] integrate(log((a*x + b)*c/x)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(\frac{c\,(b+a\,x)}{x}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((c*(b + a*x))/x)^2,x)
[Out] int(log((c*(b + a*x))/x)^2, x)
```

3.95 $\int \log^3\left(\frac{c(b+ax)}{x}\right)\,dx$

Optimal. Leaf size=97

$$\frac{(b+ax)\log^3\left(ac+\frac{bc}{x}\right)}{a}-\frac{3b\log^2\left(c\left(a+\frac{b}{x}\right)\right)\log\left(-\frac{b}{ax}\right)}{a}-\frac{6b\log\left(c\left(a+\frac{b}{x}\right)\right)\mathrm{Li}_2\left(1+\frac{b}{ax}\right)}{a}+\frac{6b\mathrm{Li}_3\left(1+\frac{b}{ax}\right)}{a}$$

```
[Out] (a*x+b)*ln(a*c+b*c/x)^3/a-3*b*ln(c*(a+b/x))^2*ln(-b/a/x)/a-6*b*ln(c*(a+b/x)
)*polylog(2,1+b/a/x)/a+6*b*polylog(3,1+b/a/x)/a
```

Rubi [A]
time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2503, 2499, 2504, 2443, 2481, 2421, 6724}

$$-\frac{6b\text{PolyLog}\left(2,\frac{b}{ax}+1\right)\log\left(c\left(a+\frac{b}{x}\right)\right)}{a}+\frac{6b\text{PolyLog}\left(3,\frac{b}{ax}+1\right)}{a}+\frac{(ax+b)\log^3\left(ac+\frac{bc}{x}\right)}{a}-\frac{3b\log\left(-\frac{b}{ax}\right)\log^2\left(c\left(a+\frac{b}{x}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*(b + a*x))/x]^3,x]
```

```
[Out] ((b + a*x)*Log[a*c + (b*c)/x]^3)/a - (3*b*Log[c*(a + b/x)]^2*Log[-(b/(a*x))
])/a - (6*b*Log[c*(a + b/x)]*PolyLog[2, 1 + b/(a*x)])/a + (6*b*PolyLog[3, 1
 + b/(a*x)])/a
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
```

```
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2499

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)/(x_))^(p_.)]*(b_.))^(q_), x_Symbol] :>
 Simp[(e + d*x)*((a + b*Log[c*(d + e/x)^p])^q/d), x] + Dist[b*e*p*(q/d), In
t[(a + b*Log[c*(d + e/x)^p])^(q - 1)/x, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && IGtQ[q, 0]
```

Rule 2503

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.), x_Symbol] :> Int[(a + b*Lo
g[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b, c, p, q}, x] && BinomialQ[v
, x] &&  !BinomialMatchQ[v, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \log^3\left(\frac{c(b+ax)}{x}\right)\,dx = \int \log^3\left(ac+\frac{bc}{x}\right)\,dx$$
$$= \frac{(b+ax)\log^3\left(ac+\frac{bc}{x}\right)}{a} + \frac{(3b)\int \frac{\log^2\left(ac+\frac{bc}{x}\right)}{x}\,dx}{a}$$
$$= \frac{(b+ax)\log^3\left(ac+\frac{bc}{x}\right)}{a} - \frac{(3b)\text{Subst}\left(\int \frac{\log^2(ac+bcx)}{x}\,dx, x, \frac{1}{x}\right)}{a}$$
$$= \frac{(b+ax)\log^3\left(ac+\frac{bc}{x}\right)}{a} - \frac{3b\log^2\left(c\left(a+\frac{b}{x}\right)\right)\log\left(-\frac{b}{ax}\right)}{a} + \frac{(6b^2c)\,\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right.}{a}\right.}{a}$$
$$= \frac{(b+ax)\log^3\left(ac+\frac{bc}{x}\right)}{a} - \frac{3b\log^2\left(c\left(a+\frac{b}{x}\right)\right)\log\left(-\frac{b}{ax}\right)}{a} + \frac{(6b)\text{Subst}\left(\int \frac{\log(x)\log\left(\right.}{}\right.}{}$$
$$= \frac{(b+ax)\log^3\left(ac+\frac{bc}{x}\right)}{a} - \frac{3b\log^2\left(c\left(a+\frac{b}{x}\right)\right)\log\left(-\frac{b}{ax}\right)}{a} - \frac{6b\log\left(c\left(a+\frac{b}{x}\right)\right)\text{Li}_2\left(1\right.}{a}$$
$$= \frac{(b+ax)\log^3\left(ac+\frac{bc}{x}\right)}{a} - \frac{3b\log^2\left(c\left(a+\frac{b}{x}\right)\right)\log\left(-\frac{b}{ax}\right)}{a} - \frac{6b\log\left(c\left(a+\frac{b}{x}\right)\right)\text{Li}_2\left(1\right.}{a}$$

Mathematica [A]
time = 0.02, size = 91, normalized size = 0.94

$$\frac{\log^2\left(\frac{c(b+ax)}{x}\right)\left(-3b\log\left(-\frac{b}{ax}\right)+(b+ax)\log\left(\frac{c(b+ax)}{x}\right)\right)-6b\log\left(\frac{c(b+ax)}{x}\right)\text{Li}_2\left(1+\frac{b}{ax}\right)+6b\text{Li}_3\left(1+\frac{b}{ax}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*(b + a*x))/x]^3,x]
```

```
[Out] (Log[(c*(b + a*x))/x]^2*(-3*b*Log[-(b/(a*x))] + (b + a*x)*Log[(c*(b + a*x))
/x]) - 6*b*Log[(c*(b + a*x))/x]*PolyLog[2, 1 + b/(a*x)] + 6*b*PolyLog[3, 1
+ b/(a*x)])/a
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \ln\left(\frac{c(ax+b)}{x}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a*x+b)/x)^3,x)
```

```
[Out] int(ln(c*(a*x+b)/x)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)/x)^3,x, algorithm="maxima")
```

```
[Out] ((a*x + b)*log(a*x + b)^3 + 3*(a*x*log(c) - a*x*log(x))*log(a*x + b)^2)/a +
 integrate((a*x*log(c)^3 + b*log(c)^3 - (a*x + b)*log(x)^3 + 3*(a*x*log(c)
+ b*log(c))*log(x)^2 + 3*((log(c)^2 - 2*log(c))*a*x + b*log(c)^2 + (a*x + b
)*log(x)^2 - 2*(a*x*(log(c) - 1) + b*log(c))*log(x))*log(a*x + b) - 3*(a*x*
log(c)^2 + b*log(c)^2)*log(x))/(a*x + b), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)/x)^3,x, algorithm="fricas")
```

```
[Out] integral(log((a*c*x + b*c)/x)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$3b\int\frac{\log\left(ac+\frac{bc}{x}\right)^{2}}{ax+b}\,dx+x\log\left(\frac{c(ax+b)}{x}\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a*x+b)/x)**3,x)
```

```
[Out] 3*b*Integral(log(a*c + b*c/x)**2/(a*x + b), x) + x*log(c*(a*x + b)/x)**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)/x)^3,x, algorithm="giac")
```

```
[Out] integrate(log((a*x + b)*c/x)^3, x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(\frac{c\,(b+a\,x)}{x}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((c*(b + a*x))/x)^3,x)
```

```
[Out] int(log((c*(b + a*x))/x)^3, x)
```

3.96 $\int \log\left(\frac{c(b+ax)^2}{x^2}\right) dx$

Optimal. Leaf size=28

$$\frac{2b\log(b+ax)}{a} + x\log\left(\frac{c(b+ax)^2}{x^2}\right)$$

[Out] 2*b*ln(a*x+b)/a+x*ln(c*(a*x+b)^2/x^2)

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2536, 31}

$$x\log\left(\frac{c(ax+b)^2}{x^2}\right) + \frac{2b\log(ax+b)}{a}$$

Antiderivative was successfully verified.

[In] Int[Log[(c*(b + a*x)^2)/x^2],x]

[Out] (2*b*Log[b + a*x])/a + x*Log[(c*(b + a*x)^2)/x^2]

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2536

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b
*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \log\left(\frac{c(b+ax)^2}{x^2}\right) dx &= x\log\left(\frac{c(b+ax)^2}{x^2}\right) + (2b)\int \frac{1}{b+ax}\, dx \\
&= \frac{2b\log(b+ax)}{a} + x\log\left(\frac{c(b+ax)^2}{x^2}\right)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.00

$$\frac{2b\log(b+ax)}{a}+x\log\left(\frac{c(b+ax)^2}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*(b + a*x)^2)/x^2],x]
```

```
[Out] (2*b*Log[b + a*x])/a + x*Log[(c*(b + a*x)^2)/x^2]
```

Maple [A]
time = 0.21, size = 41, normalized size = 1.46

method	result	size
risch	$\frac{2b\ln(ax+b)}{a}+x\ln\left(\frac{c(ax+b)^2}{x^2}\right)$	29
derivativedivides	$x\ln\left(c\left(a+\frac{b}{x}\right)^2\right)-2b\left(-\frac{\ln\left(a+\frac{b}{x}\right)}{a}+\frac{\ln\left(\frac{1}{x}\right)}{a}\right)$	41
default	$x\ln\left(c\left(a+\frac{b}{x}\right)^2\right)-2b\left(-\frac{\ln\left(a+\frac{b}{x}\right)}{a}+\frac{\ln\left(\frac{1}{x}\right)}{a}\right)$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a*x+b)^2/x^2),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(c*(a+b/x)^2)-2*b*(-1/a*ln(a+b/x)+1/a*ln(1/x))
```

Maxima [A]
time = 0.27, size = 28, normalized size = 1.00

$$x\log\left(\frac{(ax+b)^2c}{x^2}\right)+\frac{2\,b\log(ax+b)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)^2/x^2),x, algorithm="maxima")
```

```
[Out] x*log((a*x + b)^2*c/x^2) + 2*b*log(a*x + b)/a
```

Fricas [A]
time = 0.32, size = 42, normalized size = 1.50

$$\frac{ax\log\left(\frac{a^2cx^2+2\,abcx+b^2c}{x^2}\right)+2\,b\log(ax+b)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)^2/x^2),x, algorithm="fricas")
```

```
[Out] (a*x*log((a^2*c*x^2 + 2*a*b*c*x + b^2*c)/x^2) + 2*b*log(a*x + b))/a
```

Sympy [A]
time = 0.05, size = 26, normalized size = 0.93

$$x\log\left(\frac{c(ax+b)^2}{x^2}\right)+\frac{2b\log\left(ax+b\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a*x+b)**2/x**2),x)
```

```
[Out] x*log(c*(a*x + b)**2/x**2) + 2*b*log(a*x + b)/a
```

Giac [A]
time = 3.96, size = 29, normalized size = 1.04

$$x\log\left(\frac{(ax+b)^2c}{x^2}\right)+\frac{2\,b\log\left(|ax+b|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)^2/x^2),x, algorithm="giac")
```

```
[Out] x*log((a*x + b)^2*c/x^2) + 2*b*log(abs(a*x + b))/a
```

Mupad [B]
time = 0.15, size = 28, normalized size = 1.00

$$x\,\ln\left(\frac{c\,(b+a\,x)^2}{x^2}\right)+\frac{2\,b\,\ln\left(b+a\,x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((c*(b + a*x)^2)/x^2),x)
```

```
[Out] x*log((c*(b + a*x)^2)/x^2) + (2*b*log(b + a*x))/a
```

3.97 $\int \log^2\left(\frac{c(b+ax)^2}{x^2}\right)\,dx$

Optimal. Leaf size=67

$$-\frac{4b\log\left(\frac{b}{b+ax}\right)\log\left(\frac{c(b+ax)^2}{x^2}\right)}{a}+x\log^2\left(\frac{c(b+ax)^2}{x^2}\right)+\frac{8b\mathrm{Li}_2\left(1-\frac{b}{b+ax}\right)}{a}$$

```
[Out] -4*b*ln(b/(a*x+b))*ln(c*(a*x+b)^2/x^2)/a+x*ln(c*(a*x+b)^2/x^2)^2+8*b*polylo
g(2,1-b/(a*x+b))/a
```

Rubi [A]
time = 0.10, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2536, 2542, 2458, 2378, 2370, 2352}

$$\frac{8b\text{PolyLog}\left(2,1-\frac{b}{ax+b}\right)}{a}+x\log^2\left(\frac{c(ax+b)^2}{x^2}\right)-\frac{4b\log\left(\frac{b}{ax+b}\right)\log\left(\frac{c(ax+b)^2}{x^2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*(b + a*x)^2)/x^2]^2,x]
```

```
[Out] (-4*b*Log[b/(b + a*x)]*Log[(c*(b + a*x)^2)/x^2])/a + x*Log[(c*(b + a*x)^2)/
x^2]^2 + (8*b*PolyLog[2, 1 - b/(b + a*x)])/a
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
```

```
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2536

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b
*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rule 2542

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a
+ b*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Dist[B*n*((b*c
 - a*d)/g), Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x],
x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
 - a*d, 0] && EqQ[b*f - a*g, 0]
```

Rubi steps

$$\int \log^2\left(\frac{c(b+ax)^2}{x^2}\right) dx = x\log^2\left(\frac{c(b+ax)^2}{x^2}\right) + (4b)\int \frac{\log\left(\frac{c(b+ax)^2}{x^2}\right)}{b+ax}\,dx$$

$$= -\frac{4b\log\left(\frac{b}{b+ax}\right)\log\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x\log^2\left(\frac{c(b+ax)^2}{x^2}\right) - \frac{(8b^2)\int \frac{\log\left(\frac{b}{b+ax}\right)}{x(b+ax)}\,dx}{a}$$

$$= -\frac{4b\log\left(\frac{b}{b+ax}\right)\log\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x\log^2\left(\frac{c(b+ax)^2}{x^2}\right) - \frac{(8b^2)\,\mathrm{Subst}\left(\int \frac{\log\left(\frac{b}{x}\right)}{x\left(-\frac{b}{a}+\frac{x}{a}\right)}\,d\right.}{a^2}$$

$$= -\frac{4b\log\left(\frac{b}{b+ax}\right)\log\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x\log^2\left(\frac{c(b+ax)^2}{x^2}\right) + \frac{(8b^2)\,\mathrm{Subst}\left(\int \frac{\log(bx)}{\left(-\frac{b}{a}+\frac{1}{ax}\right)x}\right.}{a^2}$$

$$= -\frac{4b\log\left(\frac{b}{b+ax}\right)\log\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x\log^2\left(\frac{c(b+ax)^2}{x^2}\right) + \frac{(8b^2)\,\mathrm{Subst}\left(\int \frac{\log(bx)}{\frac{1}{a}-\frac{bx}{a}}\,dx, x\right.}{a^2}$$

$$= -\frac{4b\log\left(\frac{b}{b+ax}\right)\log\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x\log^2\left(\frac{c(b+ax)^2}{x^2}\right) + \frac{8b\,\mathrm{Li}_2\left(\frac{ax}{b+ax}\right)}{a}$$

Mathematica [A]

time = 0.01, size = 106, normalized size = 1.58

$$-\frac{8b\log\left(-\frac{ax}{b}\right)\log\left(\frac{b}{b+ax}\right)}{a}-\frac{4b\log^2\left(\frac{b}{b+ax}\right)}{a}-\frac{4b\log\left(\frac{b}{b+ax}\right)\log\left(\frac{c(b+ax)^2}{x^2}\right)}{a}+x\log^2\left(\frac{c(b+ax)^2}{x^2}\right)+\frac{8b\mathrm{Li}_2\left(\frac{b+ax}{b}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*(b + a*x)^2)/x^2]^2,x]
```

```
[Out] (-8*b*Log[-((a*x)/b)]*Log[b/(b + a*x)])/a - (4*b*Log[b/(b + a*x)]^2)/a - (4
*b*Log[b/(b + a*x)]*Log[(c*(b + a*x)^2)/x^2])/a + x*Log[(c*(b + a*x)^2)/x^2
]^2 + (8*b*PolyLog[2, (b + a*x)/b])/a
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \ln\left(\frac{c(ax+b)^2}{x^2}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a*x+b)^2/x^2)^2,x)
```

```
[Out] int(ln(c*(a*x+b)^2/x^2)^2,x)
```

Maxima [A]
time = 0.28, size = 118, normalized size = 1.76

$$x\log\left(\frac{(ax+b)^2c}{x^2}\right)^2+\frac{4\,b\log(ax+b)\log\left(\frac{(ax+b)^2c}{x^2}\right)}{a}+\frac{4\left(\left(\frac{c\log(ax+b)^2}{a}-\frac{2\left(\log\left(\frac{ax}{b}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{ax}{b}\right)\right)c}{a}\right)b-\frac{2\left(c\log(ax+b)-c\log(x)\right)b\log(ax+b)}{a}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] x*log((a*x + b)^2*c/x^2)^2 + 4*b*log(a*x + b)*log((a*x + b)^2*c/x^2)/a + 4*
((c*log(a*x + b)^2/a - 2*(log(a*x/b + 1)*log(x) + dilog(-a*x/b))*c/a)*b - 2
*(c*log(a*x + b) - c*log(x))*b*log(a*x + b)/a)/c
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] integral(log((a^2*c*x^2 + 2*a*b*c*x + b^2*c)/x^2)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$4b\int \frac{\log\left(a^2c+\frac{2abc}{x}+\frac{b^2c}{x^2}\right)}{ax+b}\,dx + x\log\left(\frac{c(ax+b)^2}{x^2}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a*x+b)**2/x**2)**2,x)
```

```
[Out] 4*b*Integral(log(a**2*c + 2*a*b*c/x + b**2*c/x**2)/(a*x + b), x) + x*log(c*
(a*x + b)**2/x**2)**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)^2/x^2)^2,x, algorithm="giac")
```

```
[Out] integrate(log((a*x + b)^2*c/x^2)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(\frac{c\,(b+a\,x)^2}{x^2}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((c*(b + a*x)^2)/x^2)^2,x)
```

```
[Out] int(log((c*(b + a*x)^2)/x^2)^2, x)
```

3.98 $\int \log^3\left(\frac{c(b+ax)^2}{x^2}\right)\,dx$

Optimal. Leaf size=102

$$x\log^3\left(\frac{c(b+ax)^2}{x^2}\right)-\frac{6b\log^2\left(\frac{c(b+ax)^2}{x^2}\right)\log\left(1-\frac{ax}{b+ax}\right)}{a}+\frac{24b\log\left(\frac{c(b+ax)^2}{x^2}\right)\mathrm{Li}_2\left(\frac{ax}{b+ax}\right)}{a}+\frac{48b\mathrm{Li}_3\left(\frac{ax}{b+ax}\right)}{a}$$

```
[Out] x*ln(c*(a*x+b)^2/x^2)^3-6*b*ln(c*(a*x+b)^2/x^2)^2*ln(1-a*x/(a*x+b))/a+24*b*
ln(c*(a*x+b)^2/x^2)*polylog(2,a*x/(a*x+b))/a+48*b*polylog(3,a*x/(a*x+b))/a
```

Rubi [A]
time = 0.07, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2536, 2550, 2379, 2421, 6724}

$$\frac{24b\text{PolyLog}\left(2,\frac{ax}{ax+b}\right)\log\left(\frac{c(ax+b)^2}{x^2}\right)}{a}+\frac{48b\text{PolyLog}\left(3,\frac{ax}{ax+b}\right)}{a}+x\log^3\left(\frac{c(ax+b)^2}{x^2}\right)-\frac{6b\log\left(1-\frac{ax}{ax+b}\right)\log^2\left(\frac{c(ax+b)^2}{x^2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*(b + a*x)^2)/x^2]^3,x]
```

```
[Out] x*Log[(c*(b + a*x)^2)/x^2]^3 - (6*b*Log[(c*(b + a*x)^2)/x^2]^2*Log[1 - (a*x
)/(b + a*x)])/a + (24*b*Log[(c*(b + a*x)^2)/x^2]*PolyLog[2, (a*x)/(b + a*x)
])/a + (48*b*PolyLog[3, (a*x)/(b + a*x)])/a
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2536

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b
```

```
*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \log^3\left(\frac{c(b+ax)^2}{x^2}\right)dx &= x\log^3\left(\frac{c(b+ax)^2}{x^2}\right)+(6b)\int\frac{\log^2\left(\frac{c(b+ax)^2}{x^2}\right)}{b+ax}\,dx\\
&= -\frac{6b\log\left(\frac{b}{b+ax}\right)\log^2\left(\frac{c(b+ax)^2}{x^2}\right)}{a}+x\log^3\left(\frac{c(b+ax)^2}{x^2}\right)-\frac{(24b^2)\int\frac{\log\left(\frac{b}{b+ax}\right)\log\left(\frac{c(b+
\end{aligned}
$$

$$
= -\frac{6b\log\left(\frac{b}{b+ax}\right)\log^2\left(\frac{c(b+ax)^2}{x^2}\right)}{a}+x\log^3\left(\frac{c(b+ax)^2}{x^2}\right)+\frac{24b\log\left(\frac{c(b+ax)^2}{x^2}\right)\mathrm{Li}_2(1-
$$

$$
= -\frac{6b\log\left(\frac{b}{b+ax}\right)\log^2\left(\frac{c(b+ax)^2}{x^2}\right)}{a}+x\log^3\left(\frac{c(b+ax)^2}{x^2}\right)+\frac{24b\log\left(\frac{c(b+ax)^2}{x^2}\right)\mathrm{Li}_2(1-
$$

Mathematica [A]
time = 0.02, size = 98, normalized size = 0.96

$$-\frac{6b\log\left(\frac{b}{b+ax}\right)\log^2\left(\frac{c(b+ax)^2}{x^2}\right)}{a}+x\log^3\left(\frac{c(b+ax)^2}{x^2}\right)+\frac{24b\log\left(\frac{c(b+ax)^2}{x^2}\right)\mathrm{Li}_2\left(\frac{ax}{b+ax}\right)}{a}+\frac{48b\mathrm{Li}_3\left(\frac{ax}{b+ax}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*(b + a*x)^2)/x^2]^3,x]
```

```
[Out] (-6*b*Log[b/(b + a*x)]*Log[(c*(b + a*x)^2)/x^2]^2)/a + x*Log[(c*(b + a*x)^2
)/x^2]^3 + (24*b*Log[(c*(b + a*x)^2)/x^2]*PolyLog[2, (a*x)/(b + a*x)])/a +
(48*b*PolyLog[3, (a*x)/(b + a*x)])/a
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \ln \left(\frac{c(ax+b)^2}{x^2}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a*x+b)^2/x^2)^3,x)
```

```
[Out] int(ln(c*(a*x+b)^2/x^2)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] 4*(2*(a*x + b)*log(a*x + b)^3 + 3*(a*x*log(c) - 2*a*x*log(x))*log(a*x + b)^
2)/a + integrate((a*x*log(c)^3 + b*log(c)^3 - 8*(a*x + b)*log(x)^3 + 12*(a*
x*log(c) + b*log(c))*log(x)^2 + 6*((log(c)^2 - 4*log(c))*a*x + b*log(c)^2 +
 4*(a*x + b)*log(x)^2 - 4*(a*x*(log(c) - 2) + b*log(c))*log(x))*log(a*x + b
) - 6*(a*x*log(c)^2 + b*log(c)^2)*log(x))/(a*x + b), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] integral(log((a^2*c*x^2 + 2*a*b*c*x + b^2*c)/x^2)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$6b \int \frac{\log \left(a^2c + \frac{2abc}{x} + \frac{b^2c}{x^2}\right)^2}{ax+b}\, dx + x \log \left(\frac{c(ax+b)^2}{x^2}\right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a*x+b)**2/x**2)**3,x)
```

```
[Out] 6*b*Integral(log(a**2*c + 2*a*b*c/x + b**2*c/x**2)**2/(a*x + b), x) + x*log
(c*(a*x + b)**2/x**2)**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)^2/x^2)^3,x, algorithm="giac")
```

```
[Out] integrate(log((a*x + b)^2*c/x^2)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(\frac{c\,{\left(b+a\,x\right)}^2}{x^2}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((c*(b + a*x)^2)/x^2)^3,x)
```

```
[Out] int(log((c*(b + a*x)^2)/x^2)^3, x)
```

3.99 $\int \log \left(\frac{cx^2}{(b+ax)^2}\right) dx$

Optimal. Leaf size=28

$$x\log\left(\frac{cx^2}{(b+ax)^2}\right)-\frac{2b\log(b+ax)}{a}$$

```
[Out] x*ln(c*x^2/(a*x+b)^2)-2*b*ln(a*x+b)/a
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = $\{2536, 31\}$

$$x\log\left(\frac{cx^2}{(ax+b)^2}\right)-\frac{2b\log(ax+b)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*x^2)/(b + a*x)^2],x]
```

```
[Out] x*Log[(c*x^2)/(b + a*x)^2] - (2*b*Log[b + a*x])/a
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2536

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b
*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \log\left(\frac{cx^2}{(b+ax)^2}\right)dx &= x\log\left(\frac{cx^2}{(b+ax)^2}\right)-(2b)\int\frac{1}{b+ax}\,dx\\ &= x\log\left(\frac{cx^2}{(b+ax)^2}\right)-\frac{2b\log(b+ax)}{a}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.00

$$x\log\left(\frac{cx^2}{(b+ax)^2}\right)-\frac{2b\log(b+ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*x^2)/(b + a*x)^2],x]
```

```
[Out] x*Log[(c*x^2)/(b + a*x)^2] - (2*b*Log[b + a*x])/a
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(28) = 56$.
time = 0.45, size = 59, normalized size = 2.11

method	result	size
risch	$x\ln\left(\frac{cx^2}{(ax+b)^2}\right)-\frac{2b\ln(ax+b)}{a}$	29
derivativedivides	$-\frac{-(ax+b)\ln\left(\frac{c\left(\frac{b}{ax+b}-1\right)^2}{a^2}\right)+2b\left(-\ln\left(\frac{1}{ax+b}\right)+\ln\left(\frac{b}{ax+b}-1\right)\right)}{a}$	59
default	$-\frac{-(ax+b)\ln\left(\frac{c\left(\frac{b}{ax+b}-1\right)^2}{a^2}\right)+2b\left(-\ln\left(\frac{1}{ax+b}\right)+\ln\left(\frac{b}{ax+b}-1\right)\right)}{a}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^2/(a*x+b)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a*(-(a*x+b)*ln(c*(b/(a*x+b)-1)^2/a^2)+2*b*(-ln(1/(a*x+b))+ln(b/(a*x+b)-1
)))
```

Maxima [A]
time = 0.28, size = 28, normalized size = 1.00

$$x\log\left(\frac{cx^2}{(ax+b)^2}\right)-\frac{2\,b\log\left(ax+b\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(a*x+b)^2),x, algorithm="maxima")
```

```
[Out] x*log(c*x^2/(a*x + b)^2) - 2*b*log(a*x + b)/a
```

Fricas [A]
time = 0.33, size = 41, normalized size = 1.46

$$\frac{ax\log\left(\frac{cx^2}{a^2x^2+2\,abx+b^2}\right)-2\,b\log\left(ax+b\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(a*x+b)^2),x, algorithm="fricas")
```

```
[Out] (a*x*log(c*x^2/(a^2*x^2 + 2*a*b*x + b^2)) - 2*b*log(a*x + b))/a
```

Sympy [A]
time = 0.06, size = 26, normalized size = 0.93

$$x\log\left(\frac{cx^2}{(ax+b)^2}\right)-\frac{2b\log(ax+b)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**2/(a*x+b)**2),x)
```

```
[Out] x*log(c*x**2/(a*x + b)**2) - 2*b*log(a*x + b)/a
```

Giac [A]
time = 3.87, size = 29, normalized size = 1.04

$$x\log\left(\frac{cx^2}{(ax+b)^2}\right)-\frac{2\,b\log(|ax+b|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(a*x+b)^2),x, algorithm="giac")
```

```
[Out] x*log(c*x^2/(a*x + b)^2) - 2*b*log(abs(a*x + b))/a
```

Mupad [B]
time = 0.15, size = 28, normalized size = 1.00

$$x\,\ln\left(\frac{c\,x^2}{(b+a\,x)^2}\right)-\frac{2\,b\,\ln(b+a\,x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((c*x^2)/(b + a*x)^2),x)
```

```
[Out] x*log((c*x^2)/(b + a*x)^2) - (2*b*log(b + a*x))/a
```

3.100 $\int \log^2\left(\frac{cx^2}{(b+ax)^2}\right)\,dx$

Optimal. Leaf size=67

$$x\log^2\left(\frac{cx^2}{(b+ax)^2}\right)+\frac{4b\log\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a}+\frac{8b\text{Li}_2\left(1-\frac{b}{b+ax}\right)}{a}$$

```
[Out] x*ln(c*x^2/(a*x+b)^2)^2+4*b*ln(c*x^2/(a*x+b)^2)*ln(b/(a*x+b))/a+8*b*polylog
(2,1-b/(a*x+b))/a
```

Rubi [A]
time = 0.10, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2536, 2542, 2458, 2378, 2370, 2352}

$$\frac{8b\,\text{PolyLog}\left(2,1-\frac{b}{ax+b}\right)}{a}+x\log^2\left(\frac{cx^2}{(ax+b)^2}\right)+\frac{4b\log\left(\frac{b}{ax+b}\right)\log\left(\frac{cx^2}{(ax+b)^2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*x^2)/(b + a*x)^2]^2,x]
```

```
[Out] x*Log[(c*x^2)/(b + a*x)^2]^2 + (4*b*Log[(c*x^2)/(b + a*x)^2]*Log[b/(b + a*x
)])/a + (8*b*PolyLog[2, 1 - b/(b + a*x)])/a
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
```

```
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2536

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b
*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rule 2542

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a
+ b*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Dist[B*n*((b*c
 - a*d)/g), Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x],
x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
 - a*d, 0] && EqQ[b*f - a*g, 0]
```

Rubi steps

$$\begin{aligned}
\int \log^2\left(\frac{cx^2}{(b+ax)^2}\right)\,dx &= x\log^2\left(\frac{cx^2}{(b+ax)^2}\right) - (4b)\int \frac{\log\left(\frac{cx^2}{(b+ax)^2}\right)}{b+ax}\,dx \\
&= x\log^2\left(\frac{cx^2}{(b+ax)^2}\right) + \frac{4b\log\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a} - \frac{(8b^2)\int \frac{\log\left(\frac{b}{b+ax}\right)}{x(b+ax)}\,dx}{a} \\
&= x\log^2\left(\frac{cx^2}{(b+ax)^2}\right) + \frac{4b\log\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a} - \frac{(8b^2)\,\text{Subst}\left(\int \frac{\log\left(\frac{b}{x}\right)}{x\left(-\frac{b}{a}+\frac{x}{a}\right)}\,dx, x\right.}{a^2} \\
&= x\log^2\left(\frac{cx^2}{(b+ax)^2}\right) + \frac{4b\log\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a} + \frac{(8b^2)\,\text{Subst}\left(\int \frac{\log(bx)}{\left(-\frac{b}{a}+\frac{1}{ax}\right)x}\,dx,\right.}{a^2} \\
&= x\log^2\left(\frac{cx^2}{(b+ax)^2}\right) + \frac{4b\log\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a} + \frac{(8b^2)\,\text{Subst}\left(\int \frac{\log(bx)}{\frac{1}{a}-\frac{bx}{a}}\,dx, x, \right.}{a^2} \\
&= x\log^2\left(\frac{cx^2}{(b+ax)^2}\right) + \frac{4b\log\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a} + \frac{8b\text{Li}_2\left(\frac{ax}{b+ax}\right)}{a}
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 106, normalized size = 1.58

$$x\log^2\left(\frac{cx^2}{(b+ax)^2}\right)-\frac{8b\log\left(-\frac{ax}{b}\right)\log\left(\frac{b}{b+ax}\right)}{a}+\frac{4b\log\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a}-\frac{4b\log^2\left(\frac{b}{b+ax}\right)}{a}+\frac{8b\mathrm{Li}_2\left(\frac{b+ax}{b}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*x^2)/(b + a*x)^2]^2,x]
```

```
[Out] x*Log[(c*x^2)/(b + a*x)^2]^2 - (8*b*Log[-((a*x)/b)]*Log[b/(b + a*x)])/a + (
4*b*Log[(c*x^2)/(b + a*x)^2]*Log[b/(b + a*x)])/a - (4*b*Log[b/(b + a*x)]^2)
/a + (8*b*PolyLog[2, (b + a*x)/b])/a
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \ln\left(\frac{c\,x^2}{(ax+b)^2}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^2/(a*x+b)^2)^2,x)
```

```
[Out] int(ln(c*x^2/(a*x+b)^2)^2,x)
```

Maxima [A]
time = 0.28, size = 118, normalized size = 1.76

$$x\log\left(\frac{cx^2}{(ax+b)^2}\right)^2-\frac{4\,b\log(ax+b)\log\left(\frac{cx^2}{(ax+b)^2}\right)}{a}+\frac{4\left(\left(\frac{c\log(ax+b)^2}{a}-\frac{2\left(\log\left(\frac{ax}{b}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{ax}{b}\right)\right)c}{a}\right)b-\frac{2\left(c\log(ax+b)-c\log(x)\right)b\log(ax+b)}{a}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(a*x+b)^2)^2,x, algorithm="maxima")
```

```
[Out] x*log(c*x^2/(a*x + b)^2)^2 - 4*b*log(a*x + b)*log(c*x^2/(a*x + b)^2)/a + 4*
((c*log(a*x + b)^2/a - 2*(log(a*x/b + 1)*log(x) + dilog(-a*x/b))*c/a)*b - 2
*(c*log(a*x + b) - c*log(x))*b*log(a*x + b)/a)/c
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(a*x+b)^2)^2,x, algorithm="fricas")
```

```
[Out] integral(log(c*x^2/(a^2*x^2 + 2*a*b*x + b^2))^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-4b\int\frac{\log\left(\frac{cx^2}{a^2x^2+2abx+b^2}\right)}{ax+b}\,dx+x\log\left(\frac{cx^2}{(ax+b)^2}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**2/(a*x+b)**2)**2,x)
```

```
[Out] -4*b*Integral(log(c*x**2/(a**2*x**2 + 2*a*b*x + b**2))/(a*x + b), x) + x*lo
g(c*x**2/(a*x + b)**2)**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(a*x+b)^2)^2,x, algorithm="giac")
```

```
[Out] integrate(log(c*x^2/(a*x + b)^2)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(\frac{c\,x^2}{{\left(b+a\,x\right)}^2}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((c*x^2)/(b + a*x)^2)^2,x)
```

```
[Out] int(log((c*x^2)/(b + a*x)^2)^2, x)
```

3.101 $\int \log^3\left(\frac{cx^2}{(b+ax)^2}\right)\,dx$

Optimal. Leaf size=98

$$x\log^3\left(\frac{cx^2}{(b+ax)^2}\right)+\frac{6b\log^2\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a}+\frac{24b\log\left(\frac{cx^2}{(b+ax)^2}\right)\mathrm{Li}_2\left(\frac{ax}{b+ax}\right)}{a}-\frac{48b\mathrm{Li}_3\left(\frac{ax}{b+ax}\right)}{a}$$

```
[Out] x*ln(c*x^2/(a*x+b)^2)^3+6*b*ln(c*x^2/(a*x+b)^2)^2*ln(b/(a*x+b))/a+24*b*ln(c
*x^2/(a*x+b)^2)*polylog(2,a*x/(a*x+b))/a-48*b*polylog(3,a*x/(a*x+b))/a
```

Rubi [A]
time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2536, 2552, 2354, 2421, 6724}

$$\frac{24b\text{PolyLog}\left(2,\frac{ax}{ax+b}\right)\log\left(\frac{cx^2}{(ax+b)^2}\right)}{a}-\frac{48b\text{PolyLog}\left(3,\frac{ax}{ax+b}\right)}{a}+x\log^3\left(\frac{cx^2}{(ax+b)^2}\right)+\frac{6b\log\left(\frac{b}{ax+b}\right)\log^2\left(\frac{cx^2}{(ax+b)^2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*x^2)/(b + a*x)^2]^3,x]
```

```
[Out] x*Log[(c*x^2)/(b + a*x)^2]^3 + (6*b*Log[(c*x^2)/(b + a*x)^2]^2*Log[b/(b + a
*x)])/a + (24*b*Log[(c*x^2)/(b + a*x)^2]*PolyLog[2, (a*x)/(b + a*x)])/a - (
48*b*PolyLog[3, (a*x)/(b + a*x)])/a
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2536

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b
```

```
*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \log^3\left(\frac{cx^2}{(b+ax)^2}\right)dx &= x\log^3\left(\frac{cx^2}{(b+ax)^2}\right)-(6b)\int \frac{\log^2\left(\frac{cx^2}{(b+ax)^2}\right)}{b+ax}\,dx\\
&= x\log^3\left(\frac{cx^2}{(b+ax)^2}\right)+\frac{6b\log^2\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a}-\frac{(24b^2)\int \frac{\log\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{x(b+ax)}}{a}\\
&= x\log^3\left(\frac{cx^2}{(b+ax)^2}\right)+\frac{6b\log^2\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a}+\frac{24b\log\left(\frac{cx^2}{(b+ax)^2}\right)\mathrm{Li}_2\left(1-\frac{b}{b+}\right.}{a}\\
&= x\log^3\left(\frac{cx^2}{(b+ax)^2}\right)+\frac{6b\log^2\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a}+\frac{24b\log\left(\frac{cx^2}{(b+ax)^2}\right)\mathrm{Li}_2\left(1-\frac{b}{b+}\right.}{a}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 98, normalized size = 1.00

$$x\log^3\left(\frac{cx^2}{(b+ax)^2}\right)+\frac{6b\log^2\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a}+\frac{24b\log\left(\frac{cx^2}{(b+ax)^2}\right)\mathrm{Li}_2\left(\frac{ax}{b+ax}\right)}{a}-\frac{48b\mathrm{Li}_3\left(\frac{ax}{b+ax}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*x^2)/(b + a*x)^2]^3,x]
```

```
[Out] x*Log[(c*x^2)/(b + a*x)^2]^3 + (6*b*Log[(c*x^2)/(b + a*x)^2]^2*Log[b/(b + a
*x)])/a + (24*b*Log[(c*x^2)/(b + a*x)^2]*PolyLog[2, (a*x)/(b + a*x)])/a - (
48*b*PolyLog[3, (a*x)/(b + a*x)])/a
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \ln\left(\frac{c\,x^{2}}{\left(ax+b\right)^{2}}\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^2/(a*x+b)^2)^3,x)
```

```
[Out] int(ln(c*x^2/(a*x+b)^2)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(a*x+b)^2)^3,x, algorithm="maxima")
```

```
[Out] -4*(2*(a*x + b)*log(a*x + b)^3 - 3*(a*x*log(c) + 2*a*x*log(x))*log(a*x + b)
^2)/a - integrate(-(a*x*log(c)^3 + b*log(c)^3 + 8*(a*x + b)*log(x)^3 + 12*(
a*x*log(c) + b*log(c))*log(x)^2 - 6*((log(c)^2 + 4*log(c))*a*x + b*log(c)^2
 + 4*(a*x + b)*log(x)^2 + 4*(a*x*(log(c) + 2) + b*log(c))*log(x))*log(a*x +
 b) + 6*(a*x*log(c)^2 + b*log(c)^2)*log(x))/(a*x + b), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(a*x+b)^2)^3,x, algorithm="fricas")
```

```
[Out] integral(log(c*x^2/(a^2*x^2 + 2*a*b*x + b^2))^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-6b\int \frac{\log\left(\frac{cx^{2}}{a^{2}x^{2}+2abx+b^{2}}\right)^{2}}{ax+b}\,dx + x\log\left(\frac{cx^{2}}{\left(ax+b\right)^{2}}\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**2/(a*x+b)**2)**3,x)
```

```
[Out] -6*b*Integral(log(c*x**2/(a**2*x**2 + 2*a*b*x + b**2))**2/(a*x + b), x) + x
*log(c*x**2/(a*x + b)**2)**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(a*x+b)^2)^3,x, algorithm="giac")
```

```
[Out] integrate(log(c*x^2/(a*x + b)^2)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(\frac{c\,x^2}{{\left(b+a\,x\right)}^2}\right)^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((c*x^2)/(b + a*x)^2)^3,x)
```

```
[Out] int(log((c*x^2)/(b + a*x)^2)^3, x)
```

3.102 $\int \frac{\text{Li}_2\left(1+\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=35

$$-\frac{\text{Li}_3\left(1+\frac{bc-ad}{d(a+bx)}\right)}{bc-ad}$$

```
[Out] -polylog(3,1+(-a*d+b*c)/d/(b*x+a))/(-a*d+b*c)
```

Rubi [A]
time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {6745}

$$-\frac{\text{PolyLog}\left(3,\frac{bc-ad}{d(a+bx)}+1\right)}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)),x]
```

```
[Out] -(PolyLog[3, 1 + (b*c - a*d)/(d*(a + b*x))]/(b*c - a*d))
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\text{Li}_2\left(1+\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}\,dx=-\frac{\text{Li}_3\left(1+\frac{bc-ad}{d(a+bx)}\right)}{bc-ad}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 0.86

$$\frac{\text{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{-bc+ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)),x
]
```

```
[Out] PolyLog[3, (b*(c + d*x))/(d*(a + b*x))]/(-(b*c) + a*d)
```

Maple [A]
time = 0.68, size = 36, normalized size = 1.03

method	result	size
derivativedivides	$\frac{\operatorname{polylog}\left(3,1-\frac{ad-cb}{d(bx+a)}\right)}{ad-cb}$	36
default	$\frac{\operatorname{polylog}\left(3,1-\frac{ad-cb}{d(bx+a)}\right)}{ad-cb}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,1+(-a*d+b*c)/d/(b*x+a))/(b*x+a)/(d*x+c),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/(a*d-b*c)*polylog(3,1-(a*d-b*c)/d/(b*x+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,1+(-a*d+b*c)/d/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="m
axima")
```

```
[Out] integrate(dilog((b*c - a*d)/((b*x + a)*d) + 1)/((b*x + a)*(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,1+(-a*d+b*c)/d/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="f
ricas")
```

```
[Out] integral(dilog((b*c - a*d)/(b*d*x + a*d) + 1)/(b*d*x^2 + a*c + (b*c + a*d)*
x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Li}_{2}\left(-\frac{ad}{ad+bdx}+\frac{bc}{ad+bdx}+1\right)}{(a+bx)\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,1+(-a*d+b*c)/d/(b*x+a))/(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(polylog(2, -a*d/(a*d + b*d*x) + b*c/(a*d + b*d*x) + 1)/((a + b*x)*
(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,1+(-a*d+b*c)/d/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="g
iac")
```

```
[Out] integrate(dilog((b*c - a*d)/((b*x + a)*d) + 1)/((b*x + a)*(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\mathrm{polylog}\left(2,1-\frac{a\,d-b\,c}{d\,(a+b\,x)}\right)}{(a+b\,x)\,\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, 1 - (a*d - b*c)/(d*(a + b*x)))/((a + b*x)*(c + d*x)),x)
```

```
[Out] int(polylog(2, 1 - (a*d - b*c)/(d*(a + b*x)))/((a + b*x)*(c + d*x)), x)
```

3.103 $\int \frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=85

$$\frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\operatorname{Li}_2\left(1+\frac{bc-ad}{d(a+bx)}\right)}{bc-ad}-\frac{\operatorname{Li}_3\left(1+\frac{bc-ad}{d(a+bx)}\right)}{bc-ad}$$

```
[Out] ln(e*(d*x+c)/(b*x+a))*polylog(2,1+(-a*d+b*c)/d/(b*x+a))/(-a*d+b*c)-polylog(
3,1+(-a*d+b*c)/d/(b*x+a))/(-a*d+b*c)
```

Rubi [A]
time = 0.09, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2588, 6745}

$$\frac{\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{bc-ad}-\frac{\text{PolyLog}\left(3,\frac{bc-ad}{d(a+bx)}+1\right)}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(e*(c + d*x))/(a + b*x)])/((a +
b*x)*(c + d*x)),x]
```

```
[Out] (Log[(e*(c + d*x))/(a + b*x)]*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b
*c - a*d) - PolyLog[3, 1 + (b*c - a*d)/(d*(a + b*x))]/(b*c - a*d)
```

Rule 2588

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[(v - 1)*((c + d
*x)/(a + b*x))], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[(-h)*PolyLog[2,
 1 - v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(b*c - a*d)), x] + Dist[h*p
*r*s, Int[PolyLog[2, 1 - v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e,
 f, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\mathrm{Li}_2\left(1+\frac{bc-ad}{d(a+bx)}\right)}{bc-ad} + \int \frac{\mathrm{Li}_2\left(1-\frac{-bc+ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}\,dx$$
$$= \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\mathrm{Li}_2\left(1+\frac{bc-ad}{d(a+bx)}\right)}{bc-ad} - \frac{\mathrm{Li}_3\left(1+\frac{bc-ad}{d(a+bx)}\right)}{bc-ad}$$

Mathematica [A]
time = 0.02, size = 68, normalized size = 0.80

$$\frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\mathrm{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right) - \mathrm{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(e*(c + d*x))/(a + b*x)])/
((a + b*x)*(c + d*x)),x]
```

```
[Out] (Log[(e*(c + d*x))/(a + b*x)]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] - Pol
yLog[3, (b*(c + d*x))/(d*(a + b*x))])/(b*c - a*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(85) = 170$.
time = 3.71, size = 186, normalized size = 2.19

method	result
default	$\frac{\ln\left(\frac{e(dx+c)}{bx+a}\right)^2 \ln\left(-\frac{\frac{e(dx+c)b}{bx+a}-ed}{ed}\right)}{2ad-2cb} - \frac{\ln\left(\frac{e(dx+c)}{bx+a}\right)^2 \ln\left(1-\frac{b(dx+c)}{d(bx+a)}\right)}{2(ad-cb)} - \frac{\ln\left(\frac{e(dx+c)}{bx+a}\right)\mathrm{polylog}\left(2,\frac{b(dx+c)}{d(bx+a)}\right)}{ad-cb} + \frac{\mathrm{polylog}\left(3,\frac{b(dx+c)}{d(bx+a)}\right)}{ad-cb}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((a*d-b*c)/d/(b*x+a))*ln(e*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x,method=
_RETURNVERBOSE)
```

```
[Out] 1/2/(a*d-b*c)*ln(e*(d*x+c)/(b*x+a))^2*ln(-(e*(d*x+c)/(b*x+a)*b-e*d)/e/d)-1/
2/(a*d-b*c)*ln(e*(d*x+c)/(b*x+a))^2*ln(1-b*(d*x+c)/d/(b*x+a))-1/(a*d-b*c)*l
n(e*(d*x+c)/(b*x+a))*polylog(2,b*(d*x+c)/d/(b*x+a))+1/(a*d-b*c)*polylog(3,b
*(d*x+c)/d/(b*x+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a*d-b*c)/d/(b*x+a))*log(e*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x
, algorithm="maxima")
```

```
[Out] integrate(log((d*x + c)*e/(b*x + a))*log(-(b*c - a*d)/((b*x + a)*d))/((b*x
+ a)*(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a*d-b*c)/d/(b*x+a))*log(e*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x
, algorithm="fricas")
```

```
[Out] integral(log((d*x + c)*e/(b*x + a))*log(-(b*c - a*d)/(b*d*x + a*d))/(b*d*x^
2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{b\int\frac{\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)^2}{a+bx}\,dx}{2\left(ad-bc\right)}+\frac{\log\left(\frac{ad-bc}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)^2}{2ad-2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((a*d-b*c)/d/(b*x+a))*ln(e*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x)
```

```
[Out] b*Integral(log(c*e/(a + b*x) + d*e*x/(a + b*x))**2/(a + b*x), x)/(2*(a*d -
b*c)) + log((a*d - b*c)/(d*(a + b*x)))*log(e*(c + d*x)/(a + b*x))**2/(2*a*d
 - 2*b*c)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a*d-b*c)/d/(b*x+a))*log(e*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x
, algorithm="giac")
```

```
[Out] integrate(log((d*x + c)*e/(b*x + a))*log(-(b*c - a*d)/((b*x + a)*d))/((b*x
+ a)*(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\,\ln\left(\frac{a\,d-b\,c}{d\,(a+b\,x)}\right)}{(a+b\,x)\,\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log((e*(c + d*x))/(a + b*x))*log((a*d - b*c)/(d*(a + b*x))))/((a + b*x
)*(c + d*x)),x)
```

```
[Out] int((log((e*(c + d*x))/(a + b*x))*log((a*d - b*c)/(d*(a + b*x))))/((a + b*x
)*(c + d*x)), x)
```

3.104 $\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{a+bx}\,dx$

Optimal. Leaf size=140

$$-\frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{b}-\frac{2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b}+\frac{2\text{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b}$$

```
[Out] -ln((a*d-b*c)/d/(b*x+a))*ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/b-2*ln
((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*polylog(2,b*(d*x+c)/d/(b*x+a))/b+2*
polylog(3,b*(d*x+c)/d/(b*x+a))/b
```

Rubi [A]
time = 0.06, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2552, 2354, 2421, 6724}

$$-\frac{2\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{b}+\frac{2\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b}-\frac{\log\left(\frac{ad-bc}{d(a+bx)}\right)\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2/(a + b*x),x]
```

```
[Out] -((Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*
f)*(a + b*x))]^2)/b) - (2*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x
))]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/b + (2*PolyLog[3, (b*(c + d*x)
)/(d*(a + b*x))])/b
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
```

```
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{a+bx}\,dx &= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{b}-\frac{(2(bc-ad))\int\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)}}{b}\\ &= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{b}-\frac{2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\mathrm{Li}_2\left(1+\frac{bc-ad}{d(a+bx)}\right)}{b}-\\ &= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{b}-\frac{2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\mathrm{Li}_2\left(1+\frac{bc-ad}{d(a+bx)}\right)}{b}+\end{aligned}$$

Mathematica [A]
time = 0.03, size = 135, normalized size = 0.96

$$\frac{-\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)-2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\mathrm{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)+2\mathrm{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2/(a + b*x),
x]
```

```
[Out] (-(Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*
f)*(a + b*x))]^2) - 2*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*
PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + 2*PolyLog[3, (b*(c + d*x))/(d*(a
+ b*x))])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(140) = 280$.
time = 0.90, size = 419, normalized size = 2.99

method	result
derivativedivides	$\frac{(cf-ed)\left(\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-ed)(bx+a)}+\frac{d(af-be)}{(cf-ed)b}\right)^2 \ln\left(1-\frac{(-bcf+bde)\left(-\frac{(af-be)(ad-cb)}{b(cf-ed)(bx+a)}+\frac{d(af-be)}{(cf-ed)b}\right)}{-adf+bde}\right)+2\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-ed)(bx+a)}\right.\right.}{}$
default	$\frac{(cf-ed)\left(\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-ed)(bx+a)}+\frac{d(af-be)}{(cf-ed)b}\right)^2 \ln\left(1-\frac{(-bcf+bde)\left(-\frac{(af-be)(ad-cb)}{b(cf-ed)(bx+a)}+\frac{d(af-be)}{(cf-ed)b}\right)}{-adf+bde}\right)+2\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-ed)(bx+a)}\right.\right.}{}$
risch	$\frac{\ln\left(1-\frac{(-bcf+bde)\left(-\frac{(af-be)(ad-cb)}{b(cf-ed)(bx+a)}+\frac{d(af-be)}{(cf-ed)b}\right)}{-adf+bde}\right)\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-ed)(bx+a)}+\frac{d(af-be)}{(cf-ed)b}\right)^2 cf}{-bcf+bde}-\frac{\ln\left(1-\frac{(-bcf+bde)\left(-\frac{(af-}{b(cf-}\right.}{-a}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a),x,method=_RETURNVER
BOSE)
```

```
[Out] (c*f-d*e)/(-b*c*f+b*d*e)*(ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*
f-d*e)*(a*f-b*e)/b)^2*ln(1-(-b*c*f+b*d*e)/(-a*d*f+b*d*e)*(-(a*f-b*e)*(a*d-b
*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))+2*ln(-(a*f-b*e)*(a*d-b*c)
/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*polylog(2,(-b*c*f+b*d*e)/(-a*
d*f+b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/
b))-2*polylog(3,(-b*c*f+b*d*e)/(-a*d*f+b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-
d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a),x, algorithm
="maxima")
```

```
[Out] log(d*x + c)^3/a - integrate(-((d*log(a*f - b*e)^2 - 2*d*log(a*f - b*e)*log
(c*f - d*e) + d*log(c*f - d*e)^2)*b*x + (b*d*x + b*c)*log(b*x + a)^2 + (c*l
og(a*f - b*e)^2 - 2*c*log(a*f - b*e)*log(c*f - d*e) + c*log(c*f - d*e)^2)*b
 - 2*((d*log(a*f - b*e) - d*log(c*f - d*e))*b*x + (c*log(a*f - b*e) - c*log
(c*f - d*e))*b)*log(b*x + a) + 2*((d*log(a*f - b*e) - d*log(c*f - d*e))*b*x
 + (c*log(a*f - b*e) - c*log(c*f - d*e))*b - (2*b*d*x + b*c + a*d)*log(b*x
+ a))*log(d*x + c))/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a),x, algorithm
="fricas")
```

```
[Out] integral(log((a*d*f*x + a*c*f - (b*d*x + b*c)*e)/(b*c*f*x + a*c*f - (b*d*x
+ a*d)*e))^2/(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(-\frac{acf}{-acf+ade-bcfx+bdex} - \frac{adfx}{-acf+ade-bcfx+bdex} + \frac{bce}{-acf+ade-bcfx+bdex} + \frac{bdex}{-acf+ade-bcfx+bdex}\right)^2}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))**2/(b*x+a),x)
```

```
[Out] Integral(log(-a*c*f/(-a*c*f + a*d*e - b*c*f*x + b*d*e*x) - a*d*f*x/(-a*c*f
+ a*d*e - b*c*f*x + b*d*e*x) + b*c*e/(-a*c*f + a*d*e - b*c*f*x + b*d*e*x) +
 b*d*e*x/(-a*c*f + a*d*e - b*c*f*x + b*d*e*x))**2/(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a),x, algorithm
="giac")
```

```
[Out] integrate(log((a*f - b*e)*(d*x + c)/((c*f - d*e)*(b*x + a)))^2/(b*x + a), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(\frac{(a\,f-b\,e)\,(c+d\,x)}{(c\,f-d\,e)\,(a+b\,x)}\right)^2}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(((a*f - b*e)*(c + d*x))/((c*f - d*e)*(a + b*x)))^2/(a + b*x),x)
```

```
[Out] int(log(((a*f - b*e)*(c + d*x))/((c*f - d*e)*(a + b*x)))^2/(a + b*x), x)
```

3.105 $\int \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\log\left(\frac{(-bc+ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=109

$$\frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\mathrm{Li}_2\left(1+\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{bc-ad}-\frac{\mathrm{Li}_3\left(1+\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{bc-ad}$$

```
[Out] ln(e*(d*x+c)/(b*x+a))*polylog(2,1+(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/(-
a*d+b*c)-polylog(3,1+(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/(-a*d+b*c)
```

Rubi [A]
time = 0.10, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 62, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {2588, 6745}

$$\frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\mathrm{PolyLog}\left(2,\frac{(e+fx)(bc-ad)}{(a+bx)(de-cf)}+1\right)}{bc-ad}-\frac{\mathrm{PolyLog}\left(3,\frac{(e+fx)(bc-ad)}{(a+bx)(de-cf)}+1\right)}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[(e*(c + d*x))/(a + b*x)]*Log[((-(b*c) + a*d)*(e + f*x))/((d*e - c*
f)*(a + b*x))])/((a + b*x)*(c + d*x)),x]
```

```
[Out] (Log[(e*(c + d*x))/(a + b*x)]*PolyLog[2, 1 + ((b*c - a*d)*(e + f*x))/((d*e
- c*f)*(a + b*x))])/(b*c - a*d) - PolyLog[3, 1 + ((b*c - a*d)*(e + f*x))/((
d*e - c*f)*(a + b*x))]/(b*c - a*d)
```

Rule 2588

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[(v - 1)*((c + d
*x)/(a + b*x))], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[(-h)*PolyLog[2,
 1 - v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(b*c - a*d)), x] + Dist[h*p
*r*s, Int[PolyLog[2, 1 - v]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e,
 f, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\log\left(\frac{(-bc+ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)}\,dx = \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\operatorname{Li}_2\left(1+\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{bc-ad} + \int \frac{\operatorname{Li}_2\left(1-\frac{(-bc+ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)}\,d$$

$$= \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\operatorname{Li}_2\left(1+\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{bc-ad} - \frac{\operatorname{Li}_3\left(1+\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{bc-ad}$$

Mathematica [A]
time = 0.02, size = 96, normalized size = 0.88

$$\frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\operatorname{Li}_2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)-\operatorname{Li}_3\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[(e*(c + d*x))/(a + b*x)]*Log[((-(b*c) + a*d)*(e + f*x))/((d*
e - c*f)*(a + b*x))])/((a + b*x)*(c + d*x)),x]
```

```
[Out] (Log[(e*(c + d*x))/(a + b*x)]*PolyLog[2, ((b*e - a*f)*(c + d*x))/((d*e - c*
f)*(a + b*x))] - PolyLog[3, ((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))
])/(b*c - a*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 523 vs. 2(109) = 218.
time = 8.53, size = 524, normalized size = 4.81

method	result
default	$\frac{\ln\left(\frac{e(dx+c)}{bx+a}\right)^2\ln\left(-\frac{\frac{e(dx+c)af}{bx+a}-\frac{e^2(dx+c)b}{bx+a}-cef+de^2}{e(cf-ed)}\right)}{2ad-2cb}-\frac{af\ln\left(\frac{e(dx+c)}{bx+a}\right)^2\ln\left(1-\frac{(af-be)e(dx+c)}{(bx+a)\left(cef-de^2\right)}\right)}{2(ad-cb)(af-be)}-\frac{af\ln\left(\frac{e(dx+c)}{bx+a}\right)\operatorname{polylog}\left(2\right.}{(ad-cb)(a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(d*x+c)/(b*x+a))*ln((a*d-b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/(b*x+a)/
(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(a*d-b*c)*ln(e*(d*x+c)/(b*x+a))^2*ln(-(e*(d*x+c)/(b*x+a)*a*f-e^2*(d*x+c
)/(b*x+a)*b-c*e*f+d*e^2)/e/(c*f-d*e))-1/2/(a*d-b*c)*a*f/(a*f-b*e)*ln(e*(d*x
+c)/(b*x+a))^2*ln(1-(a*f-b*e)*e*(d*x+c)/(b*x+a)/(c*e*f-d*e^2))-1/(a*d-b*c)*
a*f/(a*f-b*e)*ln(e*(d*x+c)/(b*x+a))*polylog(2,(a*f-b*e)*e*(d*x+c)/(b*x+a)/(
c*e*f-d*e^2))+1/(a*d-b*c)*a*f/(a*f-b*e)*polylog(3,(a*f-b*e)*e*(d*x+c)/(b*x+
a)/(c*e*f-d*e^2))+1/2/(a*d-b*c)*b*e/(a*f-b*e)*ln(e*(d*x+c)/(b*x+a))^2*ln(1-
(a*f-b*e)*e*(d*x+c)/(b*x+a)/(c*e*f-d*e^2))+1/(a*d-b*c)*b*e/(a*f-b*e)*ln(e*(
d*x+c)/(b*x+a))*polylog(2,(a*f-b*e)*e*(d*x+c)/(b*x+a)/(c*e*f-d*e^2))-1/(a*d
-b*c)*b*e/(a*f-b*e)*polylog(3,(a*f-b*e)*e*(d*x+c)/(b*x+a)/(c*e*f-d*e^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(d*x+c)/(b*x+a))*log((a*d-b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/
(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(log(b*x + a)^2 - 2*(log(b*x + a) - 1)*log(d*x + c) + log(d*x + c)^2 -
 2*log(b*x + a))*log(f*x + e)/(b*c - a*d) + integrate(1/2*((b*d*f*x^2 + a*c
*f + (3*b*c*f - a*d*f)*x + 2*(b*c - a*d)*e)*log(b*x + a)^2 + 2*((f*log(-b*c
 + a*d) - f*log(-c*f + d*e))*b*c - (d*f*log(-b*c + a*d) - d*f*log(-c*f + d*
e))*a)*x + 2*(b*c*(log(-b*c + a*d) - log(-c*f + d*e)) - (d*log(-b*c + a*d)
- d*log(-c*f + d*e))*a)*e - 2*(b*d*f*x^2 + a*c*f + ((f*log(-b*c + a*d) - f*
log(-c*f + d*e) + 2*f)*b*c - (d*f*log(-b*c + a*d) - d*f*log(-c*f + d*e))*a)
*x + (b*c*(log(-b*c + a*d) - log(-c*f + d*e) + 1) + (d*(log(-c*f + d*e) - 1
) - d*log(-b*c + a*d))*a)*e)*log(b*x + a) + 2*(b*d*f*x^2 + a*c*f + ((f*log(
-b*c + a*d) - f*log(-c*f + d*e) + f)*b*c - (d*f*log(-b*c + a*d) - (f*log(-c
*f + d*e) + f)*d)*a)*x + (b*c*(log(-b*c + a*d) - log(-c*f + d*e)) - (d*log(
-b*c + a*d) - d*log(-c*f + d*e))*a)*e - (b*d*f*x^2 + 2*b*c*f*x + a*c*f + (b
*c - a*d)*e)*log(b*x + a))*log(d*x + c))/((b^2*c*d*f - a*b*d^2*f)*x^3 + (b^
2*c^2*f - a^2*d^2*f + (b^2*c*d - a*b*d^2)*e)*x^2 + (a*b*c^2*f - a^2*c*d*f +
 (b^2*c^2 - a^2*d^2)*e)*x + (a*b*c^2 - a^2*c*d)*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(d*x+c)/(b*x+a))*log((a*d-b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/
(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(log((d*x + c)*e/(b*x + a))*log(((b*c - a*d)*f*x + (b*c - a*d)*e)/(
b*c*f*x + a*c*f - (b*d*x + a*d)*e))/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\log\left(\frac{e(c+dx)}{a+bx}\right)^2\log\left(\frac{(e+fx)(ad-bc)}{(a+bx)(-cf+de)}\right)}{2ad-2bc}-\frac{(af-be)\int\frac{\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)^2}{ae+afx+bex+bfx^2}\,dx}{2\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(d*x+c)/(b*x+a))*ln((a*d-b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/(b
*x+a)/(d*x+c),x)
```

```
[Out] log(e*(c + d*x)/(a + b*x))**2*log((e + f*x)*(a*d - b*c)/((a + b*x)*(-c*f +
d*e)))/(2*a*d - 2*b*c) - (a*f - b*e)*Integral(log(c*e/(a + b*x) + d*e*x/(a
+ b*x))**2/(a*e + a*f*x + b*e*x + b*f*x**2), x)/(2*(a*d - b*c))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(d*x+c)/(b*x+a))*log((a*d-b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/
(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(log((d*x + c)*e/(b*x + a))*log((b*c - a*d)*(f*x + e)/((c*f - d*e)
*(b*x + a)))/((b*x + a)*(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)\,\ln\left(-\frac{\left(e+f\,x\right)\,\left(a\,d-b\,c\right)}{\left(c\,f-d\,e\right)\,\left(a+b\,x\right)}\right)}{\left(a+b\,x\right)\,\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log((e*(c + d*x))/(a + b*x))*log(-((e + f*x)*(a*d - b*c))/((c*f - d*e)
*(a + b*x))))/((a + b*x)*(c + d*x)),x)
```

```
[Out] int((log((e*(c + d*x))/(a + b*x))*log(-((e + f*x)*(a*d - b*c))/((c*f - d*e)
*(a + b*x))))/((a + b*x)*(c + d*x)), x)
```

3.106 $\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(a+bx)(e+fx)}\,dx$

Optimal. Leaf size=204

$$-\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(1-\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{be-af}-\frac{2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\text{Li}_2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{be-af}+\frac{2\text{Li}_3\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{be-af}$$

```
[Out] -ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2*ln(1-(-a*f+b*e)*(d*x+c)/(-c*f+
d*e)/(b*x+a))/(-a*f+b*e)-2*ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*polylo
g(2,(-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))/(-a*f+b*e)+2*polylog(3,(-a*f+b*e
)*(d*x+c)/(-c*f+d*e)/(b*x+a))/(-a*f+b*e)
```

Rubi [A]
time = 0.11, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.082$, Rules used = {2566, 2354, 2421, 6724}

$$\frac{2\text{PolyLog}\left(3,\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{be-af}-\frac{2\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)\text{PolyLog}\left(2,\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{be-af}-\frac{\log\left(1-\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{be-af}$$

Antiderivative was successfully verified.

```
[In] Int[Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2/((a + b*x)*(e +
f*x)),x]
```

```
[Out] -((Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*Log[1 - ((b*e - a
*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/(b*e - a*f)) - (2*Log[((b*e - a*f)
*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2, ((b*e - a*f)*(c + d*x))/((d
*e - c*f)*(a + b*x))])/(b*e - a*f) + (2*PolyLog[3, ((b*e - a*f)*(c + d*x))/
((d*e - c*f)*(a + b*x))])/(b*e - a*f)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2566

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(q + 1)*(i/d)^q, Subst[Int[(b*f - a*g - (d*f - c*
g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c +
 d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n}, x] && EqQ[n + mn,
 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, q] && IGtQ[p, 0] &&
EqQ[d*h - c*i, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(a+bx)(e+fx)}\,dx &= -\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{be-af}-\frac{(2(bc-ad))\int\frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(-\right.}{(a+bx)(c+dx)}}{be-af}\\ &= -\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{be-af}-\frac{2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\mathrm{Li}_2\left(1+\frac{(bc-ad)}{(de-cf)}\right.}{be-af}\\ &= -\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{be-af}-\frac{2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\mathrm{Li}_2\left(1+\frac{(bc-ad)}{(de-cf)}\right.}{be-af}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1636 vs. $2(204) = 408$.
time = 0.31, size = 1636, normalized size = 8.02

Antiderivative was successfully verified.

```
[In] Integrate[Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2/((a + b*x)
*(e + f*x)),x]
```

```
[Out] (-2*Log[a/b + x]^3 + 3*Log[a/b + x]^2*Log[a + b*x] - 6*Log[a/b + x]*Log[c/d
 + x]*Log[a + b*x] + 3*Log[c/d + x]^2*Log[a + b*x] + 6*Log[a/b + x]*Log[c/d
 + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] - 3*Log[c/d + x]^2*Log[(d*(a + b*x)
)/(-(b*c) + a*d)] + 3*Log[a/b + x]^2*Log[(b*(c + d*x))/(b*c - a*d)] - 3*Log
[a/b + x]^2*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))] + 6*Log[a/
```

```
b + x]*Log[a + b*x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))] -
6*Log[c/d + x]*Log[a + b*x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b
*x))] + 6*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[((b*e - a*f)*(
c + d*x))/((d*e - c*f)*(a + b*x))] + 3*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Lo
g[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2 + 3*Log[a + b*x]*Log[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2 - 3*Log[a/b + x]^2*Log[e
+ f*x] + 6*Log[a/b + x]*Log[c/d + x]*Log[e + f*x] - 3*Log[c/d + x]^2*Log[e
+ f*x] - 6*Log[a/b + x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))
]*Log[e + f*x] + 6*Log[c/d + x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a
 + b*x))]*Log[e + f*x] - 3*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*
x))]^2*Log[e + f*x] + 3*Log[a/b + x]^2*Log[(b*(e + f*x))/(b*e - a*f)] - 6*L
og[a/b + x]*Log[(f*(c + d*x))/(-(d*e) + c*f)]*Log[(b*(e + f*x))/(b*e - a*f)
] + 3*Log[(f*(c + d*x))/(-(d*e) + c*f)]^2*Log[(b*(e + f*x))/(b*e - a*f)] +
6*Log[a/b + x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(b*
(e + f*x))/(b*e - a*f)] - 6*Log[(f*(c + d*x))/(-(d*e) + c*f)]*Log[((b*e - a
*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(b*(e + f*x))/(b*e - a*f)] + 3*
Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*Log[(b*(e + f*x))/(b
*e - a*f)] - 6*Log[a/b + x]*Log[c/d + x]*Log[(d*(e + f*x))/(d*e - c*f)] + 3
*Log[c/d + x]^2*Log[(d*(e + f*x))/(d*e - c*f)] + 6*Log[a/b + x]*Log[(f*(c +
 d*x))/(-(d*e) + c*f)]*Log[(d*(e + f*x))/(d*e - c*f)] - 3*Log[(f*(c + d*x))
/(-(d*e) + c*f)]^2*Log[(d*(e + f*x))/(d*e - c*f)] - 6*Log[c/d + x]*Log[((b*
e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(d*(e + f*x))/(d*e - c*f)]
 + 6*Log[(f*(c + d*x))/(-(d*e) + c*f)]*Log[((b*e - a*f)*(c + d*x))/((d*e -
c*f)*(a + b*x))]*Log[(d*(e + f*x))/(d*e - c*f)] - 3*Log[((b*e - a*f)*(c + d
*x))/((d*e - c*f)*(a + b*x))]^2*Log[((-(b*c) + a*d)*(e + f*x))/((d*e - c*f)
*(a + b*x))] + 6*Log[a/b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 6*
(Log[a/b + x] + Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])*PolyL
og[2, (b*(c + d*x))/(b*c - a*d)] + 6*Log[((b*e - a*f)*(c + d*x))/((d*e - c*
f)*(a + b*x))]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] - 6*Log[((b*e - a*f)
*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2, ((b*e - a*f)*(c + d*x))/((d
*e - c*f)*(a + b*x))] - 6*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] - 6*Poly
Log[3, (b*(c + d*x))/(b*c - a*d)] - 6*PolyLog[3, (b*(c + d*x))/(d*(a + b*x)
)] + 6*PolyLog[3, ((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/(3*b*e
- 3*a*f)
```

Maple [A]

time = 1.04, size = 338, normalized size = 1.66

method	result
derivativedivides	$\frac{\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-ed)(bx+a)}+\frac{d(af-be)}{(cf-ed)b}\right)^{2}\ln\left(\frac{(af-be)(ad-cb)}{b(cf-ed)(bx+a)}-\frac{d(af-be)}{(cf-ed)b}+1\right)+2\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-ed)(bx+a)}+\frac{d(af-be)}{(cf-ed)b}\right)\operatorname{polylog}\left(2,-\right.}{af-be}$
default	$\frac{\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-ed)(bx+a)}+\frac{d(af-be)}{(cf-ed)b}\right)^{2}\ln\left(\frac{(af-be)(ad-cb)}{b(cf-ed)(bx+a)}-\frac{d(af-be)}{(cf-ed)b}+1\right)+2\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-ed)(bx+a)}+\frac{d(af-be)}{(cf-ed)b}\right)\operatorname{polylog}\left(2,-\right.}{af-be}$
risch	$\frac{\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-ed)(bx+a)}+\frac{d(af-be)}{(cf-ed)b}\right)^{2}\ln\left(\frac{(af-be)(ad-cb)}{b(cf-ed)(bx+a)}-\frac{d(af-be)}{(cf-ed)b}+1\right)}{af-be}+\frac{2\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-ed)(bx+a)}+\frac{d(af-be)}{(cf-ed)b}\right)\operatorname{polylog}\left(2,\right.}{af-be}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a)/(f*x+e),x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/(a*f-b*e)*(ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b
*e)/b)^2*ln((a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)-d/(c*f-d*e)*(a*f-b*e)/b
+1)+2*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*
polylog(2,-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)
-2*polylog(3,-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)
/b))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a)/(f*x+e),x, a
lgorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Memory limit reached. Please ju
mp to an outer pointer, quit program and enlarge thememory limits before ex
ecuting the program again.
```

Fricas [**A**]
time = 0.42, size = 266, normalized size = 1.30

$$\frac{\log\left(\frac{adfx+acf-(bdx+bc)e}{bcfx+acf-(bdx+ad)e}\right)^2\log\left(\frac{(bc-ad)fx+(bc-ad)e}{bcfx+acf-(bdx+ad)e}\right)+2\,\mathrm{Li}_2\left(-\frac{(bc-ad)fx+(bc-ad)e}{bcfx+acf-(bdx+ad)e}+1\right)\log\left(\frac{adfx+acf-(bdx+bc)e}{bcfx+acf-(bdx+ad)e}\right)-2\,\mathrm{polylog}\left(3,\frac{adfx+acf-(bdx+bc)e}{bcfx+acf-(bdx+ad)e}\right)}{af-be}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a)/(f*x+e),x, a
lgorithm="fricas")
```

```
[Out] (log((a*d*f*x + a*c*f - (b*d*x + b*c)*e)/(b*c*f*x + a*c*f - (b*d*x + a*d)*e
))^2*log(((b*c - a*d)*f*x + (b*c - a*d)*e)/(b*c*f*x + a*c*f - (b*d*x + a*d)
*e)) + 2*dilog(-((b*c - a*d)*f*x + (b*c - a*d)*e)/(b*c*f*x + a*c*f - (b*d*x
 + a*d)*e) + 1)*log((a*d*f*x + a*c*f - (b*d*x + b*c)*e)/(b*c*f*x + a*c*f -
(b*d*x + a*d)*e)) - 2*polylog(3, (a*d*f*x + a*c*f - (b*d*x + b*c)*e)/(b*c*f
*x + a*c*f - (b*d*x + a*d)*e)))/(a*f - b*e)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(-\frac{acf}{-acf+ade-bcfx+bdex}-\frac{adfx}{-acf+ade-bcfx+bdex}+\frac{bce}{-acf+ade-bcfx+bdex}+\frac{bdex}{-acf+ade-bcfx+bdex}\right)^2}{(a+bx)\,(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))**2/(b*x+a)/(f*x+e),x)
```

```
[Out] Integral(log(-a*c*f/(-a*c*f + a*d*e - b*c*f*x + b*d*e*x) - a*d*f*x/(-a*c*f
+ a*d*e - b*c*f*x + b*d*e*x) + b*c*e/(-a*c*f + a*d*e - b*c*f*x + b*d*e*x) +
 b*d*e*x/(-a*c*f + a*d*e - b*c*f*x + b*d*e*x))**2/((a + b*x)*(e + f*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a)/(f*x+e),x, a
lgorithm="giac")
```

```
[Out] integrate(log((a*f - b*e)*(d*x + c)/((c*f - d*e)*(b*x + a)))^2/((b*x + a)*(
f*x + e)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(\frac{(a\,f-b\,e)\,(c+d\,x)}{(c\,f-d\,e)\,(a+b\,x)}\right)^2}{(e+f\,x)\,(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(((a*f - b*e)*(c + d*x))/((c*f - d*e)*(a + b*x)))^2/((e + f*x)*(a +
b*x)),x)
```

```
[Out] int(log(((a*f - b*e)*(c + d*x))/((c*f - d*e)*(a + b*x)))^2/((e + f*x)*(a +
b*x)), x)
```

3.107 $\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{e+fx}\,dx$

Optimal. Leaf size=322

$$-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f}+\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(1-\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f}-\frac{2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\mathrm{Li}_2\left(\frac{b(}{d(}\right.}{f}$$

```
[Out] -ln((a*d-b*c)/d/(b*x+a))*ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/f+ln((
-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2*ln(1-(-a*f+b*e)*(d*x+c)/(-c*f+d*e)/
(b*x+a))/f-2*ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*polylog(2,b*(d*x+c)/
d/(b*x+a))/f+2*ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*polylog(2,(-a*f+b*
e)*(d*x+c)/(-c*f+d*e)/(b*x+a))/f+2*polylog(3,b*(d*x+c)/d/(b*x+a))/f-2*polyl
og(3,(-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))/f
```

Rubi [A]
time = 0.31, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$, Rules used = {2554, 2404, 2354, 2421, 6724}

$$-\frac{2\text{PolyLog}\left(3,\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{f}-\frac{2\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{f}+\frac{2\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)\text{PolyLog}\left(2,\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{f}+\frac{2\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{f}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{f}+\frac{\log\left(1-\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2/(e + f*x),x]
```

```
[Out] -((Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*
f)*(a + b*x))]^2)/f) + (Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))
]^2*Log[1 - ((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/f - (2*Log[((
b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2, (b*(c + d*x))/(d*
(a + b*x))])/f + (2*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Po
lyLog[2, ((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/f + (2*PolyLog[3
, (b*(c + d*x))/(d*(a + b*x))])/f - (2*PolyLog[3, ((b*e - a*f)*(c + d*x))/(
(d*e - c*f)*(a + b*x))])/f
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
```

```
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{e+fx}\,dx = \frac{d\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{c+dx}\,dx}{f} - \frac{(de-cf)\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(c+dx)(e+fx)}\,dx}{f}$$

$$= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f} + \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}\right)}{f} - \cdots$$

$$= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f} + \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}\right)}{f} + \cdots$$

$$= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f} + \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}\right)}{f} + \cdots$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1080 vs. $2(322) = 644$.
time = 0.15, size = 1080, normalized size = 3.35

Antiderivative was successfully verified.

```
[In] Integrate[Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2/(e + f*x),
x]
```

```
[Out] (-(Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*
f)*(a + b*x))]^2) + Log[a/b + x]^2*Log[e + f*x] - 2*Log[a/b + x]*Log[c/d +
x]*Log[e + f*x] + Log[c/d + x]^2*Log[e + f*x] + 2*Log[a/b + x]*Log[((b*e -
a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[e + f*x] - 2*Log[c/d + x]*Log[
((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[e + f*x] + Log[((b*e -
 a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*Log[e + f*x] - Log[a/b + x]^2*L
og[(b*(e + f*x))/(b*e - a*f)] + 2*Log[a/b + x]*Log[(f*(c + d*x))/(-(d*e) +
c*f)]*Log[(b*(e + f*x))/(b*e - a*f)] - Log[(f*(c + d*x))/(-(d*e) + c*f)]^2*
Log[(b*(e + f*x))/(b*e - a*f)] - 2*Log[a/b + x]*Log[((b*e - a*f)*(c + d*x))
/((d*e - c*f)*(a + b*x))]*Log[(b*(e + f*x))/(b*e - a*f)] + 2*Log[(f*(c + d*
x))/(-(d*e) + c*f)]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Lo
g[(b*(e + f*x))/(b*e - a*f)] - Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a
+ b*x))]^2*Log[(b*(e + f*x))/(b*e - a*f)] + 2*Log[a/b + x]*Log[c/d + x]*Log
[(d*(e + f*x))/(d*e - c*f)] - Log[c/d + x]^2*Log[(d*(e + f*x))/(d*e - c*f)]
 - 2*Log[a/b + x]*Log[(f*(c + d*x))/(-(d*e) + c*f)]*Log[(d*(e + f*x))/(d*e
- c*f)] + Log[(f*(c + d*x))/(-(d*e) + c*f)]^2*Log[(d*(e + f*x))/(d*e - c*f)
] + 2*Log[c/d + x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log
[(d*(e + f*x))/(d*e - c*f)] - 2*Log[(f*(c + d*x))/(-(d*e) + c*f)]*Log[((b*e
 - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(d*(e + f*x))/(d*e - c*f)]
+ Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*Log[((-(b*c) + a*d
)*(e + f*x))/((d*e - c*f)*(a + b*x))] - 2*Log[((b*e - a*f)*(c + d*x))/((d*e
 - c*f)*(a + b*x))]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + 2*Log[((b*e -
 a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2, ((b*e - a*f)*(c + d*x)
)/((d*e - c*f)*(a + b*x))] + 2*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))] - 2*
PolyLog[3, ((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/f
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 812 vs. $2(321) = 642$.
time = 0.93, size = 813, normalized size = 2.52

method	result
derivativedivides	$(af-be)(ad-cb)\left(\frac{\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-ed)(bx+a)}+\frac{d(af-be)}{(cf-ed)b}\right)^2 \ln\left(\frac{(af-be)(ad-cb)}{b(cf-ed)(bx+a)}-\frac{d(af-be)}{(cf-ed)b}+1\right)+2\ln\left(-\frac{(af-be)(ad-cb}{b(cf-ed)(bx+a}\right.}{(af-}\right.$
default	$(af-be)(ad-cb)\left(\frac{\ln\left(-\frac{(af-be)(ad-cb)}{b(cf-ed)(bx+a)}+\frac{d(af-be)}{(cf-ed)b}\right)^2 \ln\left(\frac{(af-be)(ad-cb)}{b(cf-ed)(bx+a)}-\frac{d(af-be)}{(cf-ed)b}+1\right)+2\ln\left(-\frac{(af-be)(ad-cb}{b(cf-ed)(bx+a}\right.}{(af-}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(f*x+e),x,method=_RETURNVER
BOSE)
```

```
[Out] (a*f-b*e)*(a*d-b*c)*(1/(a*f-b*e)/f/(a*d-b*c)*(ln(-(a*f-b*e)*(a*d-b*c)/b/(c*
f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)^2*ln((a*f-b*e)*(a*d-b*c)/b/(c*f-d*e
)/(b*x+a)-d/(c*f-d*e)*(a*f-b*e)/b+1)+2*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/
(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*polylog(2,-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)
/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)-2*polylog(3,-(a*f-b*e)*(a*d-b*c)/b/(c*f-d
*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))-b*(c*f-d*e)/(b*c*f-b*d*e)/(a*f-b*e)/f
/(a*d-b*c)*(ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*
e)/b)^2*ln(1+(b*c*f-b*d*e)/(-a*d*f+b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)
/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))+2*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b
*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*polylog(2,-(b*c*f-b*d*e)/(-a*d*f+b*d*e)*(-(a
*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))-2*polylog(3
,-(b*c*f-b*d*e)/(-a*d*f+b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/
(c*f-d*e)*(a*f-b*e)/b))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(f*x+e),x, algorithm
="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Memory limit reached. Please ju
mp to an outer pointer, quit program and enlarge thememory limits before ex
ecuting the program again.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(f*x+e),x, algorithm
="fricas")
```

```
[Out] integral(log((a*d*f*x + a*c*f - (b*d*x + b*c)*e)/(b*c*f*x + a*c*f - (b*d*x
 + a*d)*e))^2/(f*x + e), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))**2/(f*x+e),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(f*x+e),x, algorithm
="giac")
```

```
[Out] integrate(log((a*f - b*e)*(d*x + c)/((c*f - d*e)*(b*x + a)))^2/(f*x + e), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(\frac{(a\,f-b\,e)\,(c+d\,x)}{(c\,f-d\,e)\,(a+b\,x)}\right)^2}{e+f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(((a*f - b*e)*(c + d*x))/((c*f - d*e)*(a + b*x)))^2/(e + f*x),x)
```

```
[Out] int(log(((a*f - b*e)*(c + d*x))/((c*f - d*e)*(a + b*x)))^2/(e + f*x), x)
```

3.108 $\int \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=433

$$-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{2(bc-ad)}-\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{2(bc-ad)}+\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(1-\frac{(be-af}{(de-cf}\right.}{2(bc-ad)}$$

```
[Out] -1/2*ln((a*d-b*c)/d/(b*x+a))*ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(-
a*d+b*c)-1/2*ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2*ln(b*(f*x+e)/(-a*f
+b*e))/(-a*d+b*c)+1/2*ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2*ln(1-(-a*
f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))/(-a*d+b*c)-ln((-a*f+b*e)*(d*x+c)/(-c*f+d
*e)/(b*x+a))*polylog(2,b*(d*x+c)/d/(b*x+a))/(-a*d+b*c)+ln((-a*f+b*e)*(d*x+c
)/(-c*f+d*e)/(b*x+a))*polylog(2,(-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))/(-a*
d+b*c)+polylog(3,b*(d*x+c)/d/(b*x+a))/(-a*d+b*c)-polylog(3,(-a*f+b*e)*(d*x+
c)/(-c*f+d*e)/(b*x+a))/(-a*d+b*c)
```

Rubi [A]
time = 0.36, antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 65, $\frac{\text{number of rules}}{\text{integrand size}} = 0.092$, Rules used = {2589, 2554, 2404, 2354, 2421, 6724}

$$-\frac{\text{PolyLog}\left(3,\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{bc-ad}-\frac{\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{bc-ad}+\frac{\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)\text{PolyLog}\left(2,\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{bc-ad}+\frac{\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{bc-ad}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{2(bc-ad)}-\frac{\log\left(\frac{b(e+fx)}{be-af}\right)\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{2(bc-ad)}+\frac{\log\left(1-\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(b*(e + f*x))
/(b*e - a*f)])/((a + b*x)*(c + d*x)),x]
```

```
[Out] -1/2*(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[((b*e - a*f)*(c + d*x))/((d*e -
 c*f)*(a + b*x))]^2)/(b*c - a*d) - (Log[((b*e - a*f)*(c + d*x))/((d*e - c*f
)*(a + b*x))]^2*Log[(b*(e + f*x))/(b*e - a*f)])/(2*(b*c - a*d)) + (Log[((b*
e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*Log[1 - ((b*e - a*f)*(c + d*
x))/((d*e - c*f)*(a + b*x))])/(2*(b*c - a*d)) - (Log[((b*e - a*f)*(c + d*x)
)/((d*e - c*f)*(a + b*x))]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(b*c -
a*d) + (Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2, ((b
*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/(b*c - a*d) + PolyLog[3, (b*
(c + d*x))/(d*(a + b*x))]/(b*c - a*d) - PolyLog[3, ((b*e - a*f)*(c + d*x))/
((d*e - c*f)*(a + b*x))]/(b*c - a*d)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rule 2589

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*Log[(i_.)*((j_.)*((g_.) + (h_.)*(x_))^(t_.))^(u_.)]*(v_), x_S
ymbol] :> With[{k = Simplify[v*(a + b*x)*(c + d*x)]}, Simp[k*Log[i*(j*(g +
h*x)^t)^u]*(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s + 1)/(p*r*(s + 1)*(b*c
- a*d))), x] - Dist[k*h*t*(u/(p*r*(s + 1)*(b*c - a*d))), Int[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^(s + 1)/(g + h*x), x], x] /; FreeQ[k, x]] /; FreeQ[{
a, b, c, d, e, f, g, h, i, j, p, q, r, s, t, u}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[p + q, 0] && NeQ[s, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{(a+bx)(c+dx)}\,dx = -\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{2(bc-ad)} + \frac{f\int\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{e+fx}dx}{2(bc-ad)}$$

$$= -\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{2(bc-ad)} + \frac{d\int\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{c+dx}dx}{2(bc-ad)} - \frac{(de \cdots}{}$$

$$= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{2(bc-ad)} - \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+f \cdots}{be-a \cdots}\right.}{2(bc-ad)}$$

$$= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{2(bc-ad)} - \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+f \cdots}{be-a \cdots}\right.}{2(bc-ad)}$$

$$= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{2(bc-ad)} - \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+f \cdots}{be-a \cdots}\right.}{2(bc-ad)}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 908 vs. $2(433) = 866$.
time $= 0.51$, size $= 908$, normalized size $= 2.10$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(b*(e +
 f*x))/(b*e - a*f)])/((a + b*x)*(c + d*x)),x]
```

```
[Out] (-(Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*
f)*(a + b*x))]^2) - 2*Log[a + b*x]*Log[c + d*x]*Log[(b*(e + f*x))/(b*e - a*
f)] + Log[c + d*x]^2*Log[(b*(e + f*x))/(b*e - a*f)] + 2*Log[a + b*x]*Log[(f
*(c + d*x))/(-(d*e) + c*f)]*Log[(b*(e + f*x))/(b*e - a*f)] - Log[(f*(c + d*
x))/(-(d*e) + c*f)]^2*Log[(b*(e + f*x))/(b*e - a*f)] - 2*Log[c + d*x]*Log[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(b*(e + f*x))/(b*e - a*
f)] + 2*Log[(f*(c + d*x))/(-(d*e) + c*f)]*Log[((b*e - a*f)*(c + d*x))/((d*e
 - c*f)*(a + b*x))]*Log[(b*(e + f*x))/(b*e - a*f)] - Log[((b*e - a*f)*(c +
d*x))/((d*e - c*f)*(a + b*x))]^2*Log[(b*(e + f*x))/(b*e - a*f)] + 2*Log[a +
 b*x]*Log[c + d*x]*Log[(d*(e + f*x))/(d*e - c*f)] - Log[c + d*x]^2*Log[(d*(
e + f*x))/(d*e - c*f)] - 2*Log[a + b*x]*Log[(f*(c + d*x))/(-(d*e) + c*f)]*L
og[(d*(e + f*x))/(d*e - c*f)] + Log[(f*(c + d*x))/(-(d*e) + c*f)]^2*Log[(d*
(e + f*x))/(d*e - c*f)] + 2*Log[c + d*x]*Log[((b*e - a*f)*(c + d*x))/((d*e
 - c*f)*(a + b*x))]*Log[(d*(e + f*x))/(d*e - c*f)] - 2*Log[(f*(c + d*x))/(-(
d*e) + c*f)]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(d*(e
 + f*x))/(d*e - c*f)] + Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))
```

```
]^2*Log[((-(b*c) + a*d)*(e + f*x))/((d*e - c*f)*(a + b*x))] - 2*Log[((b*e -
 a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2, (b*(c + d*x))/(d*(a +
b*x))] + 2*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2,
((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))] + 2*PolyLog[3, (b*(c + d*x
))/(d*(a + b*x))] - 2*PolyLog[3, ((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a +
b*x))])/(2*b*c - 2*a*d)
```

Maple [F]
time = 1.15, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(\frac{(-af+be)(dx+c)}{(-cf+ed)(bx+a)}\right)\ln\left(\frac{b(fx+e)}{-af+be}\right)}{(bx+a)\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*ln(b*(f*x+e)/(-a*f+b*e))/(b*x
+a)/(d*x+c),x)
```

```
[Out] int(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*ln(b*(f*x+e)/(-a*f+b*e))/(b*x
+a)/(d*x+c),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*log(b*(f*x+e)/(-a*f+b*
e))/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Memory limit reached. Please ju
mp to an outer pointer, quit program and enlarge thememory limits before ex
ecuting the program again.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*log(b*(f*x+e)/(-a*f+b*
e))/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(log((a*d*f*x + a*c*f - (b*d*x + b*c)*e)/(b*c*f*x + a*c*f - (b*d*x
 + a*d)*e))*log(-(b*f*x + b*e)/(a*f - b*e))/(b*d*x^2 + a*c + (b*c + a*d)*x),
 x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*ln(b*(f*x+e)/(-a*f+b*e)
)/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*log(b*(f*x+e)/(-a*f+b*
e))/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(log(-(f*x + e)*b/(a*f - b*e))*log((a*f - b*e)*(d*x + c)/((c*f - d
*e)*(b*x + a)))/((b*x + a)*(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(-\frac{b\,(e+f\,x)}{a\,f-b\,e}\right)\,\ln\left(\frac{(a\,f-b\,e)\,(c+d\,x)}{(c\,f-d\,e)\,(a+b\,x)}\right)}{(a+b\,x)\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(-(b*(e + f*x))/(a*f - b*e))*log(((a*f - b*e)*(c + d*x))/((c*f - d*
e)*(a + b*x))))/((a + b*x)*(c + d*x)),x)
```

```
[Out] int((log(-(b*(e + f*x))/(a*f - b*e))*log(((a*f - b*e)*(c + d*x))/((c*f - d*
e)*(a + b*x))))/((a + b*x)*(c + d*x)), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                    convert(ExpnType_result,string)," vs. order ",
                    convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
        ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022 Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
  elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
  elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
  elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
  elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
  elif str(expn).find("Integral") != -1: #this will never happen, since it
          #is checked before calling the grading function that is passed.
          #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
def grade_antiderivative(result,optimal):


  if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

  leaf_count_result  = tree_size(result) #leaf_count(result)
  leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

  #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


  expnType_result  = expnType(result)
  expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```